Online appendix for "Endogenous Plucking Through Networks: The Plucking Paradox"

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A Derivations for the static equilibrium

This appendix derives the static equilibrium conditions reported in Section 2. We assume perfect competition in the simple sector and monopolistic competition for varieties sold to the retailer. All firms are symmetric within each sector.

Marginal costs. The network firm minimizes the cost of the Cobb–Douglas aggregator in (1). Let λ^N denote unit cost for a network firm and λ^S the unit cost for a simple firm. With competitive input prices and symmetry,

$$\ln \lambda^N = \alpha \ln \lambda^{N,\text{agg}} + (1-\alpha) \ln \lambda^{S,\text{agg}} - \ln A, \tag{1}$$

$$\ln \lambda^S = \ln w - \ln A. \tag{2}$$

Prices. Equating $\lambda^S, \lambda^N, \alpha$ in (2) with $\lambda^{S,\text{agg}}, \lambda^{N,\text{agg}}, \alpha^{\text{agg}}$ since the model features representative firms, we can solve for the sectoral prices as functions of A and w

$$\ln \lambda^{N,\text{agg}} = \ln w - \frac{2-\alpha^{\text{agg}}}{1-\alpha^{\text{agg}}} \ln A, \tag{3}$$

$$\ln \lambda^{S,\text{agg}} = \ln w - \ln A. \tag{4}$$

Retailer's cost minimization. The retailer faces constant-elasticity demand within each block, so the sectoral price indices satisfy

$$P = (p^{N,\text{agg}})^{\zeta} (p^{S,\text{agg}})^{1-\zeta} = \left(\frac{\sigma}{\sigma-1}\right)^{\zeta} (\lambda^{N,\text{agg}})^{\zeta} (\lambda^{S,\text{agg}})^{1-\zeta}. \tag{5}$$

Normalizing $P \equiv 1$ and substituting (1)–(2) yields

$$\ln w = \left(\frac{\zeta}{1-\alpha^{\text{agg}}} + 1\right) \ln A - \zeta \ln \frac{\sigma}{\sigma-1}, \tag{6}$$

which is Equation (9) in the text.

Accounting and profits. Let TC denote total cost of network intermediates used by network firms. A fraction αTC is network goods and $(1-\alpha)TC$ is simple goods. Total cost of simple goods is wL, so the retailer spends $wL - (1-\alpha)TC$ on simple goods and, by CES expenditure shares, $\frac{\zeta}{1-\zeta}(wL - (1-\alpha)TC)$ on network goods. Converting retailer spending on network goods into costs using the sectoral markup implies

$$\alpha TC + \frac{\zeta}{1-\zeta}\frac{\sigma-1}{\sigma}\big(wL - (1-\alpha)TC\big) = TC, \tag{7}$$

so

$$TC = \frac{\zeta(\sigma-1)}{(1-\alpha)(\sigma-\zeta)}wL. \tag{8}$$

Retailer spending on network goods is then $\frac{\sigma\zeta}{\sigma-\zeta}wL$, and network profits equal revenue minus cost,

$$\pi^N = \frac{\zeta}{\sigma-\zeta}wL, \tag{9}$$

while perfect competition in the simple sector implies

$$\pi^S = 0. \tag{10}$$

Finally, total consumption equals total revenue,

$$C = \frac{\sigma}{\sigma-\zeta}wL = wL + \pi^N, \tag{11}$$

which matches the aggregate consumption defined in (8).

Firm-specific profits. Let $X = [\alpha^{\text{agg}}, A]$ and consider a firm that chooses its own network intensity α while taking aggregate prices as given. Using (1) with aggregate prices (3)–(4) and (6), the firm's unit cost satisfies

$$\ln \lambda^N(\alpha; X) = \left(\frac{\zeta - \alpha}{1 - \alpha^{\text{agg}}} - 1 \right) \ln A - \zeta \ln \frac{\sigma}{\sigma - 1}, \tag{12}$$

The aggregate $\lambda^{N,\text{agg}}(X)$ satisfies

$$\ln \lambda^{N,\text{agg}}(X) = \left(\frac{\zeta - 1}{1 - \alpha^{\text{agg}}} \right) \ln A - \zeta \ln \frac{\sigma}{\sigma - 1}. \tag{13}$$

Under CES demand, revenue is proportional to $(p(\alpha; X)/p^{N,\text{agg}}(X))^{1-\sigma}$, implying

$$\pi^N(\alpha; X) = \pi^{N,\text{agg}}(X) \left(\frac{p(\alpha; X)}{p^{N,\text{agg}}(X)} \right)^{1-\sigma} \tag{14}$$

$$= \pi^{N,\text{agg}}(X) \left(\frac{\lambda^N(\alpha; X)}{\lambda^{N,\text{agg}}(X)} \right)^{1-\sigma} \tag{15}$$

$$= \frac{\zeta}{\sigma - \zeta} \left(\frac{\sigma - 1}{\sigma} \right)^{\zeta} A^{\frac{\zeta + (1-(1-\delta)\alpha)(1-\sigma)}{1-(1-\delta)\alpha^{\text{agg}}} + \sigma} L, \tag{16}$$

which is consistent with the main text.

B Derivations for the optimal fiscal policy

This appendix derives the tax rule that decentralizes the planner allocation by equating the competitive-equilibrium (CE) and social-planner (SPP) Euler equations.

Competitive Equilibrium Euler Equation. Let $\tilde{c}(S)$ denote household consumption in state S. The firm Euler in CE with a proportional tax/subsidy τ on network profits is

$$\mathbb{E}\left[\beta \frac{(\tilde{c}'(S'))^{-\rho}}{(\tilde{c}(S))^{-\rho}} \left(\pi_1(\alpha'; S')(1 + \tau') - \Phi_1(\alpha', \alpha'') \right) \middle| S \right] = \Phi_2(\alpha, \alpha') - \lambda_{CE} + \mu_{CE}, \tag{17}$$

where, using the static equilibrium,

$$w(S) = \left(\frac{\sigma-1}{\sigma}\right)^{\zeta} A^{\frac{\zeta}{1-(1-\delta)\alpha}+1}, \tag{18}$$

$$n(S) = \left(\frac{w(S)}{\eta}\right)^{\chi} = \eta^{-\chi}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta\chi} A^{\left(\frac{\zeta}{1-(1-\delta)\alpha}+1\right)\chi}, \tag{19}$$

$$\pi(\alpha;S) = \frac{\zeta}{\sigma-\zeta}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta} A^{\frac{\zeta}{1-(1-\delta)\alpha}+1} n(S). \tag{20}$$

Thus

$$\pi_1(\alpha;S) = \pi(\alpha;S)\,\log(A)\,\frac{(\sigma-1)(1-\delta)}{1-(1-\delta)\alpha}. \tag{21}$$

$$= \frac{\zeta}{\sigma-\zeta}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta} A^{\frac{\zeta}{1-(1-\delta)\alpha}+1}\eta^{-\chi}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta\chi} A^{\left(\frac{\zeta}{1-(1-\delta)\alpha}+1\right)\chi}\log(A)\,\frac{(\sigma-1)(1-\delta)}{1-(1-\delta)\alpha}, \tag{22}$$

$$= \frac{\zeta}{\sigma-\zeta}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta(1+\chi)} \eta^{-\chi} A^{\left(\frac{\zeta}{1-(1-\delta)\alpha}+1\right)(1+\chi)}\log(A)\,\frac{(\sigma-1)(1-\delta)}{1-(1-\delta)\alpha}, \tag{23}$$

Planner's Euler Equation. The SPP Euler equation is

$$\begin{aligned}
&\beta\mathbb{E}\Bigg[(\tilde{c}')^{-\rho}\Bigg(\left(\frac{\sigma}{\sigma-\zeta}\right)^{1+\chi}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta(1+\chi)}\eta^{-\chi}(A')^{\left(\frac{\zeta}{1-(1-\delta)\alpha'}+1\right)(1+\chi)}\log(A')\,\frac{\zeta(1-\delta)}{(1-(1-\delta)\alpha')^2} \\
&-\Phi_1(\alpha',\alpha'')\Bigg)\Bigg] \\
&= (\tilde{c})^{-\rho}\Phi_2(\alpha,\alpha') - \lambda_{SP} + \mu_{SP}.
\end{aligned} \tag{24}$$

Tax rule. Equating (17) and (24) (and matching multipliers via $(\tilde{c})^{\rho}\lambda_{SP} = \lambda_{CE}$ and $(\tilde{c})^{\rho}\mu_{CE} = \mu_{SP}$) yields

$$1+\tau' = \frac{\sigma-\zeta}{\sigma-1}\left(\frac{\sigma}{\sigma-\zeta}\right)^{1+\chi}\frac{1}{1-(1-\delta)\alpha'} \tag{25}$$

Budget balance. The subsidy is rebated lump-sum to households. Using $c = wn + \pi(1+\tau) - \Phi(\alpha, \alpha') - \tau\pi$, the resource constraint reduces to

$$c = wn + \pi - \Phi(\alpha, \alpha'), \tag{26}$$

so the tax leaves aggregate resources unchanged.

C Additional empirical results

In this section, we present additional empirical results that further support the model's mechanism and its implications for sectoral employment dynamics during recessions. Table C.1 reports the results from the regression of the depth of employment contraction on material share (with energy cost included), controlling for various industry characteristics. The coefficient on material share remains positive and statistically significant across all specifications, reinforcing the finding that industries with higher material intensity experience deeper contractions during recessions. Table C.2 presents the results from the trough-level regressions, showing that industries with higher material share (with energy cost included) not only experience deeper contractions but also reach lower levels of employment and total payroll at the trough. This further corroborates the model's mechanism, as a more network-intensive production structure exacerbates the depth of downturns, leading to more pronounced declines in key economic indicators during recessions.

Table C.1: Material share (including energy) and peak-to-trough drop [Data]

Drop^Y, Y=	(1) emp	(2) pay	(3) emp	(4) pay	(5) emp	(6) pay
Material Cost Share	0.132	0.140			0.125	0.123
	(0.039)	(0.042)			(0.041)	(0.043)
Booming Duration			0.014	0.011	0.014	0.011
			(0.002)	(0.002)	(0.002)	(0.002)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Peak year FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.349	0.352	0.363	0.365	0.365	0.367
N	5882	5882	5518	5518	5518	5518

Notes: Dependent variable is the peak-to-trough drop in employment, payroll, value added. Material share is measured at the peak year (pre-determined). Industry and peak-year fixed effects are included. Capital stock and the outcome variable at the peak level is included as control; standard errors are clustered by industry. Standard errors in parentheses.

Table C.2: Trough level regression (including energy cost in material cost) [Data]

Drop^Y, Y=	(1) emp	(2) pay	(3) emp	(4) pay	(5) emp	(6) pay
Material Cost Share	-0.131	-0.141			-0.122	-0.123
	(0.039)	(0.043)			(0.041)	(0.044)
Booming Duration			-0.014	-0.011	-0.014	-0.011
			(0.002)	(0.002)	(0.002)	(0.002)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Peak year FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.989	0.992	0.989	0.991	0.989	0.991
N	5882	5882	5518	5518	5518	5518

Notes: Dependent variable is the drop in employment, payroll. Material share is measured at the peak year (pre-determined). Industry and peak-year fixed effects are included. Capital stock at the peak level is included as control; standard errors are clustered by industry. Standard errors in parentheses.

D Canonical RBC model for benchmark

This appendix describes the canonical RBC model used to construct the benchmark conditional saddle paths in Figure 5(a). A representative household solves

$$v(a; K, A) = \max_{c,a'} \quad \log(c) + \beta \mathbb{E}\left[v(a'; K', A') \mid A\right] \tag{27}$$

$$\text{s.t.} \quad c + a' = a(1 + r(K, A)) + w(K, A) \tag{28}$$

$$a' \geq 0, \tag{29}$$

where v is the value function, a is wealth at the beginning of the period, K is aggregate capital, and A is aggregate TFP. The aggregate state evolves according to

$$K' = \Gamma_{\text{endo}}(K, A), \qquad A' \sim \Gamma_{\text{exo}}(\cdot \mid A), \tag{30}$$

where Γ_{endo} is the equilibrium law of motion for capital. TFP follows a two-state Markov chain between $A \in \{B, G\}$ with $G > B$ and transition matrix

$$\Gamma_{\text{exo}} = \begin{bmatrix} \pi_{BB} & \pi_{BG} \\ \pi_{GB} & \pi_{GG} \end{bmatrix}. \tag{31}$$

Production is Cobb-Douglas with constant returns to scale. Competitive factor prices are

$$r(K, A) = \alpha A K^{\alpha - 1} - \delta, \tag{32}$$

$$w(K, A) = (1 - \alpha) A K^{\alpha}, \tag{33}$$

where δ is the depreciation rate. In equilibrium, the representative household's asset holdings equal the aggregate capital stock: $a = K$.