Striking While the Iron Is Cold:
Fragility after a Surge of Lumpy Investments[†]

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Abstract

This paper studies how large firms' synchronized lumpy investments endogenously shape an economy's fragility to negative TFP shocks. I develop a heterogeneous-firm real business cycle model that matches the empirical interest rate elasticities of investment for both large and small firms. In the model, large firms' lumpy investments become persistently synchronized due to their low sensitivity to general equilibrium effects, generating investment surges. Following these surges, TFP-induced recessions are particularly severe, and the semi-elasticity of aggregate investment drops significantly, consistent with the data.

Keywords: Business cycle, state dependence, lumpy investment, interest rate elasticity.

JEL codes: E32, E22, D25.

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1 Introduction

This paper studies an economy's *endogenous fragility* to a negative TFP shock shaped by large firms' investment dynamics. Large firms' investments are lumpy, large in scale, and inelastic to changes in the interest rate. These distinctive features generate an endogenous state dependence in the economy through their synchronized large-scale investments, which has been studied only scantily in the literature. Using a calibrated heterogeneous-firm business cycle model, I substantiate that this channel leads to a significant variation in the aggregate allocations' sensitivity to the aggregate TFP shocks.

The contribution of this paper is threefold. First, this paper documents and quantifies novel state dependent investment dynamics driven by large firms. Using firm-level and macro data, I demonstrate that aggregate investment sensitivity to negative TFP shocks significantly increases when a great portion of large firms have recently completed lumpy investments. I identify the low interest rate elasticity of large firms as the key mechanism driving this observed state dependence. Quantitative analysis reveals that approximately 23% of investment rate declines during recent recessions can be attributed to fluctuations in large firms' synchronized lumpy investments. A one-standard-deviation increase in past synchronization reduces the aggregate investment semi-elasticity to interest rate changes by around 3.4% compared to steady-state levels. Furthermore, I show that such synchronization emerges endogenously within the recursive competitive equilibrium (RCE) path, enabling analysis of state dependence without requiring multiple steady states exogenously.

Second, this paper develops a fragility index that measures the economy's vulnerability to negative TFP shocks through their effects on aggregate investment and output. The index is calculated from the proportion of large firms that have recently completed lumpy investments, effectively capturing the degree of recent synchronization. This index has three key advantages: 1) it relies solely on data from large firms, 2) depends only on past observations, and 3) possesses predictive power for contemporaneous investment responsiveness. These features make the index particularly valuable for practical applications, as large firms' investment activities are easily traceable through public financial disclosures, eliminating the need to monitor the entire universe of firms.

Third, this paper develops a heterogeneous-firm model that correctly captures the empirically observed interest rate elasticity patterns across firm sizes. Standard models with fixed and convex adjustment costs fail in a crucial dimension: when calibrated to match aggregate moments, they counterfactually predict that large firms are more responsive to interest rates than small firms — the opposite of what we observe in the data. This inversion occurs because standard fixed adjustment costs are too small relative to large firms' scale. I address this by introducing size-dependent fixed costs with a curvature parameter ζ that governs how the adjustment costs increase with firm size, microfounded as coordination costs across establishments within multi-establishment firms. When calibrated to match the cross-sectional elasticity evidence ($\zeta = 3.5$), the model correctly generates interest-inelastic large firms and interest-elastic small firms. This correct specification produces substantially greater nonlinearity and state dependence in aggregate investment dynamics compared to existing models.

A key equilibrium prediction of my model is the synchronization of large firms' investment timings across the business cycle. When negative aggregate TFP shocks occur, firms tend to simultaneously pause lumpy investment projects as persistent poor economic prospects make these investments unattractive. This simultaneous pause leads to synchronized implementation in subsequent periods — analogous to pedestrians stopping together at a red light and proceeding in unison when it turns green.[1] In standard models with strong general equilibrium smoothing forces, as in Khan and Thomas (2003, 2008), such synchronization tends to be dissipated by the general equilibrium effect, which effectively disperses investment timings. However, in my baseline model, the low interest rate elasticity of large firms weakens this smoothing mechanism, allowing surges of synchronized lumpy investments even in general equilibrium.

This endogenous synchronization drives state-dependent investment sensitivity. During

[1] While positive shocks can also cause synchronization, they are less effective since accelerating investments requires paying fixed adjustment costs, whereas pausing is costless.

normal times when firms are dispersed across their Ss cycles, negative TFP shocks still trigger some investment: large firms near their lumpy investment thresholds (Ss triggers) proceed with planned adjustments since expected benefits exceed costs despite poor macroeconomic conditions. This continued investment partially buffers the negative shock's impact through capital accumulation. However, following a surge of lumpy investments, most large firms have recently completed their adjustments and thus remain far from their Ss triggers (early in their Ss cycles). With fewer firms positioned near investment thresholds, the same negative TFP shock is less buffered by offsetting investments, resulting in sharper aggregate investment declines and deeper recessions. This variation in the distribution of firms across their Ss cycles — not just their average position — constitutes the primary mechanism behind state-dependent investment responsiveness.

To solve the model, I employ a sequence-space-based nonlinear global solution method developed concurrently in Lee (2025). This approach eliminates the need to functionally specify the law of motion for endogenous aggregate states. This methodological improvement is crucial because the surges of lumpy investments — only partially smoothed by general equilibrium effects — create inherently nonlinear dynamics in aggregate states that resist functional form specification required by traditional state-space methods (Krusell and Smith, 1997, 1998). The method avoids these limitations while efficiently computing global nonlinear solution paths without requiring additional computational loops for period-by-period market clearing conditions. Based on this global solution, I provide a sharp quantification of endogenous state dependence in aggregate investment fluctuations and conduct generalized impulse response analyses (Koop et al., 1996; Andreasen et al., 2017).

In the model, aggregate investment's interest rate elasticity varies with the fragility index throughout the business cycle. This finding suggests that monetary policy effectiveness diminishes following surges in large firms' lumpy investments. It also provides a microfounded explanation for why monetary policy has been less effective during recessions, particularly through the business investment channel, as documented by Tenreyro and Thwaites (2016).

I validate these policy implications by analyzing the corresponding generalized impulse response functions (GIRF) for an extended model with exogenous stochastic discount factor shifters.

Related literature This paper is related to the literature that studies how firm-level investments shape the aggregate investment over the business cycle. The literature investigated under which conditions the firm-level nonlinear investment dynamics are relevant to the aggregate investment dynamics (Caplin and Spulber, 1987; Caballero and Engel, 1993; Elsby and Michaels, 2019) and its macroeconomic implications over the business cycle (Caballero and Engel, 1991, 1999; Cooper et al., 1999). Building upon these findings, the recent strands of the literature have studied the rich heterogeneous-firm environments, of which the complicated endogenous distributional dynamics are summarized by the sufficient statistics (Baley and Blanco, 2021) based on the novel analytical framework (Alvarez and Lippi, 2022).

My paper's fragility index builds upon the sufficient statistic approach by Bachmann et al. (2013) and Baley and Blanco (2021). Similar to the sufficient statistics, the fragility index is constructed from the cross-sectional firm-level data and captures how large a portion of firms are close to the re-adjustment point in the Ss cycle. However, the fragility index is based on the distribution of large firms instead of the entire distribution. Also, my paper analyzes the endogenous state dependence predicted by the fragility index using a global nonlinear solution method, away from the stationary equilibrium.

An unsettled debate yet in the literature is the role of general equilibrium effect in neutralizing the firm-level lumpy adjustment patterns. Using a canonical model with a fixed adjustment cost, Thomas (2002) has shown that the general equilibrium effect almost fully neutralizes the firm-level lumpiness once aggregated. Khan and Thomas (2003, 2008) have shown that the inclusion of the firm-level heterogeneity does not mitigate the strong neutralizing force of the general equilibrium. According to House (2014), this is due to the near-infinite interest rate elasticity of the firm-level capital adjustment in the extensive margin in the models with the fixed adjustment cost.

To this point, Gourio and Kashyap (2007) shows that the close-to-perfect neutralization is not a generic nature of the general equilibrium, and it depends on the parametric setup in the model such as the assumption on the distribution of the fixed adjustment cost. Bachmann et al. (2013) shows that when the maintenance investment demand is considered, the general equilibrium effect cannot perfectly smoothen the lumpiness of the aggregate investments, leading to the state-dependent responsiveness. Their firm-level maintenance demand essentially lowers the interest rate elasticity of investment, which weakens the general equilibrium effect. Similarly, in the models of Winberry (2021) and Koby and Wolf (2020), the firm-level investments feature realistic interest rate elasticity of investment on average, due to the presence of the convex adjustment cost, leading to the nonlinear aggregate investment dynamics.

Related to this literature, my paper shows that the models with plain-vanilla fixed and convex adjustment costs flip the cross-sectional ranking of the elasticities between the small and large firms. Therefore, the nonlinearity in the aggregate dynamics studied in the existing models has been counterfactually driven by the non-smoothed small firms' investments rather than the large firms' investments. I show that once the cross-sectional ranking is corrected, the state dependence in the aggregate investment dynamics becomes substantially stronger due to the unsmoothed large firms' lumpy investments.[2]

My paper's key mechanism is closely related to the investment hangover studied in Rognlie et al. (2018). Their model predicts that the investment response is state-dependent and that a deep and prolonged drop in investment can happen after the over-accumulation of capital stock. Building on this insight, I demonstrate that capital stock surges lead to varying hangover intensities depending on the investment elasticity of firms driving the preceding surge. When surges are primarily driven by interest-inelastic large firms, the hangover effect becomes more severe — corresponding to the high fragility state in my

[2] This nonlinear effect is significantly large even if the compared models share the same average interest rate elasticity at the firm level as the baseline at the steady state. That is, in the nonlinear model, the cross-section of the elasticity matters on top of the average elasticity.

framework. The lumpiness of firm-level investment generates slow recovery as large firms that drove the preceding surge remain inactive during their Ss cycle. Importantly, I show that this pattern of surges followed by deep recessions emerges endogenously within the recursive competitive equilibrium.

Lastly, this paper is related to the literature studying the state-dependent effectiveness of monetary policy. Vavra (2014) has studied the state-dependent monetary policy effectiveness based on the volatility state: high volatility lowers the effectiveness due to the increased aggregate price flexibility. Baley and Blanco (2019) shows that the real effects of nominal shocks are amplified when cross-sectional firm-level uncertainty is high. Berger et al. (2021) has shown that the monetary policy is path dependent due to the household-level mortgage prepayment channel. The most closely related paper to my paper is Tenreyro and Thwaites (2016), which shows that business investment and durables expenditure are less responsive to monetary policies during recessions. Related to this, Gnewuch and Zhang (2025) shows that when inelastic old firms take a greater portion of the market, such as in downturns, the effectiveness of monetary policy declines. My paper shows that the interest-elasticity of aggregate investment significantly decreases in the fragility index. This provides a microfounded explanation of why monetary policy has not been effective during the past recessions that were preceded by the surges of large firms' lumpy investments.

2 Motivating facts and empirical analysis

2.1 Data and the definitions

In this section, I empirically analyze the cyclical pattern of large firms' lumpy investments. I use the U.S. Compustat data for the firm-level empirical analysis. While Compustat data covers only public firms, this limitation is not concerning because the focus is on large firms, most of which are listed. Throughout the empirical analysis, large firms are defined as firms that hold capital stocks greater than the 40th percentile of the capital distribution in each industry. This percentile is calculated within each two-digit NAICS code in the

Compustat data. The firm-level real capital stock is obtained by applying the perpetual inventory method to deflated net investment. The net investment is obtained from the lag difference of the balance sheet item Property, Plant, and Equipment (Net). Intangibles are not included in the investment. The industry is categorized by the first two-digit NAICS code.[3] The choice of the 40th percentile is for consistency with the definition in Zwick and Mahon (2017), of which the estimated interest rate elasticity is one of the main calibration targets of my baseline model.[4] The sample period covers from 1980 to 2016. The further detailed description of the data and variables is available in Appendix A.

2.2 Surges of large firms' lumpy investments and the fragility index

In the following analysis, I empirically analyze the relationship between large firms' lumpy investments and the timing of recessions. I define an investment spike as a firm-specific event where a firm makes a large-scale investment greater than 20% of the firm's existing capital stock.[5] I refer to these investment spikes as lumpy investments or capital adjustments in the extensive margin interchangeably. Then, I define spike ratio as follows:

$$\text{Spike ratio}_{j,t} := \frac{\sum_{i \in j} \mathbb{I}\{i_{it}/k_{it} > 0.2\}}{\#\text{ of } j\text{-type firms at } t}, \quad j \in \{small, large\} \tag{1}$$

The numerator counts all the instances of investment spikes from firm type $j \in \{small, large\}$ at time t, and it is normalized by the total number of j-type firms. The spike ratio captures the degree of investment timing synchronization.

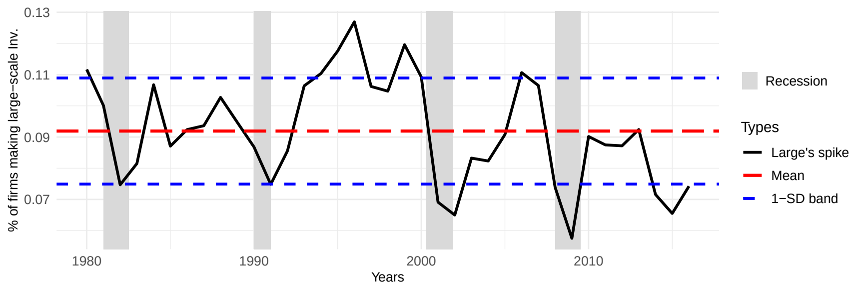
Figure 1 plots the time series of the spike ratio of large firms. On average, 9.2% of large

[3] If only SIC code is available for a firm, I imputed the NAICS code following online appendix D.2 of Autor et al. (2020). If both NAICS and SIC are missing, I filled in the next available industry code for the firm.

[4] In Zwick and Mahon (2017), large and small firms are defined by the cutoffs of (15.4M, 48.8M) in terms of sales in the years 1998 through 2000 and 2005 through 2007 (Table B.1, panel (d)). I compute the corresponding capital size cutoffs in Compustat.

[5] 20% cutoff is from the literature that studies the role of non-convex adjustment cost in the firm dynamics (Cooper and Haltiwanger, 2006; Gourio and Kashyap, 2007; Khan and Thomas, 2003, 2008). If a firm's acquired capital stock is greater than 5% of existing capital stock in a certain year, I rule out the observation from the sample due to possible noise in the reported items in the balance sheet during the acquisition year. Appendix K includes robustness checks for different cutoffs of 18% and 22% for the investment spikes.

Figure 1: Three surges of large firms' lumpy investments before recessions



Notes: The firm-level large-scale investment is defined as an investment greater than 20% of the existing capital stock. The solid line plots the time series of the fraction of large firms making large-scale investments. The grey areas indicate the NBER recession periods.

firms adjust their existing capital stocks in the extensive margin in a year. As can be seen from Figure 1, since 1980, there have been only four periods (1980, 1996, 1998, and 2007) where the fraction of large firms making spiky investments surged beyond one standard deviation. Three out of the four events were followed by recessions within two years.

Conversely, there were four recessions in the U.S. over the same period, and three out of four recessions were preceded by surges in large firms' lumpy investments. The exception was the recession in 1990, and it was the mildest recession among the four recessions.

Consistent with this pattern, I show that aggregate investment rate is conditionally heteroskedastic on the average lagged spike ratio of large firms in Appendix A.1. That is, the residualized volatility of aggregate investment rate is high if a great portion of large firms have recently made lumpy investments synchronously.

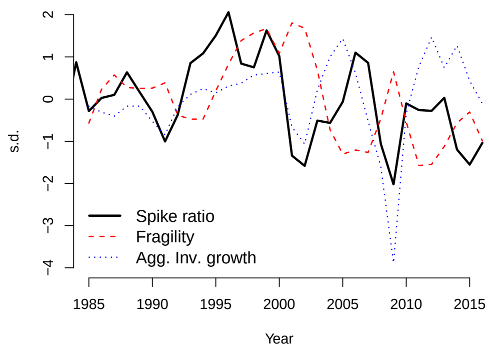
Fragility index Motivated from the investment surges preceding the recessions, I introduce a fragility index that captures the fraction of large firms that have recently completed large-scale investments:

$$Fragility_t := \frac{\sum \mathbb{I}\{s_{it} \leq \overline{s}\}\mathbb{I}\{k_{it} > \overline{k}\}}{\sum \mathbb{I}\{k_{it} > \overline{k}\}} \tag{2}$$

where s_{it} is the time from the last lumpy investment of firm i; $\overline{s}$ is the time threshold for s_{it} to be counted as a recent event; $\overline{k}$ is the size threshold of large firms.[6]

[6] The median duration between two lumpy investments is around 6 years. In the regression that includes

Figure 2: Fragility, spike ratio, and the aggregate investments



Notes: The solid line plots the time series of the spike ratio. The dashed line is the time series of the fragility index. The dotted line is time series of the aggregate investment growth rate.

When a great fraction of large firms have just finished a large-scale investment, a relatively small fraction of large firms are willing to make another round subsequently. This occurs because firm-level lumpy investment entails a fixed adjustment cost. Therefore, there is a lead-and-lag relationship between the spike ratio and the fragility index, as can be seen from Figure 2. When the fragility index is regressed on the three-year lagged spike ratio, the coefficient is 0.75 (statistically significant at the 1% level).[7] Because large firms' lumpy investments comprise a substantial portion of aggregate investment, the spike ratio displays significant positive co-movement with aggregate investment growth. The fragility index's lead-and-lag relationship with the spike ratio naturally generates predictability of aggregate investment dynamics.

A one-standard-deviation increase in the fragility index is associated with a 0.46 standard deviation decrease in investment growth (statistically significant at the 10% level), indicating that the predetermined fragility index negatively predicts contemporaneous investment growth rates. The full regression results are available in Appendix I.2. The calibrated heterogeneous-firm business cycle model developed later explains the economic

the fragility index, reported in Table 5, I found $\overline{s} = 3$ maximizes the fitness of the regression. Thus, in the following analysis, I use $\overline{s} = 3$ for the fragility index. The main results are not significantly affected by the choice of $\overline{s}$.

[7]The correlation is strongest with a three-year lag. Table I.10 reports the full regression results with different lags.

mechanisms underlying this predictability and analyzes how it interacts with exogenous TFP fluctuations.

2.3 How different are the large firms' investments from the small firms?

This section compares the lumpy investment patterns and investment elasticities between large and small firms.

Lumpiness Table 1 reports the inaction-related moments in the first part and the moments based on lumpy investments in the second part. The time to lumpy investment is defined as the time distance between two neighboring lumpy investments.

Table 1: Comparison of lumpy investment patterns between large and small firms

	Large	Small
Inaction moments (all in yrs.)		
Unconditional mean of time to lumpy investment	6.892	6.460
	(0.148)	(0.191)
Mean of average firm-level time to lumpy investment	7.687	6.933
	(0.196)	(0.245)
Lumpy investment moments (all in %)		
Dollar share of lumpy investments in total investments	21.050	28.364
	(2.524)	(2.785)
Average spike ratio	9.192	16.813
	(0.401)	(0.924)

Notes: The statistics are from the US Compustat firm-level data. The numbers in the bracket are heteroskedasticity and autocorrelation consistent (HAC) standard errors.

The first part of the table shows that large firms' Ss cycle is slightly longer than that of small firms, as their unconditional and cross-sectional means of firm-level time to lumpy investments show. The second part of the table shows that small firms' lumpy investments account for a greater portion of total investments than large firms. However, large firms' lumpy investments still account for 21% of the entire investments. The spike ratio defined in Equation (1) is smaller for large firms than small firms, which is consistent with the inaction moments that indicate large firms' lumpy investments feature a longer Ss cycle. Therefore, large firms' lumpy investments are less frequent but substantially large in size.

If a lumpy investment is made only at an establishment level, large firms that own many establishments should display a smoothed investment pattern. However, the statistics above show that large firms' investments are also lumpy, and their Ss cycle tends to be slightly lengthier than that of small firms.

The observed lumpiness in large firms' capital adjustment likely stems from real frictions rather than financial constraints. Since large firms typically face fewer financial constraints than small firms, their lumpy investment patterns suggest the presence of real adjustment costs even with good access to capital markets. This motivates this paper's focus on understanding how these real frictions shape investment responses to economic conditions.

Interest rate elasticities I empirically estimate the elasticity of firm-level investment using firm-level balance sheet data and monetary policy shocks from the literature. I estimate the elasticities of small and large firms' spike ratios (extensive margin) in addition to gross investment, which I use to guide my baseline model to capture the empirically-supported investment patterns. I use the following empirical specification, separately for large and small firms:

$$f(k_{it}, k_{it+1}) = \beta MP_t + \alpha_i + \alpha_{sy} + Controls_{it} + \epsilon_{it}$$

where MP_t is the monetary policy shock; α_i is firm fixed effect; α_{sy} is sector-year fixed effect. The control variables include lagged current account (ACT_{t-1}), lagged total debt (DT_{t-1}), and operating profit ($OIBDP_t$) normalized by lagged total asset (AT_{t-1}), log of lagged capital stock, and log of employment (EMP_t). The standard errors are two-way clustered across firms and years.

The monetary policy shock is computed and merged with the Compustat data following Ottonello and Winberry (2020) and Jeenas (2018). The monetary policy data on the timings of the FOMC announcement and the high-frequency surprise are from Gurkaynak et al. (2005) and Gorodnichenko and Weber (2016), covering the sample period from March 1990 until December 2009. This sample contains a sufficient number of shocks with meaningful

Table 2: Investment sensitivities to the monetary policy shocks (narrow window)

	Dependent variables:			
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$	
	Large	Small	Large	Small
MP_t (in *p.p.*)	-2.201	-7.025	-0.870	-2.072
	(0.606)	(2.41)	(0.367)	(0.676)
Obs.	29,400	7,903	29,400	7,903
R^2	0.929	0.791	0.603	0.558
Firm FE	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes

Notes: The independent variables include the monetary policy shock (tight window) measured in percentage points, fixed effects (firm and sector-year), and firm-level control variables (lagged current account (ACT_{t-1}), lagged total debt (DT_{t-1}), and operating profit ($OIBDP_t$) normalized by lagged total asset (AT_{t-1}), log of lagged capital stock, and log of employment (EMP_t)). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by firm and year.

variation, while largely predating the zero-lower-bound era. The monetary policy shock is measured in percentage points. The details on the computation of the monetary policy shock and merging steps are available in Appendix D.

Table 2 reports the coefficient of monetary policy shock (MP_t) for large and small firms across different choices of dependent variables.[8] As can be seen from the first two columns, the elasticity of the investment is significantly lower in large firms than in small firms. This is consistent with the empirical results in the literature (Zwick and Mahon, 2017) and contradictory to the elasticities implied in the existing models, which will be investigated in the following section. Also, the sensitivity of the spike ratio is significantly lower in large firms than small firms, as reported in the third and fourth columns. Two estimates are statistically different at the 5% significance level. A set of extended regression results is available in Appendix B.2.

[8]Table 2 is based on the tight-window monetary policy shock. The results for the wide-window shock and extended results are all available in Appendix B.2. Also, I provide the robustness check based on a 10% cutoff in Appendix B.2.

In summary, the empirical findings indicate that large firms' investment (i) is lumpy and (ii) is significantly more inelastic to interest-rate changes than that of small firms.

3 Model

I develop and analyze a heterogeneous-firm real business cycle model in which the cross-section of the interest elasticities of firm-level investment matches the empirical estimates. In the model, time is discrete and lasts forever. There is a continuum of measure one of firms that own capital, produce business outputs, and make investments. The business output can be reinvested as capital after a firm pays adjustment costs.

3.1 Technology

A firm produces goods using capital and labor inputs, which can be converted to a unit of capital after paying an adjustment cost. The production technology is a Cobb-Douglas function with decreasing returns to scale:

$$z_{it}A_t f(k_{it}, l_{it}) = z_{it}A_t k_{it}^{\alpha} l_{it}^{\gamma}, \quad \alpha + \gamma < 1 \tag{3}$$

where k_{it} is firm i's capital stock at the beginning of period t; l_{it} is labor input; z_{it} is idiosyncratic productivity; A_t is aggregate TFP. Idiosyncratic productivity, z_{it}, and aggregate TFP, A_t, follow the stochastic processes as specified below:

$$ln(z_{it+1}) = \rho_z ln(z_{it}) + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim_{iid} N(0, \sigma_z) \tag{4}$$

$$ln(A_{t+1}) = \rho_A ln(A_t) + \epsilon_{A,t+1}, \quad \epsilon_{A,t+1} \sim_{iid} N(0, \sigma_A) \tag{5}$$

where ρ_s and σ_s are persistence and standard deviation of the $i.i.d$ innovation in each process $s \in \{z, A\}$, respectively. Both stochastic processes are discretized using the Tauchen method in computation.

3.1.1 Investment and adjustment costs

I assume firm-level large-scale investment can be made only after paying a total adjustment cost, Ψ_{it}, which varies across firm-level allocations. The total adjustment cost is a function of capital stock, k_{it}, investment size I_{it}, and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \overline{\xi}]$ as in Winberry (2021). This total adjustment cost is composed of two additively separable parts: a convex adjustment cost and a fixed adjustment cost. The convex adjustment cost is a function of the current capital stock, k_{it}, and the investment I_{it} as assumed in the literature.The fixed adjustment cost, F_{it}, is a function of the current capital stock k_{it} and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \overline{\xi}]$. The fixed cost is not incurred if a firm adjusts capital within a moderate range $I_{it} \in \Omega(k_{it}) := [-\nu k_{it}, \nu k_{it}]$, where $\nu < \delta$. The range does not fully cover the depreciated portion, requiring firms to pay the fixed cost to maintain the existing capital stock, as in Khan and Thomas (2008). The fixed cost is assumed to be overhead labor cost, so it varies over the business cycle due to wage fluctuations.[9]

To summarize, I assume the following total adjustment cost structures:[10]

$$\Psi_{it} = \Psi(k_{it}, I_{it}, \xi_{it}; w_t) \tag{6}$$

$$= \mu \left(\frac{I_{it}}{k_{it}}\right)^2 k_{it} + F(k_{it}, \xi_{it}) w_t \tag{7}$$

$$F(k_{it}, \xi_{it}) = \begin{cases} \xi_{it} k^{\zeta} & \text{if } I_{it} \notin \Omega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \\ 0 & \text{if } I_{it} \in \Omega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \end{cases} \tag{8}$$

This model's key difference from the existing literature is the size-dependent fixed cost parametrized by the extensive-margin elasticity dispersion parameter, ζ. As ζ increases,

[9] The labor cost assumption follows Khan and Thomas (2003, 2008), Miao (2019), and Winberry (2021). Alternatively, the literature has considered fixed costs scaling with productivity (Baley and Blanco, 2021), or profit (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006). These papers typically assume linear or near-linear scaling. However, as demonstrated in Appendix B.1, such weak scaling fails to capture the empirical cross-sectional pattern where large firms exhibit substantially lower interest elasticities than small firms. The stronger curvature ($\zeta = 3.5$) in my specification is necessary to match this empirical regularity.

[10] The convex adjustment cost parameter μ has units of final output per unit of capital. When applied to the quadratic function $(\mu/2)(i/k)^2 k$, it yields total costs in units of final output. The fixed adjustment cost ξk^{ζ} represents the overhead labor headcount required for coordination across establishments, where k^{ζ} captures the number of coordinations required and ξ has units of labor per coordination. When multiplied by wage $w(S)$, this gives costs in final output units. The maintenance range ν is expressed as a fraction of capital stock, constrained to be less than the depreciation rate δ.

the extensive-margin elasticity gap between small and large firms widens.[11] In Section 4, I quantitatively investigate how ζ affects the cross-sectional distribution of interest-elasticity and the macroeconomic allocations.

3.1.2 Size-dependent fixed cost: microfoundation

The primary motivation for the size-dependent fixed cost specification is to match the empirical evidence on investment elasticities across firm sizes. As documented in Appendix B.1, this specification successfully captures the observed pattern where large firms exhibit lower interest elasticities than small firms—a pattern that standard models fail to reproduce. While other mechanisms might also generate these elasticities, this section provides a suggestive theoretical foundation based on coordination costs across establishments that offers a plausible microfoundation for the size dependence.

The presence of fixed costs in firm investment has been widely accepted in the literature. However, relatively little research has been conducted on whether the fixed cost occurs at the establishment or firm levels. Depending on the model specification and the granularity of the data, researchers have flexibly chosen between establishment and firm-level specifications. My paper incorporates the fixed cost at the firm level, but its functional form is grounded in coordination costs across the establishments.[12] I argue that if a firm decides to make a large-scale investment by expanding establishments, the total fixed cost increases in the number of establishments due to interdependence across the establishments. For example, if a new establishment is constructed, the production lines in the existing establishments have to be adjusted to coordinate with the new one, and managers need to be reallocated across different production units.[13] Therefore, intuitively, firm-level fixed cost increases in

[11] Fang (2023) shows that a plain-vanilla fixed adjustment cost following uniform distribution makes it difficult to match the empirically observed levels of mean and variance of the adjustment costs. The size dependence in the fixed adjustment cost effectively handles this problem through the additional variation in the cost side determined by the endogenous firm size dispersion.

[12] Consistent with Kehrig and Vincent (2025), the establishment-based fixed adjustment cost would decrease the dispersion of the marginal product of capital across the establishment within a firm. However, this is beyond the scope of my paper, as my model abstracts from the establishment-level dynamics.

[13] I assume the fixed cost is in the unit of labor, necessitating multiplication by wage $w(S)$ in the full cost specification (Thomas, 2002; Gourio and Kashyap, 2007; Khan and Thomas, 2008; Winberry, 2021).

the number of establishments and the degree of interdependence across the establishments.

To sharpen the theoretical points, let's assume a firm has n establishments and plans to expand a new factory. Then, if establishments are coordinated pairwise, and if the fixed cost of each coordinated pair is ξ, the total firm-level fixed cost F_2 is as follows:[14]

$$F_2 = \binom{n}{2} \times \xi = \frac{n(n-1)}{2}\xi, \tag{9}$$

which quadratically increases in the number of establishments. This is when each establishment is interdependent pairwise. Then, if an establishment's operation is dependent on $\zeta - 1$ number of other establishments on average, the firm-level fixed cost becomes as follows:

$$F_\zeta = \binom{n}{\zeta} \times \xi = \frac{n(n-1)(n-2)\dots(n-\zeta+1)}{\zeta!}\xi \tag{10}$$

The firm-level fixed cost F_ζ increases with the number of establishments raised to the power of ζ. For a higher interdependence across the establishments, the fixed cost increases faster. This simple theoretical result shows that the number of the basic operation units (e.g., establishment, department or team) convexly increases the internal complexity in term of the interactions under the interdependence. Then, it increases the firm-level fixed cost when the firm makes a large-scale capital adjustment.

I proxy the number of establishments (or basic production units) by the total capital stock k_{it} based on the empirical evidence from Cao et al. (2019). Using the US administrative data, they point out that the firm growth is dominantly driven by the expansion in the number of establishments.

When the cost is assumed to be in the unit of firm-level outputs, thereby removing cyclical fluctuations, the baseline model's state dependence is amplified by around 3%.

[14] The subscript 2 indicates the degree of the interdependence, which is 2 (pairwise) here. Note that ξ represents the cost per coordination requirement, not per establishment.

3.2 Household

I consider a stand-in household which consumes, supplies labor, and saves in the equity portfolio. In the beginning of a period, the household has an equity portfolio a and the information on the contemporaneous distribution of firms Φ and the aggregate TFP level A. The household problem in the recursive form is as follows:

$$V(a;S) = \max_{c,a',l_H} \quad log(c) - \eta l_H + \beta \mathbb{E} V(a';S') \tag{11}$$

$$\text{s.t. } c + \int \int \Gamma_{A,A'} q(S,S') J(k',z';S') a(k',z') d[k' \times z'] dA' \tag{12}$$

$$= w(S) l_H + \int J(k,z;S) a(k,z) d[k \times z] \tag{13}$$

$$G_\Phi(S) = \Phi', \quad \mathbb{P}(A'|A) = \Gamma_{A,A'}, \quad S = \{\Phi, A\} \tag{14}$$

where V is the value function of the household; $\Gamma_{A,A'}$ is the aggregate state transition probability; c is consumption; a' is a future saving portfolio; l_H is labor supply; w is wage, and r is real interest rate.

From the household's first-order condition and the envelope condition, I obtain the following characterization of the stochastic discount factor $q(S,S')$:

$$q(S,S') = \beta \frac{C(S)}{C(S')} \tag{15}$$

I define $p(S) := 1/C(S)$. In the recursive formulation of a firms' problem in the next section, I use $p(S)$ to normalize the firm's value function, following Khan and Thomas (2008).

3.3 A firm's problem: Recursive formulation

In this section, I formulate a firm's problem in the recursive form. For notational brevity, I abstract the time and individual subscripts and denote future allocations by an apostrophe. In the beginning of a period, a firm starts with capital k, an idiosyncratic productivity z, and the information on the contemporaneous distribution of firms Φ and the aggregate TFP level A. For each period, the firm determines investment level I and labor demand l. A firm's problem is formulated in the following recursive form:

$$J(k,z;S) = \pi(k,z;S) + (1-\delta)k \tag{16}$$

$$+ \int_0^{\bar{\xi}} \max\left\{R^*(k,z;S) - F(k,\xi)w(S), R^c(k,z;S)\right\} dG_\xi(\xi) \tag{17}$$

$$R^*(k,z;S) = \max_{k' \geq 0} \; -k' - c(k,k') + \mathbb{E}q(S,S')J(k',z';S') \tag{18}$$

$$R^c(k,z;S) = \max_{k^c-(1-\delta)k \in \Omega(k)} \; -k^c - c(k,k^c) + \mathbb{E}q(S,S')J(k^c,z';S') \tag{19}$$

$$\text{(Operating profit)} \qquad \pi(k,z;S) := \max_l zAk^\alpha l^\gamma - w(S)l \text{ } (l\text{: labor demand}) \tag{20}$$

$$\text{(Convex adjustment cost)} \qquad c(k,k') := \left(\mu^I/2\right)\left((k'-(1-\delta)k)/k\right)^2 k \tag{21}$$

$$\text{(Size-dependent fixed cost)} \qquad F(k,\xi) := \xi k^\zeta \tag{22}$$

$$\text{(Constrained investment)} \qquad k^c - (1-\delta)k \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta) \tag{23}$$

$$\text{(Idiosyncratic productivity)} \qquad z' = G_z(z) \text{ (AR(1) process)} \tag{24}$$

$$\text{(Stochastic discount factor)} \qquad q(S,S') = \beta\left(C(S)/C(S')\right) \tag{25}$$

$$\text{(Aggregate states)} \qquad S = \{A, \Phi\} \tag{26}$$

$$\text{(Aggregate law of motion)} \qquad \Phi' := H(S),\ A' = G_A(A) \text{ (AR(1) process)}, \tag{27}$$

Then, I multiply $p(S) = 1/C(S)$ on the both sides of line (17) to normalize the value functions, following Khan and Thomas (2008). The detailed explanation is available in Appendix J.1.

A firm makes a large-scale investment only if $R^*(k,z;S) > R^c(k,z;S)$. Therefore, a firm-level extensive-margin investment decision can be characterized by the threshold rule, g_{ξ^*}, as follows:

$$g_{\xi^*}(k,z;S) = \min\left\{\frac{R^*(k,z;S) - R^c(k,z;S)}{w(S)k^\zeta}, \bar{\xi}\right\}, \tag{28}$$

where firms invest in the extensive margin only when $\xi \in [0, g_{\xi^*}(k,z;S))$. This threshold rule is distinguished from the ones in the literature in that it includes the capital stock in the denominator. This lowers the threshold level more for large firms than for small firms, helping capture the empirically supported cross-section of interest elasticities — large firms are less elastic than small firms. I quantitatively show this in Section 4.

I denote g_{k^*} as the optimal future capital stock conditional on the extensive-margin investment, g_{k^c} as the optimal future capital stock conditional on the small-scale investment, and g_k as the unconditional optimal capital stock. Then, the capital adjustment policy can be summarized as follows:

$$g_k(k, z; S) = \begin{cases} g_{k^*}(k, z; S) & \text{if } \xi < g_{\xi^*}(k, z; S) \\ g_{k^c}(k, z; S) & \text{if } \xi \geq g_{\xi^*}(k, z; S). \end{cases} \tag{29}$$

The standard recursive competitive equilibrium is considered for the analysis of the global equilibrium dynamics. The equilibrium consists of: i) household inter- and intra-temporal policy functions, ii) firm policy functions satisfying individual optimality conditions, iii) market-clearing prices, and iv) a perceived aggregate law of motion consistent with the aggregate dynamics implied by these policy functions. The formal definition is available in Appendix J.2.

4 Fragility after a surge of lumpy investments

This section quantitatively analyzes the macroeconomic implications of the synchronized lumpy investments of large firms. First, I discipline the baseline model by calibrating the parameters to fit the data moments. Especially, the different interest elasticities between small and large firms are the key moments to be fitted, which are hardly captured in alternative models.

4.1 Calibration

In this section, I elaborate on how the model is fitted to the data and the corresponding parameter levels. Table 3 reports the target and untargeted moments from the data and the simulated moments in the model. Table 4 reports the calibrated parameters given the fixed parameters reported in Table I.9. In the simulation step, I use the non-stochastic method in Young (2010).

Table 3: Fitted moments

Moments	Data	Model	Reference
Targeted moments			
Semi-elasticity of investment (%)	7.20	6.60	Zwick and Mahon (2017)
Cross-sectional semi-elasticity ratio (%)	1.95	1.81	Zwick and Mahon (2017)
Cross-sectional average of i_{it}/k_{it} ratio	0.10	0.10	Zwick and Mahon (2017)
Cross-sectional dispersion of i_{it}/k_{it} $(s.d.)$	0.16	0.16	Zwick and Mahon (2017)
Cross-sectional average spike ratio	0.14	0.14	Zwick and Mahon (2017)
Positive investment rate	0.86	0.86	Winberry (2021)
Time-series volatility of $log(Y_t)$	0.06	0.07	NIPA data (Annual)
Labor hours	0.33	0.33	8 working hours per day
Untargeted moments (all in yrs.)			
Average inaction periods	6.07	7.88	Compustat data
Dispersion of inaction periods	4.87	5.65	Compustat data
Average of lag diff. of inaction periods	0.49	0.69	Compustat data
Dispersion of lag diff. of inaction periods	6.34	8.57	Compustat data
Large firm's average spike ratio	9.19	9.58	Compustat data
Small firm's average spike ratio	16.81	13.29	Compustat data

Notes: The data moments are from the sources specified in the reference column. The same sample restriction as in the empirical analysis applies. I use linearly detrended real GDP from the National Income and Product Accounts at the annual frequency for the aggregate output volatility.

The target semi-elasticity of average investment comes from Zwick and Mahon (2017). The cross-sectional semi-elasticity ratio is also from the same paper, which documents that small firms' investments are around twice elastic as large firms towards the interest rate change. The cross-sectional average and dispersion of the investment-to-capital ratio and the average spike ratio are targeted to match the levels in Zwick and Mahon (2017) as in Winberry (2021) and Koby and Wolf (2020). Consistent with the literature, I define the spike ratio as the fraction of firms investing greater than 20% of the existing capital stock. The target of positive investment rate is from Winberry (2021). The positive investment rate is defined as the fraction of firms with an investment that is greater than 1% but smaller than 20% of existing capital stock. Only a negligible fraction of firms makes negative investment in both data and the model. To discipline the aggregate TFP-driven fluctuations in the model, I target the output volatility calculated from annual National Income and Product Accounts (NIPA) data.

Table 4: Calibrated parameters

Parameters	Description	Value
Internally calibrated parameters		
ζ	Fixed cost curvature	3.500
$\overline{\xi}$	Fixed cost upperbound	0.440
μ^I	Capital adjustment cost	0.760
ν	Small investment range	0.041
σ	Standard deviation of idiosyncratic TFP	0.130
σ_A	Standard deviation of aggregate TFP shock	0.025
η	Labor disutility	2.400
Externally estimated parameters		
ρ	Persistence of idiosyncratic TFP	0.750

Notes: Parameters in the upper part of the table are calibrated to match the moments in Table 3. The persistence of idiosyncratic TFP is directly computed from fitting the estimated firm-level TFP (Compustat) into AR(1) process. The firm-level TFP is estimated following Ackerberg et al. (2015) using US Compustat data.

In the model, variations in the fixed cost parameter and convex adjustment cost parameters lead to sharply divergent effects on the dispersion of the investment rate (investment-to-capital ratio), while both lower the average investment rate. The dispersion of the investment rate increases with the fixed cost parameter, as the difference in the investment rate between extensive-margin adjusters and non-adjusters increases.[15] On the other hand, a higher convex adjustment cost lowers the investment rate for all firms, leading to a lower dispersion in the investment rate. These two divergent effects, together with the average investment rate, jointly identify the levels of the fixed and convex adjustment cost parameters.

The investment range parameter ν is jointly identified from the spike ratio and the dispersion of the investment rate. A higher investment range leads to a lower spike rate due to a greater feasibility of the low-cost investment and to a lower investment rate disper-

[15] If the fixed cost is too high, the portion of adjusters become too small to have meaningful contribution to the investment rate dispersion.

sion. ν is assumed to be bounded above by the depreciation rate δ to prevent firms from permanently avoiding fixed costs through maintenance alone.

The fixed cost curvature parameter ζ is identified from the cross-sectional semi-elasticity ratio between small and large firms. As ζ increases beyond unity, the large firms' interest rate elasticity decreases due to the lengthened Ss band.[16] The calibrated level of ζ is 3.5, suggesting that 3.5 establishments are involved per production line on average.

The calibrated baseline model correctly captures the cross-sectional elasticity ratio between small and large firms. This makes it a suitable framework for analyzing the role of large firms' investment in dynamic stochastic general equilibrium — a key contribution to the literature, as existing models fail to capture this cross-sectional pattern.[17] Throughout the quantitative analyses, this calibrated model will be referred to as the baseline model.

Untargeted moments The model also matches several untargeted moments, providing external validation of the mechanism. Average inaction is around 6.07 years in the data, while in the model it is 7.88 years. The standard deviation of inaction periods is 4.87 years in the data, and the model counterpart is 5.65 years. The model also captures the different lumpy investment patterns between large and small firms—the model-implied spike ratios are 9.58% and 13.29% for large and small firms, respectively, while the data counterparts are 9.19% and 16.81%.[18] These close matches to untargeted moments, particularly the spike ratio differences, suggest that the results would be robust to alternative calibration strategies (such as directly targeting the spike ratio differences to discipline the ζ parameter).

Time-series validation I validate the calibrated baseline model by comparing model-implied spike ratios for large firms with their observed counterparts from Compustat. Using the measured TFP series from the Bureau of Labor Statistics (BLS) and Fernald (2014)

[16] A contemporaneous work, Gnewuch and Zhang (2025) studies how monetary policy shock affects the distribution of investment rates, and they document that young firms are more sensitive to the shock than old firms. Regarding this elasticity difference, they conclude that the extensive-margin sensitivity plays a crucial role, consistent with the results in my paper.

[17] Appendix B.1 quantitatively demonstrates that conventional fixed adjustment cost models counterfactually reverse the interest rate elasticity ranking, making large firms more responsive than small firms.

[18] Large and small firms' spike ratios are statistically different at 1% significance level both in the data and the model.

within the global solution framework, I simulate model-based spike ratios that are directly comparable to the data. As shown in Figure C.3 in Appendix C, the model and data spike ratios exhibit closely correlated patterns, with correlations of 0.79 (BLS) and 0.72 (Fernald) after controlling for pre-dot-com outliers. This validation confirms that the calibrated model successfully captures the cyclical dynamics of large firms' lumpy investments. Appendix C provides further details and visualizations of this analysis.

Business cycle statistics Finally, I compare the baseline model's business cycle moments with aggregate data from NIPA (1955–present, annual). As shown in Table I.12, the model matches key second moments well: output volatility is targeted, while investment autocorrelation (0.740 in the model vs 0.742 in the data) and its correlation with output (0.796 in the model vs 0.795 in the data) are closely replicated despite not being targeted. The model slightly understates the relative volatilities of consumption and investment.

What if financial frictions? The baseline model does not explicitly incorporate financial frictions; their effects are instead indirectly absorbed into adjustment costs. By placing firm-level capital k_{it} in the denominator of the convex adjustment cost as in Cooper and Haltiwanger (2006), the model implicitly captures large firms' financing advantages. Introducing an explicit borrowing constraint or external finance premium would primarily affect small firms, reducing their investment elasticity to interest rate changes. To maintain the observed average semi-elasticity, the convex adjustment cost would need to be lowered to compensate for the additional reduction in small firms' elasticity. Meanwhile, the fixed cost curvature parameter would need to be increased to preserve the cross-sectional elasticity ratio. This leads to a wider inaction band for large firms, implying greater investment lumpiness and a larger role for the extensive margin. Therefore, the state dependence documented in this paper, driven primarily by large firms' synchronized investment cycles, would likely be amplified by a hypothetical inclusion of financial frictions.

4.2 The global nonlinear solution method

I solve the model with the aggregate uncertainty using a sequence-space-based global nonlinear solution framework called the repeated transition method. Due to the nonlinear aggregate dynamics, it is difficult to correctly specify the law of motion for the endogenous aggregate variable (the firm distribution) in the state-space-based method. Instead, the concurrently developed method in Lee (2025) solves nonlinear dynamic stochastic general equilibrium in the sequence space.

The method exploits the ergodicity of the recursive competitive equilibrium: if the simulated equilibrium path is long enough, all possible equilibrium states are realized on the path. This enables sharp computation of conditional expectations by using realized value functions from periods with similar endogenous states. Notably, the conditional expectation is computed without specifying parametric laws of motion. Instead, the method relies on a similarity metric across state realizations.

In addition to computational accuracy, the method achieves substantial computational efficiency due to its efficient treatment of the non-trivial market clearing conditions. Khan and Thomas (2008) has introduced an internal loop for the period-by-period market clearing condition following Krusell and Smith (1997), which substantially increases the computational cost. The sequence-space-based method bypasses the market clearing condition over the iteration by using the implied prices rather than market clearing prices, and the former converges to the latter in the limit, simultaneously when the equilibrium allocations are computed. The detailed method is described in Appendix E.

By adopting the global equilibrium analysis, my paper tracks the endogenous formation of the fragility condition, as well as the global impulse response analyses conditional on different fragility levels. Thus, the results do not rely on differently calibrated steady states or exogenously given conditions — all the results are based on the subpaths of the integrated recursive competitive equilibrium framework. Within this equilibrium, the state-dependent responsiveness is endogenously shaped.

4.3 Synchronization and fragility

Based on the global nonlinear equilibrium dynamics, I study how the synchronized investment timings of large firms affect the aggregate investment dynamics over the business cycle. This section comprises three parts: 1) synchronization mechanism, 2) fragility after the synchronization, and 3) mapping the model prediction to the data. In the equilibrium, firm-level investment timings are endogenously synchronized due to the past aggregate TFP shock history. In the first part, I elaborate on the endogenous synchronization mechanism in the model. Second, I study the economy's different responses to the same negative shock depending on the contemporaneous synchronization patterns, which is captured by the fragility index dynamics.[19] Lastly, I analyze how much of the drop in aggregate investments during recessions are accounted for by the fragility index in the data.

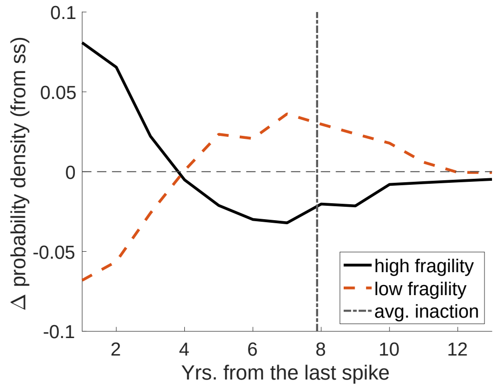
Synchronization When an aggregate TFP shock hits, firms accelerate or postpone their investment plans based on their location in the Ss cycle. For example, when a negative aggregate TFP shock hits, firms scheduled to implement lumpy investments tend to postpone them due to the expected poor economic conditions. Then, investment timings of these postponing firms and those who plan to invest in the subsequent periods become synchronized.

In the recursive competitive equilibrium, the stochastic aggregate TFP process endogenously generates significant fluctuations in synchronization patterns, reflected in the fluctuations in the fragility index. Figure 3 shows the probability distribution over the years from the last lumpy investments for the highest (solid line) and lowest (dashed line) fragility periods among the 5,000 simulated periods.[20] In particular, it plots a level deviation (the

[19] The sufficiency of the fragility index combined with the aggregate capital K in capturing the aggregate law of motion is computationally shown in Appendix E.

[20] Based on the equilibrium path of the firm-level investments, I construct the fragility index consistent with the empirical counterpart in Section 2. It is worth noting that the fragility index is constructed from the readily observable micro-level variables: the past investment history of large firms, most of which are listed and subject to financial reporting regulations. Therefore, the index can be measured in a timely manner and can help predict near-term aggregate investment. This feature is starkly contrasted with the existing indices in the literature based on the joint distribution between capital stock and productivity that is not directly observable (Caballero and Engel, 1993; Bachmann et al., 2013). In this respect, the sufficient-statistics approach of Baley and Blanco (2021) overcomes the limited observability of the joint distribution.

Figure 3: Distributions over the Ss band for the most and least fragile states in the RCE



Notes: The solid line is the probability density deviation from the stationary equilibrium counterpart for the highest fragility state in the simulated recursive competitive equilibrium. The dashed line is for the lowest fragility state.

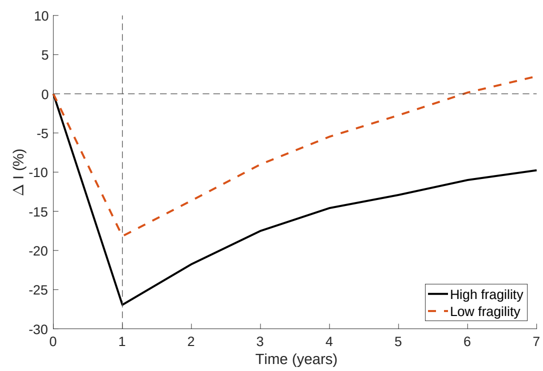
difference in the probability mass) from the stationary equilibrium's counterpart. Due to the stochastic nature of the Ss band in the model, there is no deterministic trigger point (small s) in the Ss cycle. However, firms with similar individual states tend to invest at a similar time after their last lumpy investment. If a long period has passed since a firm made its last investment, the firm is likely closer to the trigger point, and vice versa. Thus, the years from the last lumpy investment capture the location at the Ss band.

During the period of highest fragility, a significant portion of large firms have recently completed lumpy investments, indicating that many firms are further away from the trigger point in the Ss band (the small 's' threshold). Specifically, the average time since the last spike is 22% lower than the stationary equilibrium level in high-fragility periods, while it is 16% higher in low-fragility periods. In Appendix G, I show that small firms display significantly dampened synchronization compared to large firms (see Figure G.10) – in their highest and lowest fragility states, the average years from the last spike are 9.78% lower and 2.61% higher than the steady state, respectively. These endogenous fluctuations in firms' positions within the Ss band generate significantly different investment responses to aggregate TFP shocks, as demonstrated in the following analysis.[21]

[21] Appendix G provides detailed analysis of synchronization dynamics after an MIT shock, including the evolution of cross-sectional distributions on the Ss band.

Fragility after the surge Figure 4 plots the generalized impulse response function (GIRF), which is computed by combining the sub-paths of the recursive competitive equilibrium. A negative two-standard-deviation TFP shock is considered for the states with the fragility index two standard deviations above (solid line) and below (dashed line) the stationary equilibrium level. The vertical axis captures the percentage deviation from the level before the shock arrival. Despite the same exogenous shock, the responsiveness is significantly different between the two states: the high fragility state shows approximately 32% larger decline than the low fragility state (-26.94 vs. -18.18).

Figure 4: Generalized impulse response of investment



Notes: The solid line is the impulse response to the TFP shock when the fragility index is two standard deviations above the stationary equilibrium level. The dashed line is for the state where the fragility index is two standard deviations below the stationary equilibrium.

Using the linear regression, I obtain the following negative relationship between the contemporaneous aggregate investment response and the fragility index:

$$\mathit{\Delta I}_t \text{ (\% w.r.t. s.s. response)} = \underset{(0.173)}{-\,7.875} * \mathit{Fragility}_t \ (s.d.) + \epsilon_t, \quad R^2 = 0.677 \tag{30}$$

When the fragility index increases by one standard deviation, the contemporaneous response of the aggregate investment to the negative one-standard-deviation shock is amplified by 7.875% compared to the steady-state response. On the other hand, in the model with convex and constant fixed adjustment costs, the coefficient is -2.193, which is significantly lower in absolute value than the baseline. This demonstrates that the baseline model, in which large firms' inelastic adjustment drives the nonlinearity, generates greater endogenous fragility than the canonical model with convex and constant fixed adjustment costs. Lastly, if fragility increases by one standard deviation, the future output decreases by 0.322 percentage points through the amplified aggregate investment response to the negative aggregate shock. In Appendix F.1, I compare the state dependence across the different models and visualize the relationship between the responsiveness and the fragility through a scatter plot.

Fragility effect in the data How much of the investment drops in recessions are accounted for by the fragility effect? To examine the state dependence of shock responsiveness in the data with a close comparison to the model, I run the following regression analysis where the interaction between the fragility and the output shock is the key variable of interest:

$$g_t^I = \alpha + \beta^{Shock} OutputShock_t + \beta^{Fragility} OutputShock_t \times Fragility_t + \epsilon_t. \tag{31}$$

g_t^I is the *aggregate* investment growth rate. $OutputShock_t$ is a shock to the log output, obtained from the residuals in the AR(1) fitting of the log output time series. The aggregate investment and output data are from National Income and Product Accounts data. In this specification, $OutputShock_t$ exogenously arrives at t, while the $Fragility_t$ is determined at $t-1$. Therefore, the two variables are independent of each other.

In Table 5, the coefficient estimates from the model and data are statistically indistinguishable, while each coefficient is statistically significant. When the fragility index increases by one standard deviation: (i) for a one-standard-deviation negative output shock, aggregate investment growth decreases by an additional 1.5% (model) and 2.4% (data); (ii) for a one-standard-deviation positive output shock, aggregate investment growth increases by only 2.0% (model) and 1.5% (data), compared to the low-fragility case. The amplifying effect of the negative output shock and the mitigating effect of the positive output shock

Table 5: State-dependent sensitivity of the aggregate investment growth

	Dependent variable: $\Delta log(I_t)$ (p.p.)			
	$(-)$ $OutputShock_t$		$(+)$ $OutputShock_t$	
	Model	Data	Model	Data
$OutputShock_t$ $(s.d.)$	9.389	5.818	8.490	6.937
	(0.066)	(1.338)	(0.064)	(1.221)
$OutputShock_t \times Fragility_t$ $(s.d.)$	1.537	2.430	-2.011	-1.486
	(0.042)	(1.311)	(0.045)	(0.495)
Constant	Yes	Yes	Yes	Yes
Observations	2,296	16	2,705	18
R^2	0.908	0.790	0.884	0.705
Adjusted R^2	0.908	0.755	0.884	0.663

Notes: The dependent variable is the growth rate of aggregate investment. The independent variables are output shocks obtained from fitting output series into an AR(1) process and the interaction between the output shock and the fragility index. The fragility index is based on the years since the last lumpy investment of large firms. The first two columns report the regression coefficients from the simulated data and Compustat data when the negative output shock hits. The third and fourth columns report the regression coefficients when the positive output shock hits. The numbers in the brackets are standard errors.

under the high fragility state are both due to the absence of lumpy investments of large firms. That is, after a surge of lumpy investments of large firms, a negative shock leads to a deeper drop in aggregate investment, while a positive shock leads to a dampened increase in aggregate investment.[22]

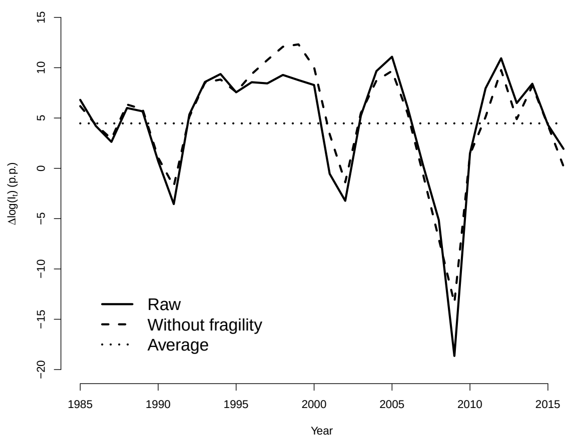
To quantify the economic significance of these findings, I use the estimates from the data in Table 5 to measure the portion of the investment growth rate that is accounted for by the interaction between the output shock and the fragility index. Specifically, the fragility-adjusted investment growth rate $g_t^{adj,I}$ is obtained as follows:

$$g_t^{adj,I} = g_t^I - \widehat{\beta}^{Fragility} \cdot OutputShock_t \times Fragility_t. \tag{32}$$

Figure 5 plots the time series of the raw aggregate investment growth rate (solid line)

[22] In Table I.11, I report the additional regression results under different specifications. When the output shock is the only independent variable in the regression, around 73% and 52% of the investment growth rate variations are explained, respectively, for negative and positive shocks in the data. Once the fragility fluctuation is considered, R^2 values improve to 79% and 71%.

Figure 5: Fragility-adjusted investment growth



Notes: The solid line is the aggregate investment growth rate from NIPA. The dashed line is the fragility-adjusted investment growth. The dotted line is the average level of the aggregate investment growth rate.

and the fragility-adjusted investment growth rate (dashed line). After the adjustment, the investment drops during the three recessions are mitigated. Table 6 compares the deviations from the average level for the raw and the fragility-adjusted investment growth rates in the three most recent recessions of the sample period. Around 23% of the deviation from the average level is accounted for by the fragility effect during recessions. The standard deviations show that around 30% of aggregate investment volatility can be explained by the interaction effect ($0.30 \approxeq 0.018/0.060$).

Table 6: Investment growth rates during recessions

	Distance between investment growth rate and average: $\Delta log(I_t)$ (p.p)		
	Raw data (NIPA)	Without fragility	Adjusted portion (%)
Recession-1991	-8.019	-6.239	22.197
Recession-2001	-7.695	-5.852	23.951
Recession-2009	-23.112	-17.847	22.780

Notes: The first column reports the investment growth rate (%) at recession years of 1991, 2001, and 2009 minus the average investment growth ($\approxeq 4.5\%$). The second column reports the adjusted investment growth rate after removing the predicted component from the fragility indices using the coefficients of Table 5. The third column reports the adjusted portion (%).

4.4 Compositional heterogeneity in the fragility effect

This section analyzes how the fragility effect operates primarily through large firms, with small firms displaying minimal state dependence. Rognlie et al. (2018) shows that an over-accumulation of capital stock results in a sharp drop in the aggregate investment. Building upon this insight, the fragility mechanism in this paper predicts that the intensity of this hangover effect depends on which firms drive the investment surge. The negative relationship between the responsiveness and the synchronization (fragility) is significantly starker for large firms than for small firms. As shown in the linearly fitted regression results below (33) - (34), there are significant differences between the two groups in terms of the slope and the goodness of fit. In Appendix F.2, I visualize these two different negative relationships.

$$\Delta I_t^{Large} \text{ (\% w.r.t. s.s. response)} = \underset{(0.142)}{-\,10.418} * Fragility_t \ (s.d.) + \epsilon_t, \quad R^2 = 0.844 \tag{33}$$

$$\Delta I_t^{Small} \text{ (\% w.r.t. s.s. response)} = \underset{(0.216)}{-\,3.140} * Fragility_t \ (s.d) + \epsilon_t, \quad R^2 = 0.175 \tag{34}$$

5 Policy implication: state-dependent interest rate elasticity of aggregate investment

In this section, I discuss the policy implications of the fluctuations of the fragility index over the business cycle. In the baseline model economy, aggregate investment features a strong history dependence. This history dependence not only affects the aggregate investment's response to the TFP shock but also affects its elasticity to the interest rate change.

Using linear regression, I obtain the following result:

$$\mathit{\Delta Elasticity_t}\ (\%\ \text{w.r.t. s.s.}) = -\underset{(0.032)}{3.350} * \mathit{Fragility_t}\ (s.d) + \epsilon_t, \quad R^2 = 0.689 \tag{35}$$

A one-standard-deviation increase in the fragility index decreases the interest rate elasticity of aggregate investment by around 3.350% compared to the steady-state level. The intuitive explanation is that when the fragility index is high, few large firms are positioned to flexibly undertake large-scale investments. Therefore, aggregate investment's responsiveness to interest rate changes decreases in high-fragility states. In Appendix H.2, I show that the state-dependent elasticity is driven by large firms through a comparison with small firms' effects.

When the fragility index increases by one standard deviation, large firms' investment elasticity decreases by around 5.257%. On the other hand, the same variation in the fragility index decreases small firms' elasticity by 1.244%, and the difference is statistically significant. This result shows that large firms dominantly drive the stark negative relationship between the interest elasticities of aggregate investment and the fragility index.

5.1 An augmented model with the SDF shifter

The baseline model does not include a monetary policy block, which precludes direct analysis of monetary transmission mechanisms. However, I can examine how interest rate changes affect investment dynamics in general equilibrium by introducing a preference shock process φ that exogenously shifts the stochastic discount factor, following Christiano et al. (2014). This approach allows me to analyze state-dependent investment responses to interest rate variations while maintaining the model's general equilibrium structure. Note that these shocks affect the real interest rate rather than nominal rates, and thus capture only one dimension of monetary policy transmission.[23]

[23] This differs from the partial equilibrium analysis in Figure H.12 (Appendix H.1), which shows the negative relationship between interest rate elasticity and the fragility index through Equation (35). A full

$$\sum_{t=0}^{T} \beta^t \varphi_t u(c_t) \implies q(S,S') = \beta \frac{\varphi'}{\varphi} \frac{u(c(S'))}{u(c(S))}, \tag{36}$$

$$\text{where the aggregate state } S \text{ is augmented as } S = [\Phi, A, \varphi]. \tag{37}$$

I assume φ follows a three-state Markov process where the states are $G_\varphi = [0.99, 1, 1.01]$, where the two non-unity values correspond to a 1% increase and decrease in the interest rate. The preference shift is assumed to happen with 10% probability and to be short-lived: once a non-unity value is visited, I assume it reverts to unity in the following period with probability one.[24]

Based on this augmentation, I recompute the recursive competitive equilibrium using the sequence-space global solution method. Figure 6 plots the generalized impulse response functions of aggregate investment to a positive SDF shock ($\varphi = 1.00 \to 1.01$) that is approximately equivalent to 1% interest rate drop. The same shock is considered for two different states. One has fragility two standard deviations above steady state, while the other has fragility two standard deviations below. I assume the TFP level at shock arrival is one standard deviation lower than average, representing a slight downturn where lowering the interest rate would be a more realistic policy consideration.

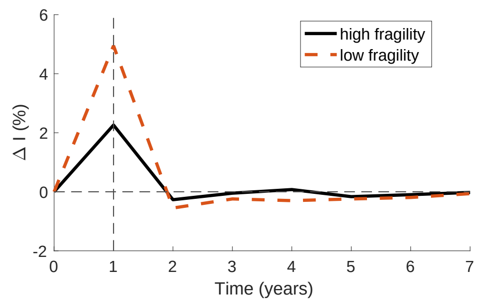
Despite the same exogenous shock, the responses at the two states are significantly different. When fragility is high, the contemporaneous responsiveness of aggregate investment is dampened by more than half compared to the low fragility state (2.2 compared to 4.9). Table 7 decomposes different responses for high and low fragility states into intensive and extensive margins.[25] For this analysis, I first fix the extensive margin decisions at the counterfactual equilibrium path where the shock did not arrive, then measure the response of aggregate investment to extract the intensive-margin variation. Then, the residual vari-

monetary analysis would require incorporating nominal rigidities and a central bank reaction function, which is beyond the scope of this paper.

[24] The quantitative result is robust over the choice of the event probability due to the i.i.d. assumption. I assumed 10% to let the shifting events happen sufficiently many times in the simulated path.

[25] Table 7 is motivated by Table 6 of Vavra (2014).

Figure 6: Generalized impulse response of investment to a positive SDF shock



Notes: The solid line (high fragility) is the impulse response to the TFP shock when the fragility index is two standard deviations above the stationary equilibrium level. The dashed line (low fragility) is for the state where the fragility index is two standard deviations below the stationary equilibrium.

ations are accounted for by the extensive-margin variations. While the intensive margin response of the high fragility state is lower than that of the low fragility state by around 42.6%, the extensive margin is lower by around 47.6%. This result shows the extent to which the lowered interest rate elasticity is driven by the extensive-margin investment decision, as captured by the marginal distribution of firms concentrated at the early stages of the Ss band in Figure 3.

Table 7: State-dependent investment responses to the interest change: decomposition

Response (%)	Total	Intensive margin	Extensive margin
High fragility	2.260	0.818	1.442
Low fragility	4.948	1.920	3.029

Notes: The first column reports the instantaneous investment responses to the stochastic discount factor shock. The second column reports the instantaneous responses when the extensive-margin investment is muted. The last column reports the residual variation after the intensive-margin-only variation (the second column).

Discussion The analyses above show that when fragility is high, monetary policy cannot effectively operate through the firm-level investment channel. Given that recent recessions occurred during periods of high fragility, this finding supports Tenreyro and Thwaites (2016), who document that conventional monetary policies have been less powerful during recessions, especially through business investment channels. Moreover, my paper adds to the related literature by providing an endogenous mechanism that explains the state dependence of monetary policy effectiveness. Importantly, while the fragility index is constructed from past investment history, it has forward-looking predictive power and can be easily measured using readily observable large firms' data. Therefore, the fragility index can potentially contribute to optimal monetary policy design in practice.

5.2 Testing the policy implications from the data

The model predicts that the monetary policy effect on the investment response is lower for the high fragility states. To empirically test this model prediction, I use the quarterly monetary policy shock series and the fragility index as in Section 2. The regression specification is as follows:

$$g_t^I = \alpha + \beta_1 MP_{t-h} + \beta_2 dTFP_t + \epsilon_t \tag{38}$$

The dependent variable g_t^I is the aggregate investment growth rate. $dTFP_t$ is the utilization-adjusted aggregate TFP growth rate from Fernald (2014). The aggregate-level regression is subject to timing complications. A subtle mixed lead-lag relationship exists between monetary policy implementation and investment accounting. Therefore, I consider a lag of four quarters ($h = 4$) for the monetary policy shock and further lags ($h = 5, 6$) for robustness check. Table 8 reports the regression results conditional on the periods with high (first three columns) and low fragilities (last three columns) measured at each period contemporaneously. High and low fragility periods are the top 40% and bottom 40% fragility index periods among the whole sample.[26] The responsiveness of aggregate investment is statistically not distinguishable from zero during high fragility periods. However, during the low fragility periods, the aggregate investment significantly negatively responds to monetary policy shocks, and the goodness of fit substantially improves. These empirical results

[26]The result stays unaffected over the different choices of the cutoffs around 40%.

support the model's equilibrium prediction at the macro level.

Table 8: State-dependent monetary policy effectiveness

	Dependent variable: g_t^I (%)					
	High fragility			Low fragility		
	$h=4$	$h=5$	$h=6$	$h=4$	$h=5$	$h=6$
$MP_{Tight,t-h}$	0.528	0.222	0.380	-0.530	-0.683	-0.643
	(0.47)	(0.399)	(0.342)	(0.353)	(0.325)	(0.318)
Obs.	28	28	28	28	28	28
R^2	0.056	0.020	0.055	0.155	0.218	0.208
Constant	Yes	Yes	Yes	Yes	Yes	Yes
TFP growth control	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The dependent variable is the aggregate investment growth rate from NIPA. Independent variables include the monetary policy shock and the TFP growth rate from Fernald (2014). h indicates the lagged timing (in quarter) of the monetary policy shock. High fragility periods refer to the top 40% of the sample periods in terms of fragility, and the low fragility periods are the bottom 40%.

Additionally, I also show that the extensive-margin response of investment at the firm level is substantially muted during high fragility periods in Table I.13 of Appendix I.5. This evidence substantiates the model prediction from the firm-level angle.

6 Concluding remarks

This paper analyzes the endogenous state dependence in aggregate investment dynamics driven by synchronized lumpy investments of large firms. Following a surge of large firms' lumpy investments, an economy becomes substantially more fragile to negative aggregate shocks. The economic significance of this channel is quantified in a heterogeneous-firm real business cycle model in which the cross-section of the elasticities of firm-level investment is matched with the empirical estimates. In the model, aggregate investment features significant state dependence in the interest elasticities, driven by fragility index fluctuations. This implies that after a surge of large firms' lumpy investments, the effectiveness of monetary policy can substantially fall due to the lowered interest rate elasticity of aggregate investment.

These findings open new avenues for policy design. Since general equilibrium forces only partially smooth firm-level investment timing, policymakers could mitigate aggregate fluctuations by preventing excessive synchronization among large firms. Crucially, this paper's results demonstrate that both the timing and targeting of such policies matter. Future work could explore how targeted investment credits or other interventions might optimally smooth investment cycles, particularly for large firms whose synchronized actions drive fragility.

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