# Striking While the Iron Is Cold: Fragility after a Surge of Lumpy Investments[†]

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**Abstract**

This paper studies an economy's endogenous fragility to a negative TFP shock shaped by large firms' synchronized lumpy investments. I develop a heterogeneous-firm real business cycle model in which the interest elasticities of large and small firms' investments are matched with the empirical estimates. In the model, the timings of large firms' lumpy investments are persistently synchronized due to the low sensitivity to the general equilibrium effect, leading to surges of lumpy investments. After the surge, TFP-induced recessions are especially severe, and the semi-elasticity of the aggregate investment drops significantly, consistent with the data.

**Keywords**: Business cycle, state dependence, lumpy investment, interest elasticity.

**JEL codes**: E32, E22, D25.

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# 1 Introduction

This paper studies an economy's *endogenous* fragility to a negative TFP shock shaped by large firms' investment dynamics. Large firms' investments are lumpy, large in its scale, and inelastic to the interest rate change. These distinctive natures generate an endogenous state dependence in the economy through their synchronized large-scale investments, which has been only scantly studied in the literature. Using a calibrated heterogeneous-firm business cycle model, I substantiate that this channel leads to a significant variation in the aggregate allocations' sensitivity to the aggregate TFP shocks.

The contribution of this paper can be summarized in three folds. The first is identifying and quantifying the novel large-firm-driven aggregate-level state dependence. Based on the firm-level and macro-level data, I show that the aggregate investment sensitivity to a TFP shock significantly increases in the portion of large firms that have recently completed large-scale investments. I deduce that the low interest elasticity of large firms is key to capturing this observed state dependence, using the comparative-static analysis based on the general equilibrium model. According to the quantitative analysis, around 23% of the investment rate drops during the recent recessions are accounted for by the fluctuations in the large firms' synchronized lumpy investments.[1] Moreover, a one-standard-deviation increase in the past synchronization leads to a decline in the aggregate investment semi-elasticity to the interest rate change by around 3.4% compared to the steady-state level.

Secondly, this paper develops a fragility index that indicates the aggregate investment and output responsiveness to a negative TFP shock. The index is based on the portion of large firms that have recently completed lumpy investments.[2] Therefore, 1) it is constructed from the data on the large firms, 2) determined by the past observations, and 3) has a predictive power on the contemporaneous investment re-

[1] The sample periods exclude the pandemic period.

[2] Thus, this index measures the degree of the recent synchronization.

sponsiveness. These three features serve a great practical value, as most of the large firms' data is public, so their recent investment activities are easily traceable.[3]

Lastly, this paper develops a heterogeneous firm model that can correctly capture the cross-section of the interest elasticity. The existing models in the literature have incorporated fixed and convex adjustment costs to capture the empirically supported firm-level investment dynamics. However, the calibrated version of these models counterfactually flips the cross-sectional ranking of the interest-elasticities of investment between large and small firms: the large becomes more interest-elastic than the small. This is because the cost of the extensive margin investment is too cheap for large firms in those models, allowing them to respond more sensitively to the interest rate changes than the small firms. I fix this problem by introducing and calibrating a parameter that governs how the size affects the inaction cutoffs through the size-dependent fixed adjustment cost. I provide a micro-founded structural implication for this parameter, which is a degree of interdependence across the establishments within a firm. The calibrated baseline model correctly captures the inelastic large firms and elastic small firms, which results in a significantly greater nonlinearity and state dependence in the aggregate investment dynamics compared to the existing models.

Over the business cycle, the large firms' investment timings are persistently synchronized in the baseline model, as observed in the data. This is because firms tend to pause lumpy investment projects synchronously when a negative aggregate TFP shock hits, as future business prospects are not promising due to the persistence of the shock.[4] Since they stop together, their implementation timings tend to be synchronized in the following periods. This is similar to pedestrians stopping altogether at the crossing when the red light turns on and walking together synchronized when

[3] This is due to the fact that large firms are mostly listed, which makes them subject to financial disclosure regulations mandated by the U.S. Securities and Exchange Commission (SEC). Therefore, the fragility index does not require to track the entire set of firms.

[4] Similar synchronization can happen for a positive aggregate TFP shock. However, the positive shock is not as effective as the negative shock in terms of synchronization because the accelerating implementation of a project entails paying a fixed adjustment cost while stopping is costless.

the light turns green. However, if a general equilibrium's smoothing force is strong enough, as in Khan and Thomas (2003, 2008), such synchronization can be resolved, encouraging firms to pursue relatively dispersed investment timings. The baseline model's low-interest elasticity of large firms makes the interest rate's smoothing force less effective, preserving surges of large firms' lumpy investments even in general equilibrium.

When a negative aggregate shock hits, some of the large firms still undergo planned lumpy investments, as their marginal benefit of investment exceeds the cost despite the bad economic prospects. These firms' investments buffer the exogenous negative impact of the TFP shock on the economy through the added capital.[5] However, after surges of large firms' lumpy investments, relatively fewer large firms are willing to undertake such counter-cyclical lumpy investments. In other words, relatively more large firms are in the early stages of their Ss band. Therefore, if a negative aggregate TFP shock hits after the surges, the response of the aggregate investment is sharper, leading to a deeper recession. This effect is the main source of the state-dependent responsiveness of the aggregate investment.

For the computation of the model, I use the sequence-space-based nonlinear global solution method concurrently developed in Lee (2024), which dispenses with the functional specification of the law of motion for the endogenous aggregate state. Due to the surges of lumpy investments that are only partially smoothed out by the general equilibrium effect, the true law of motion in the aggregate states is nonlinear. This makes it difficult to specify its functional form to apply the state-space-based method (Krusell and Smith, 1997, 1998). However, the method I use in this paper is free from this concern and quickly computes the global nonlinear solution path without an extra loop for the period-by-period non-trivial market clearing condition. Using the method, the endogenous state dependence in the aggregate investment fluctuation is accurately quantified.

[5]Some of the small firms also make counter-cyclical lumpy investments, but they are quantitatively less meaningful due to their small size.

In the model, the aggregate investment's interest elasticity depends on the level of the fragility index over the business cycle. This result implies that the monetary policy's effectiveness can be low after a surge of large firms' lumpy investments.[6] Also, it provides a micro-founded explanation of why monetary policy has not been effective during the recessions, especially through the business investment channel (Tenreyro and Thwaites, 2016).

**Related literature** This paper is related to the literature that studies how firm-level investments shape the aggregate investment over the business cycle. The literature investigated under which condition the firm-level nonlinear investment dynamics are relevant to the aggregate investment dynamics (Caplin and Spulber, 1987; Caballero and Engel, 1993; Elsby and Michaels, 2019) and its macro-economic implications over the business cycle (Caballero and Engel, 1991, 1999; Cooper et al., 1999). Building upon these findings, the recent strands of the literature has studied the rich heterogeneous-firm environments, of which the complicated endogenous distributional dynamics are summarized by the sufficient statistics (Baley and Blanco, 2021) based on the novel analytical framework (Alvarez and Lippi, 2022).

My paper's fragility index builds upon the sufficient statistic approach by Baley and Blanco (2021). Similar to the sufficient statistics, the fragility index is constructed from the cross-sectional firm-level data and captures how large a portion of firms are close to the re-adjustment point in the Ss cycle. However, the fragility index is based on the distribution of large firms instead of the entire distribution. Also, my paper analyzes the endogenous state dependence predicted by the fragility index using a global nonlinear solution method, away from the stationary equilibrium.

An unsettled debate yet in the literature is the role of general equilibrium effect in neutralizing the firm-level lumpy adjustment patterns. Using a canonical model with a fixed adjustment cost, Thomas (2002) has shown that the general equilibrium

[6]The policy implication is limited to a positive implication, as the model does not include a monetary policy block.

effect almost fully neutralizes the firm-level lumpiness once aggregated. Khan and Thomas (2003, 2008) have shown that the inclusion of the firm-level heterogeneity does not mitigate the strong neutralizing force of the general equilibrium. According to House (2014), this is due to the near-infinite interest elasticity of the firm-level capital adjustment in the extensive margin in the models with the fixed adjustment cost.

To this point, Gourio and Kashyap (2007) shows that the close-to-perfect neutralization is not a generic nature of the general equilibrium, and it depends on the parametric setup in the model such as the assumption on the distribution of the fixed adjustment cost. Bachmann et al. (2013) shows that when the maintenance investment demand is considered, the general equilibrium effect cannot perfectly smoothen the lumpiness of the aggregate investments, leading to the state-dependent responsiveness. Their firm-level maintenance demand essentially lowers the interest elasticity of investment, which weakens the general equilibrium effect. Similarly, in the models of Winberry (2021) and Koby and Wolf (2020), the firm-level investments feature realistic interest elasticity of investment on average, due to the presence of the convex adjustment cost, leading to the nonlinear aggregate investment dynamics.

Related to this literature, my paper shows that the models with plain-vanilla fixed and convex adjustment costs flip the cross-sectional ranking of the elasticities between the small and large firms. Therefore, the nonlinearity in the aggregate dynamics studied in the existing models has been counterfactually driven by the non-smoothed small firms' investments rather than the large firms' investments. I show that once the cross-sectional ranking is corrected, the negative skewness and the state dependence in aggregate investments become substantially stronger due to the unsmoothed large firms' lumpy investments.[7]

Lastly, this paper is related to the literature studying the state-dependent effec-

[7]This nonlinear effect is significantly large even if the compared models share the same average interest elasticity at the firm level as the baseline at the steady state. That is, in the nonlinear model, the cross-section of the elasticity matters on top of the average elasticity.

tiveness of monetary policy. The most closely related paper is Tenreyro and Thwaites (2016), which shows that business investment and durables expenditure are less responsive to monetary policies during recessions. Related to this, Gnewuch and Zhang (Gnewuch and Zhang) shows that when inelastic old firms take a greater portion of the market, such as in downturns, the effectiveness of monetary policy declines. My paper shows that the interest-elasticity of aggregate investment significantly decreases in the fragility index. This provides a micro-founded explanation of why monetary policy has not been effective during the past recessions that were preceded by the surges of large firms' lumpy investments.

**Roadmap** Section 2 shows motivating facts about surges of large firms' lumpy investments that proceeded the recessions. Section 3 develops a heterogeneous-firm business cycle model where the cross-section of the interest-elasticities is matched with the empirical estimates. Section 4 analyzes the macroeconomic implications of the calibrated model. Section 5 concludes.

# 2 Motivating facts

## 2.1 Data and the definitions

In this section, I empirically analyze the cyclical pattern of large firms' lumpy investments. I use U.S. Compustat data for the firm-level empirical analysis. While Compustat data covers only public firms, its coverage is relatively less of an issue in this analysis because the focus is on top large firms, most of which are listed. Throughout the empirical analysis, large firms are defined as firms that hold capital stocks greater than the 40th percentile of the capital distribution in each industry of the two-digit NAICS code in the Compustat data. The choice of the 40th percentile is for consistency with the definition in Zwick and Mahon (2017), which estimated the interest-elasticities of firm-level investments.[8] The sample period covers from 1980 to 2016. Firms with negative assets and zero employment are excluded from the sample. All the firm-level variables except capital stock and investment are deflated by the GDP deflator. Investment is deflated by non-residential fixed investment deflator available from National Income and Product Accounts data (NIPA Table 1.1.9, line 9). The firm-level real capital stock is obtained by applying the perpetual inventory method to net real investment. The industry is categorized by the first two-digit NAICS code.[9]

## 2.2 Surges of large firms' lumpy investments and recessions

In the following analysis, I empirically analyze the relationship between large firms' lumpy investments and the timing of recessions. I define an investment spike as a firm-specific event where a firm makes a large-scale investment greater than 20% of the firm's existing capital stock.[10] I refer to this investment spike as a lumpy investment or capital adjustment in the extensive margin interchangeably. Then, I define spike ratio as follows:

$$\text{Spike ratio}_{j,t} := \frac{\sum_{i\in j} \mathbb{I}\{i_{it}/k_{it} > 0.2\}}{\#\text{ of } j\text{-type firms at } t}, \quad j \in \{small, large\} \tag{1}$$

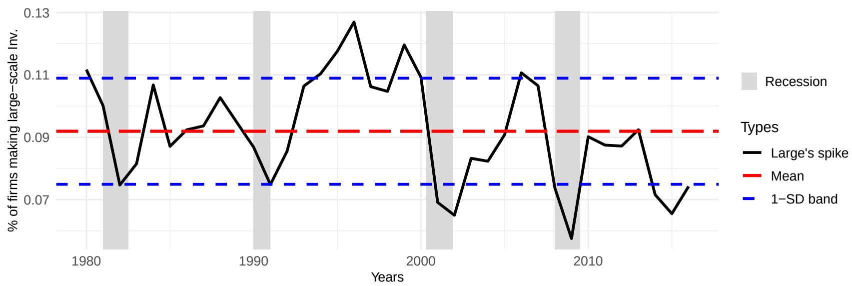
[8] In Zwick and Mahon (2017), large and small firms are defined by the cutoffs of (15.4M, 48.8M) in terms of sales in the years 1998 through 2000 and 2005 through 2007 (Table B.1, panel (d)). I compute the corresponding capital size cutoffs in Compustat.

[9] If only SIC code is available for a firm, I imputed the NAICS code following online appendix D.2 of Autor et al. (2020). If both NAICS and SIC are missing, I filled in the next available industry code for the firm.

[10] 20% cutoff is from the literature that studies the role of non-convex adjustment cost in the firm dynamics (Cooper and Haltiwanger, 2006; Gourio and Kashyap, 2007; Khan and Thomas, 2003, 2008). If a firm's acquired capital stock is greater than 5% of existing capital stock in a certain year, I rule out the observation from the sample due to possible noise in the reported items in the balance sheet during the acquisition year.

The numerator counts all the incidences of investment spikes from firm type $j \in \{small, large\}$ at time $t$, and it is normalized by the total number of $j$-type firms.

Figure 1: Three surges of large firms' lumpy investments before recessions




*Notes:* The firm-level large-scale investment is defined as an investment greater than 20% of the existing capital stock. The solid line plots the time series of the fraction of large firms making large-scale investments. The grey areas indicate the NBER recession periods.

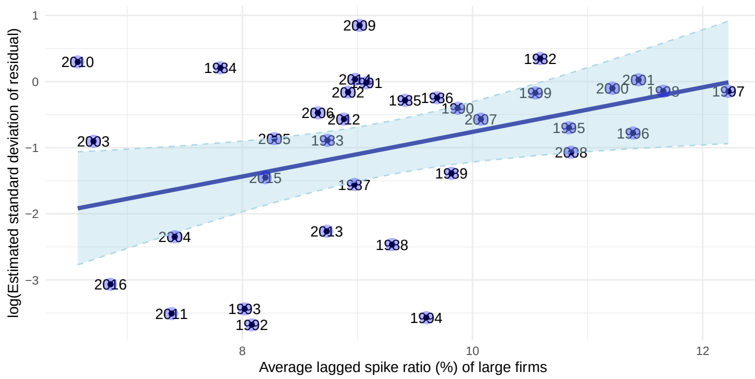
Figure 1 plots the time series of the spike ratio of large firms. On average, 9.2% of large firms adjust their existing capital stocks in the extensive margin in a year. As can be seen from Figure 1, since 1980, there have been only four periods (1980, 1996, 1998, and 2007) where the fraction of large firms making spiky investments surged beyond one-standard deviation. Three out of the four events were followed by recessions within two years.

Conversely, there were four recessions in the U.S. over the same period, and three out of four recessions were preceded by the surge of large firms' lumpy investments. The exception was the recession in 1990, and it was the mildest recession among the four recessions.

In the following analysis, I show aggregate investment rate is conditionally heteroskedastic on the average lagged spike ratio of large firms. That is, the residualized volatility of aggregate investment rate is high if a great portion of large firms have recently made lumpy investments.

For this analysis, I use aggregate data on non-residential investment (NIPA Table 1.1.5, line 9) and aggregate capital (Fixed Asset Accounts Table 1.1, line 4) from BEA. The thick line in Figure 2 plots the estimates of the log standard deviation of

Figure 2: Conditional heteroskedasticity of aggregate investment




*Notes:* The estimated standard deviation of the residual (y-axis) is obtained from fitting the aggregate investment-to-capital ratio (%) into an autoregressive process with four lags. The average lagged spike ratio of large firms (%) is obtained by averaging the most recent past two spike ratios for each observation of residualized investments. The years overlaid on the dots are the observation year of the residualized investment-to-capital ratios.

residuals from the autoregression of aggregate investment rates as a function of the recent average of large firms' spike ratio.[11] The recent average is based on the average spike ratio of the past two years. As can be seen from this figure, aggregate investment rates are heteroskedastic conditional on the lagged average spike ratio. Table E.9 reports the regression coefficients for the fitted line. According to the regression result, a one-standard-deviation increase (1.47%) in the large firms' past spike ratio is associated with a one-standard-deviation increase (0.50%) in the aggregate investment's residualized volatility. Consistent with the patterns in Figure 1, the three recession years of interest are located at the top-right corner in Figure 2.

## 2.3 Lumpiness of large and small firms' investments

This section compares the lumpy investment patterns between large and small firms. Table 1 reports the inaction-related moments in the first part and the moments based on the lumpy investments in the second part. The time to lumpy investment

[11] This empirical analysis is motivated from the conditional heteroskedasticity analysis in Figure 1 of Bachmann et al. (2013). Differently from theirs, the focus is on the large firms' recent lumpy investments.

is defined as the time distance between two neighboring lumpy investments.

Table 1: Comparison of lumpy investment patterns between large and small firms

| | Large | Small |
|---|---|---|
| **Inaction moments** (all in yrs.) | | |
| Unconditional mean of time to lumpy investment | 6.892 | 6.460 |
| | (0.122) | (0.157) |
| Mean of average firm-level time to lumpy investment | 7.687 | 6.933 |
| | (0.159) | (0.186) |
| **Lumpy investment moments** (all in percentage) | | |
| Dollar portion of lumpy investments out of total investments | 21.050 | 28.360 |
| | (1.080) | (1.340) |
| Average spike ratio | 9.192 | 16.813 |
| | (0.280) | (0.568) |

*Notes:* The statistics are from the US Compustat firm-level data.

The first part of the table shows that large firms' Ss cycle is longer than that of small firms, as their unconditional and cross-sectional means of firm-level time to lumpy investments show. The second part of the table shows that small firms' lumpy investments account for a greater portion of total investments than large firms. However, large firms' lumpy investments still account for 21% of the entire investments. The spike ratio defined in Equation (1) is smaller for large firms than small firms, which is consistent with the inaction moments that indicate large firms' lumpy investments feature a lengthier Ss cycle. Therefore, large firms' lumpy investments are less frequent but substantially large in its size.

If a lumpy investment is made only at an establishment level, large firms that own many establishments should display a smoothed investment pattern. However, the statistics above show that large firms' investments are also lumpy, and their Ss cycle tends to be lengthier than small firms.

# 3 Model

I develop and analyze a heterogeneous-firm real business cycle model in which the cross-section of the interest elasticities of firm-level investment matches the empirical estimates. In the model, time is discrete and lasts forever. There is a continuum of measure one of firms that own capital, produce business outputs, and make investments. The business output can be reinvested as capital after a firm pays adjustment costs.

## 3.1 Technology

A firm produces a unit of goods using capital and labor inputs, which can be converted to a unit of capital after paying an adjustment cost. The production technology is a Cobb-Douglas function with decreasing returns to scale:

$$z_{it}A_t f(k_{it}, l_{it}) = z_{it}A_t k_{it}^{\alpha} l_{it}^{\gamma}, \quad \alpha + \gamma < 1 \tag{2}$$

where $k_{it}$ is firm $i$'s capital stock at the beginning of period $t$; $l_{it}$ is labor input; $z_{it}$ is idiosyncratic productivity; $A_t$ is aggregate TFP. Idiosyncratic productivity, $z_{it}$, and aggregate TFP, $A_t$, follow the stochastic processes as specified below:

$$ln(z_{it+1}) = \rho_z ln(z_{it}) + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim_{iid} N(0, \sigma_z) \tag{3}$$

$$ln(A_{t+1}) = \rho_A ln(A_t) + \epsilon_{A,t+1}, \quad \epsilon_{A,t+1} \sim_{iid} N(0, \sigma_A) \tag{4}$$

where $\rho_s$ and $\sigma_s$ are persistence and standard deviation of $i.i.d$ innovation in each process $s \in \{z, A\}$, respectively. Both stochastic processes are discretized using the Tauchen method in computation.

### 3.1.1 Investment and adjustment

I assume a firm-level large-scale investment could be made only after paying a total adjustment cost, $\varPsi_{it}$, which varies over firm-level allocations. The total adjustment cost is a function of capital stock, $k_{it}$, investment size $I_{it}$, and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \bar{\xi}]$ as in Winberry (2021). This total adjustment cost is composed of two additively separable parts: a convex adjustment cost and a fixed adjustment cost. The convex adjustment cost is a function of the current capital stock, $k_{it}$, and the investment $I_{it}$ as assumed in the literature. The fixed adjustment cost, $F_{it}$, is a function of the current capital stock $k_{it}$ and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \bar{\xi}]$. The fixed cost does not incur if a firm adjusts capital within a moderate range ($I_{it} \in \varOmega(k_{it}) := [-\nu k_{it}, \nu k_{it}]$). A firm needs to pay a fixed cost for investment beyond this range. The fixed cost is assumed to be overhead labor cost, so it varies over the business cycle due to wage fluctuations.[12]

To summarize, I assume the following total adjustment cost structures:

$$\begin{aligned} \varPsi_{it} &= \varPsi(k_{it}, I_{it}, \xi_{it}; w_t) && (5) \\ &= \mu \left( \frac{I_{it}}{k_{it}} \right)^2 k_{it} + F(k_{it}, \xi_{it}) w_t && (6) \end{aligned}$$

$$F(k_{it}, \xi_{it}) = \begin{cases} \xi_{it} k^{\zeta} & \text{if } I_{it} \notin \varOmega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \\ 0 & \text{if } I_{it} \in \varOmega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \end{cases} \tag{7}$$

This model's key difference from the existing literature is the size-dependent fixed cost parametrized by the extensive-margin elasticity dispersion parameter, $\zeta$. As $\zeta$ increases, the extensive-margin elasticity gap between small and large firms widens. In Section 4, I quantitatively investigate how $\zeta$ affects the cross-sectional distribution of interest-elasticity and the macroeconomic allocations.

[12] This setup is from Khan and Thomas (2008) and Winberry (2021).

### 3.1.2 Size-dependent fixed cost: A theoretical explanation

This section provides a theoretical ground for the size-dependent fixed cost. The presence of a fixed cost for the firm-level investment has been widely accepted in the literature. However, relatively little research has been conducted on whether the fixed cost occurs at the establishment or firm levels. Depending on the model specification and the granularity of the data, each paper flexibly assumes a fixed cost.

My paper incorporates the fixed cost at the firm level, but its functional form is grounded on the establishment-level fixed cost. I argue that if a firm decides to make a large-scale investment by expanding establishments, fixed cost occurs at each existing establishment due to interdependence across the establishments. For example, if a new establishment is constructed, the production lines in the existing establishments have to be adjusted to coordinate with the new one, and managers have to be reallocated across the different production units. Therefore, intuitively, firm-level fixed cost increases in the number of establishments and the degree of interdependence across the establishments.

To sharpen the theoretical points, let's assume a firm has $n$ establishments and plans to expand a new factory. Then, if establishments are coordinated pairwise, and if the fixed cost of each coordinated pair is $\xi$, the total firm-level fixed cost $F_2$ is as follows:[13]

$$F_2 = \binom{n}{2} \times \xi = \frac{n(n-1)}{2}\xi, \tag{8}$$

which quadratically increases in the number of establishments. This is when each establishment is interdependent pairwise. Then, if an establishment's operation is dependent on $\zeta - 1$ number of other establishments on average, the firm-level fixed

[13] The subscript 2 indicates the degree of the interdependence, which is 2 (pairwise) here.

cost becomes as follows:

$$F_\zeta = \binom{n}{\zeta} \times \xi = \frac{n(n-1)(n-2)\dots(n-\zeta+1)}{\zeta!}\xi \tag{9}$$

The firm-level fixed cost $F_\zeta$ exponentially increases in the number of establishments to the power of $\zeta$. For a higher interdependence across the establishments, the fixed cost increases faster. This simple theoretical result shows that the number of the basic operation units (e.g., establishment, department or team) convexly raises the internal complexity in term of the interactions under the interdependence. Then, it increases the firm-level fixed cost when the firm makes a large-scale capital adjustment.

I proxy the number of establishments (or basic production units) by the total capital stock $k_{it}$ based on the empirical evidence from Cao et al. (2019). Using the US administrative data, they point out that the firm growth is dominantly driven by the expansion in the number of establishments.

## 3.2 Household

A stand-in household is considered. The household consumes, supplies labor, and saves in a complete market. In the beginning of a period, the household is given with an equity portfolio $a$, the information on the contemporaneous distribution of firms $\Phi$, and the aggregate TFP level $A$. The household problem in the recursive form is as follows:

$$V(a;S) = \max_{c,a',l_H} \quad log(c) - \eta l_H + \beta \mathbb{E} V(a';S') \tag{10}$$

$$\text{s.t. } c + \int \Gamma_{A,A'} q(S,S') a(S') dS' = w(S) l_H + a(S) \tag{11}$$

$$G_\Phi(S) = \Phi', \quad \mathbb{P}(A'|A) = \Gamma_{A,A'}, \quad S = \{\Phi, A\} \tag{12}$$

where $V$ is the value function of the household; $\Gamma_{A,A'}$ is the state transition probability; $c$ is consumption; $a'$ is a state-contingent future saving portfolio; $l_H$ is labor supply; $w$ is wage, and $r$ is real interest rate.

From the household's first-order condition and the envelope condition, I obtain the following characterization of the stochastic discount factor $q(S,S')$:

$$q(S,S') = \beta \frac{C(S)}{C(S')} \tag{13}$$

I define $p(S) := \frac{1}{C(S)}$. In the recursive formulation of a firms' problem in the next section, I use $p(S)$ to normalize the firm's value function, following Khan and Thomas (2008).

## 3.3 A firm's problem: Recursive formulation

In this section, I formulate a firm's problem in the recursive form. A firm is given with capital $k$, an idiosyncratic productivity $z$, in the beginning of a period. Also, they are given with the knowledge on the contemporaneous distribution of firms $\Phi$ and the aggregate TFP level $A$. For each period, firm determines investment level $I$ and labor demand $n_d$. A firm's problem is formulated in the following recursive form:

$$J(k,z;S) = \quad \pi(k,z;S) + (1-\delta)k \tag{14}$$

$$+ \int_0^{\bar{\xi}} \max\left\{R^*(k,z;S) - F(k,\xi)w(S), R^c(k,z;S)\right\} dG_\xi(\xi) \tag{15}$$

$$R^*(k,z;S) = \max_{k' \geq 0} \; -k' - c(k,k') + \mathbb{E}q(S,S')J(k',z';S') \tag{16}$$

$$R^c(k,z;S) = \max_{k^c \in \Omega(k)} \; -k^c - c(k,k^c) + \mathbb{E}q(S,S')J(k^c,z';S') \tag{17}$$

The following lines explain the details of each component in the value function.

$$\text{(Operating profit)} \quad \pi(z,k;S) := \max_{n_d} zAk^{\alpha}n_d^{\gamma} - w(S)n_d \ (n_d\text{: labor demand}) \tag{18}$$

$$\text{(Convex adjustment cost)} \quad c(k,k') := \left(\mu^I/2\right)\left((k'-(1-\delta)k)/k\right)^2 k \tag{19}$$

$$\text{(Size-dependent fixed cost)} \quad F(k,\xi) := \xi k^{\zeta} \tag{20}$$

$$\text{(Constrained investment)} \quad k^c \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta) \tag{21}$$

$$\text{(Idiosyncratic productivity)} \quad z' = G_z(z) \ \text{(AR(1) process)} \tag{22}$$

$$\text{(Stochastic discount factor)} \quad q(S,S') = \beta\left(C(S)/C(S')\right) \tag{23}$$

$$\text{(Aggregate states)} \quad S = \{A, \Phi\} \tag{24}$$

$$\text{(Aggregate law of motion)} \quad \Phi' := H(S), \ A' = G_A(A) \ \text{(AR(1) process)}, \tag{25}$$

Then, I multiply $p(S) = 1/C(S)$ on the both sides of line (15) to obtain

$$p(S)J(k,z;S) = \quad p(S)(\pi(k,z;S) + (1-\delta)k) \tag{26}$$

$$+ \int_0^{\bar{\xi}} \max\left\{p(S)R^*(k,z;S) - p(S)w(S)F(k,\xi), p(S)R^c(k,z;S)\right\} dG_{\xi}(\xi) \tag{27}$$

I define the normalized value functions as follows:

$$\widetilde{J}(k,z;S) := p(S)J(k,z;S) \tag{28}$$

$$\widetilde{R}^*(k,z;S) := p(S)R^*(k,z;S) \tag{29}$$

$$\widetilde{R}^c(k,z;S) := p(S)R^c(k,z;S) \tag{30}$$

It is necessary to check whether the recursive formulation naturally follows for the normalized value functions. Using $p(S)q(S,S') = \beta p(S')$,

$$\widetilde{R}^* = \max_{k' \geq 0} \ (-k' - c(k,k'))p(S) + \mathbb{E}p(S)q(S,S')J(k',z';S') \tag{31}$$

$$= \max_{k' \geq 0} \ (-k' - c(k,k'))p(S) + \mathbb{E}\beta p(S')J(k',z';S') \tag{32}$$

$$= \max_{k' \geq 0} \ (-k' - c(k,k'))p(S) + \beta\mathbb{E}\widetilde{J}(k',z';S') \tag{33}$$

Similarly,

$$\widetilde{R}^c = \max_{k^c \in \Omega(k)} \ (-k^c - c(k,k^c))p(S) + \beta\mathbb{E}\widetilde{J}(k^c,z';S'). \tag{34}$$

Therefore, the recursive form is preserved for the normalized value functions. As in Khan and Thomas (2008), the recursive form based on the normalized value function eases computation of the dynamic stochastic general equilibrium because the price, $p$, depends only on the current aggregate state variable, $S$.

A firm makes a large scale investment only if $R^*(k,z;S) > R^c(k,z;s)$. Therefore, a firm-level extensive-margin investment decision can be characterized by the threshold rule, $g_{\xi^*}$, as follows:

$$g_{\xi^*}(k,z;S) = \min\left\{\frac{\widetilde{R}^*(k,z;S) - \widetilde{R}^c(k,z;S)}{w(S)p(S)k^\varsigma}, \bar{\xi}\right\}, \tag{35}$$

where firms invest in the extensive margin only when $\xi \in [0, g_{\xi^*}(k,z;S))$. This threshold rule is distinguished from the ones in the literature in that it includes the capital stock in the denominator. This lowers the threshold level more for large firms than for small firms, helping capture the empirically supported cross-section of interest elasticities. I quantitatively show this in Section 4.

I denote $g_{k^*}$ as the optimal future capital stock conditional on the extensive-margin investment, $g_{k^c}$ as the optimal future capital stock conditional on the small-scale investment, and $g_k$ as the unconditional optimal investment. Then, the capital adjustment policy can be summarized as follows:

$$g_k(k, z; S) = \begin{cases} g_{k^*}(k, z; S) & \text{if } \xi < g_{\xi^*}(k, z; S) \\ g_{k^c}(k, z; S) & \text{if } \xi \geq g_{\xi^*}(k, z; S). \end{cases} \tag{36}$$

The standard recursive competitive equilibrium is considered for the analysis of the global equilibrium dynamics. The definition is available at Appendix F.

# 4 Quantitative analysis

This section quantitatively analyzes the macroeconomic implications of the synchronized lumpy investments of large firms. First, I discipline the baseline model by calibrating the parameters to fit the data moments. Especially, the different interest elasticities between small and large firms are the key moments to be fitted, which are hardly captured in alternative models. Second, I analyze the synchronization of large firms' lumpy investments using the impulse responses and the global equilibrium dynamics. Lastly, I quantitatively analyze the aggregate-level state dependence driven by the synchronized lumpy investments of large firms.

## 4.1 Calibration

In this section, I elaborate on how the model is fitted to the data and compare the fitness with alternative models. Table 2 reports the target and untargeted moments from the data and the simulated moments in the model. Table 3 reports the calibrated parameters given the fixed parameters reported in Table E10. In the simulation step, I use the non-stochastic method in Young (2010).

The target semi-elasticity of average investment is from Zwick and Mahon (2017). The cross-sectional semi-elasticity ratio is also from the same paper, which documents

Table 2: Fitted Moments

| Moments | Data | Model | Reference |
|---|---|---|---|
| **Targeted moments** | | | |
| Semi-elasticity of investment (%) | 7.20 | 6.60 | Zwick and Mahon (2017) |
| Cross-sectional semi-elasticity ratio (%) | 1.95 | 1.81 | Zwick and Mahon (2017) |
| Cross-sectional average of $i_{it}/k_{it}$ ratio | 0.10 | 0.10 | Zwick and Mahon (2017) |
| Cross-sectional dispersion of $i_{it}/k_{it}$ $(s.d.)$ | 0.16 | 0.16 | Zwick and Mahon (2017) |
| Cross-sectional average spike ratio | 0.14 | 0.14 | Zwick and Mahon (2017) |
| Positive investment rate | 0.86 | 0.86 | Winberry (2021) |
| Time-series volatility of $log(Y_t)$ | 0.06 | 0.07 | NIPA data (Annual) |
| **Untargeted moments** (all in yrs.) | | | |
| Average inaction periods | 6.38 | 7.88 | Compustat data |
| Dispersion of inaction periods | 4.87 | 5.65 | Compustat data |
| Average of lag diff. of inaction periods | 0.27 | 0.69 | Compustat data |
| Dispersion of lag diff. of inaction periods | 6.47 | 8.56 | Compustat data |

*Notes:* The data moments are from the sources specified in the reference column. The same sample restriction as in the empirical analysis applies to Compustat data. I use linearly detrended real GDP from the National Income and Product Accounts at the annual frequency for the aggregate output volatility.

that small firms' investments are around twice elastic as large firms towards the interest rate change. The cross-sectional average and dispersion of the investment-to-capital ratio and the average spike ratio are targeted to match the levels in Zwick and Mahon (2017) as in Winberry (2021) and Koby and Wolf (2020). Consistent with the literature, I define the spike ratio as the fraction of firms investing greater than 20% of the existing capital stock. The target of positive investment rate is from Winberry (2021). The positive investment rate is defined as the fraction of firms with an investment that is greater than 1% but smaller than 20% of existing capital stock. Only a negligible fraction of firms make a negative investment in both data and the model. To discipline the aggregate TFP-driven fluctuations in the model, I target the output volatility calculated from annual National Income and Product Accounts (NIPA) data.

In the model, variations in the fixed cost parameter and convex adjustment cost parameter lead to a sharply divergent effect on the dispersion of the investment rate

Table 3: Calibrated Parameters

| Parameters | Description | Value |
|---|---|---|
| **Internally calibrated parameters** | | |
| $\zeta$ | Fixed cost curvature | 3.500 |
| $\bar{\xi}$ | Fixed cost upperbound | 0.440 |
| $\mu^I$ | Capital adjustment cost | 0.760 |
| $\nu$ | Small investment range | 0.041 |
| $\sigma$ | Standard deviation of idiosyncratic TFP | 0.130 |
| $\sigma_A$ | Standard deviation of aggregate TFP shock | 0.025 |
| **Externally estimated parameters** | | |
| $\rho$ | Persistence of idiosyncratic TFP | 0.750 |

*Notes:* Parameters in the upper part of the table are calibrated to match the moments in Table 2. The persistence of idiosyncratic TFP is directly computed from fitting the estimated firm-level TFP (Compustat) into AR(1) process. The firm-level TFP is estimated following Ackerberg et al. (2015) using US Compustat data.

(investment-to-capital ratio), while both lowers the average investment rate. The dispersion of the investment rate increases in the fixed cost parameter, as the difference in the investment rate between extensive-margin adjusters and non-adjusters increases.[14] On the other hand, a higher convex adjustment cost mutes down the investment rate for all firms, leading to a lower dispersion in the investment rate. These two divergent variations, together with the average investment rate, identify the levels of the fixed and convex adjustment cost parameters.

The fixed cost curvature parameter $\zeta$ is identified from the cross-sectional semi-elasticity ratio between small and large firms. As $\zeta$ increases beyond unity, the large firms' interest elasticity decreases due to the lengthened $(S, s)$ band.[15] The calibrated level of $\zeta$ is 3.5, which I interpret as 3.5 establishments are involved per production line on average.[16]

[14]If a fixed cost is too high, the fraction of adjusters become too small to have meaningful contribution to the investment rate dispersion.

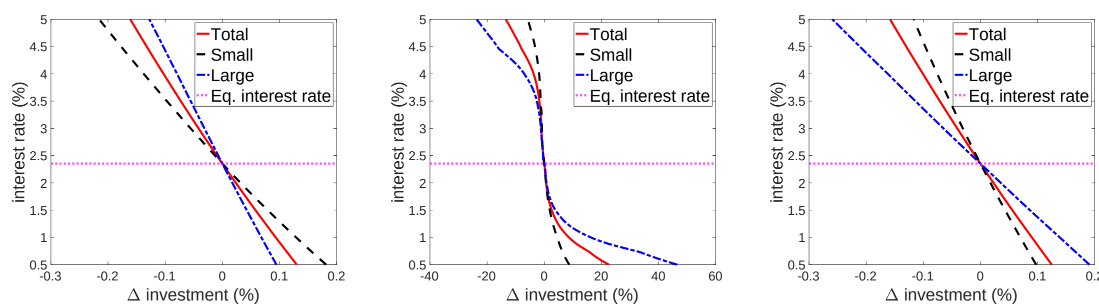
[15]A contemporaneous work, Gnewuch and Zhang (Gnewuch and Zhang) studies how monetary policy shock affects the distribution of investment rates, and they document that young firms are more sensitive to the shock than old firms. Regarding this elasticity difference, they conclude that the extensive-margin sensitivity plays a crucial role, consistent with the results in my paper.

[16]The related explanation is available in Section 3.1.2.

Moreover, the calibrated model matches untargeted moments, which are relevant indicators of firm dynamics. Average inaction is around 6.38 years in the data, and the one in the model is 7.88 years. The standard deviation of the inaction periods is 4.87 years in the data, and the model counterpart is 5.65 years.

As can be seen from Table 2, the baseline model can correctly capture the cross-sectional elasticity ratio between small and large firms. Therefore, the baseline model provides an appropriate framework for analyzing the role of large firms' investment in the dynamic stochastic general equilibrium. This is one of this paper's contributions to the literature, as the interest elasticity cross-section is not well-captured in the existing model framework.[17]

Figure 3: Semi-elasticities of investments across different models




(a) Baseline (b) Fixed only (c) Convex + Fixed

*Notes:* The figure plots the deviation of investment from the steady-state level when the interest rate changes for each different model. The vertical axis is the interest rate in per cent, and the horizontal axis is the percentage deviation from the steady-state investment. The horizontal dotted line indicates the equilibrium interest rate.

Figure 3 visualizes the large and small firms' interest elasticities for the baseline model (panel (a)), for a model with fixed cost only (panel (b)), and for a model with convex and fixed cost (panel (c)).[18] Throughout this paper, all the alternative models to the baseline are calibrated to sharply match the target moments except for the

[17] I theoretically and quantitatively point out that the cross-sectional ranking of the interest elasticities of investment between large and small firms is counterfactually flipped in existing model frameworks in Appendix A.

[18] The model with convex and fixed adjustment cost is a prototype of the models in Winberry (2021) and Koby and Wolf (2020).

cross-sectional semi-elasticity ratio.

In each panel, the vertical axis is the interest rate in per cent, and the horizontal axis is the percentage deviation from the steady-state investment. The horizontal dotted line indicates the equilibrium interest rate. As the interest rate decreases, all models' average deviation of investment from the steady-state increases. In the baseline model (panel (a)), the ranking of the interest elasticity across the firm-size group is consistent with the empirical patterns, as can be seen from the steeper curve of the large firms. However, in the model with convex and fixed adjustment cost (panel (c)), the large firms' average deviation of investment from the steady-state increases faster than small firms as the interest rate decreases. In the model with a fixed cost only (panel (b)), the interest elasticities of all groups are significantly higher than the ones in the other two models, as can be checked from the large-scale variation along the horizontal axis. The elasticities across the different models are reported and compared in Table B.5 in Appendix B.[19]

Finally, I compare the business cycle statistics implied in the baseline model with the aggregate-level data. The aggregate-level data at the annual frequency is from National Income and Product Accounts (NIPA) data, and the sample period starts from 1955. All the variables are in log and linearly detrended. Table E.11 reports the business cycle statistics from the data and the model. Among the statistics, the time-series volatility of the log output is the targeted moment in the calibration.

The correlations across the aggregate variables in the baseline model are well-matched with the observed level in the data. Notably, the autocorrelation of aggregate investment and the cross-correlation between aggregate investment and output are sharply matched, even if those are not the targeted moments. The model's moments are slightly lower than the observed level for the relative volatilities of consumption and investment.

[19]The comparison is made over the comparative statics of the fixed cost curvature parameter, while the other parameters are calibrated to match the same target moments.

## 4.2 Synchronization and the cross-section of the interest elasticity

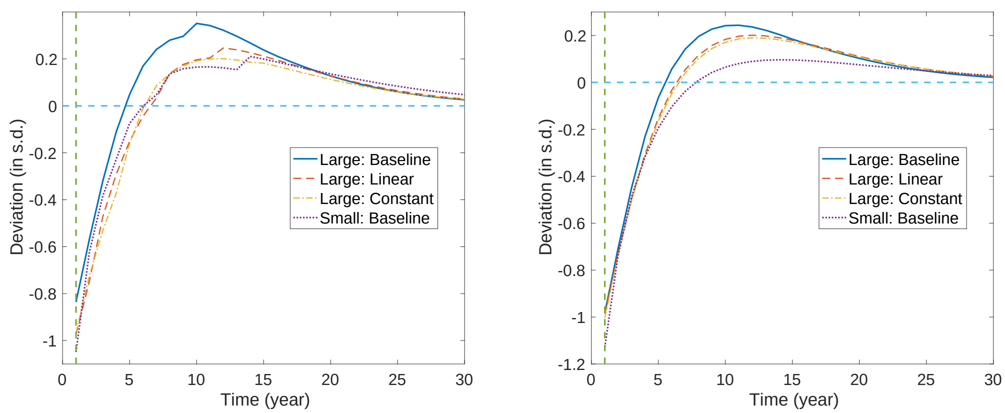
This section studies the synchronization of large firms' lumpy investment timings and how it affects the aggregate investment and other aggregate fluctuations. Figure 4 plots the impulse responses of the spike ratios (panel (a)) and $i_{it}/k_{it}$ (panel (b)) ratios in different models to a negative one-standard-deviation aggregate TFP shock. The impulse response is computed from the perfect-foresight transition path. The spike ratio is as defined in (1). Each variables' time path is normalized by its volatility (standard deviation) in the simulated path using the global nonlinear solution.

As shown in panel (a), the large firms' spike ratio in the baseline model (solid line) surges after a negative aggregate TFP shock, displaying a significant synchronization of the investment timings among large firms. This magnitude of the large firms' synchronization is substantially stronger in the baseline model (solid line) than the model with the linearly size-dependent fixed adjustment cost (dashed line) and the constant fixed adjustment cost (dash-dotted line). On the other hand, the small firms' synchronization (dotted line) is significantly weaker than the large firms' in the baseline model. Similar synchronization patterns are observed in the $i_{it}/k_{it}$ ratios (panel (b)).

This phenomenon happens because a negative aggregate TFP shock triggers a synchronous stop of large-scale investment projects. Then, as the TFP gradually recovers over the transition path, large firms tend to implement large-scale investments at a similar time to the others. If a general equilibrium effect is strong enough, these synchronized lumpy investment timings are supposed to be smoothed. However, the baseline's large firms are inelastic to the general equilibrium effect, so the synchronization survives even in the general equilibrium environment.

Next, I analyze this synchronization affects the firm and aggregate-level allocations over the business cycle. Table 4 reports the large firms' synchronization patterns over the business cycle across the models (the first block) and the corresponding high-order

Figure 4: Synchronization after a negative aggregate TFP shock




(a) Spike ratio

(b) Average $i_{it}/k_{it}$ ratio

*Notes:* The impulse response of spike ratios are obtained from the transition dynamics to the stationary equilibrium allocations after an unexpected negative one-standard-deviation aggregate TFP shock. Each time series are normalized by its standard deviation computed from the global solution.

moments of the aggregate investment (the second block) and outputs (the bottom block). All the moments are computed based on the global nonlinear solution, which I elaborate on in the following section.

The first two rows report the persistence of the time series of the spike ratio, which is obtained by fitting the series into the $AR(1)$ process. The spike ratio is most persistent in the baseline model, and its persistence decreases as the order of the size dependence in the fixed adjustment cost decreases. Once the general equilibrium effect is lifted by fixing the stochastic discount factor at the steady state (the second row), the persistence ranking is shuffled, and the model with the constant fixed adjustment cost features the strongest persistence. This result shows that the large firms' low sensitivity to the general equilibrium effect in the baseline model is the key to the persistent synchronization. On the other hand, the small firms' persistence of the synchronization is weakest in the baseline model (the third row).

The following rows in the first block show the high-order moments of the time series of the large firms' spike ratio. The spike ratio displays the largest positive skewness in the baseline model. That is, the large firms tend to be more synchronized in the baseline model than in the others. However, the model with the constant fixed adjustment cost displays the largest skewness in the partial equilibrium. This shows that the low sensitivity to the general equilibrium effect plays a crucial role in synchronizing large firms in the baseline model. For the kurtosis, regardless of the general equilibrium effect, the baseline model features the highest level, while the relative magnitude of the difference is not as substantial as the skewness differences.

Table 4: Large firms' synchronization and the aggregate dynamics

| | Baseline | Quadratic | Linear | Constant |
|---|---|---|---|---|
| ***Large firms' spike ratio*** | | | | |
| Persistence - GE | 0.769 | 0.745 | 0.737 | 0.735 |
| Persistence - PE | 0.751 | 0.746 | 0.752 | 0.762 |
| *cf.* Small firms' persistence - GE | 0.649 | 0.671 | 0.690 | 0.706 |
| Skewness - GE | 0.354 | 0.230 | 0.195 | 0.215 |
| Skewness - PE | 0.595 | 0.550 | 0.586 | 0.678 |
| Kurtosis - GE | 3.235 | 3.073 | 2.963 | 2.935 |
| Kurtosis - PE | 5.149 | 4.947 | 4.889 | 4.808 |
| ***Aggregate investment,*** $log(I_t)$ | | | | |
| Skewness | -0.124 | -0.087 | -0.063 | -0.055 |
| Kurtosis | 2.934 | 2.904 | 2.887 | 2.880 |
| ***Aggregate output,*** $log(Y_t)$ | | | | |
| Skewness | -0.017 | -0.009 | -0.001 | 0.009 |
| Kurtosis | 2.878 | 2.875 | 2.875 | 2.877 |

Notes: The table reports the firm-level and aggregate-level statistics in the baseline model, the models with quadratically and linearly size-dependant fixed costs, and the model with a constant fixed adjustment cost.

In the following block, I report the high-order moments of the aggregate investment over the business cycle. The aggregate investment displays the most negative skewness and the greatest kurtosis in the baseline model. A similar pattern is observed for the output dynamics reported in the bottom block.

The compared models share the same model structure and are sharply calibrated based on the same target moments except for the cross-section of the elasticity distribution across the large and small firms. Therefore, given the assumption that the target moments are correctly selected, the differences in the high-order moments in the aggregate allocations are driven by the differences in the only unmatched moment: the cross-section of the elasticity distribution. In the following sections, I elaborate further on how the aggregate investment and output become more negatively skewed when the large firms' interest elasticity is as low as the observed level.

## 4.3 The method for global solution

I solve the model with the aggregate uncertainty using a sequence-space-based global nonlinear solution method called the repeated transition method. Due to the nonlinear aggregate dynamics, it is difficult to correctly specifying the law of motion for the endogenous aggregate variable (the firm distribution) in the state-space-based method. So, I have contemporaneously developed the new methodology in Lee (2024), which solves nonlinear dynamic stochastic general equilibrium globally and accurately in the sequence space. The method is described in Appendix D.

## 4.4 Fragility after a surge of lumpy investments

Based on the global nonlinear equilibrium dynamics, I study how the synchronized investment timings of large firms affect the aggregate investment dynamics over the business cycle. First, I define a fragility index that captures the portion of large firms that have recently finished large-scale investments as follows:

$$Fragility_t := \frac{\sum \mathbb{I}\{s_{it} \leq \overline{s}\}\mathbb{I}\{k_{it} > \overline{k}\}}{\sum \mathbb{I}\{k_{it} > \overline{k}\}} \tag{37}$$

where $s_{it}$ is the time from the last lumpy investment of firm $i$; $\overline{s}$ is the threshold for $s_{it}$ to be counted as a recent lumpy investment; $\overline{k}$ is the size threshold of large firms. If a great fraction of large firms have just finished a large-scale investment, a relatively small fraction of large firms are willing to make a large-scale investment due to the presence of the fixed adjustment cost. Over the business cycle, the fluctuations in this index interplay with the exogenous TFP fluctuations, as the following analyses will conclude.

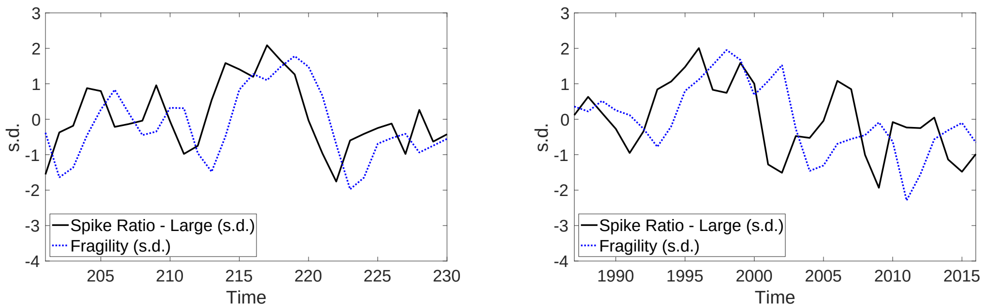
The median duration between two lumpy investments is 6 years in both the model and the data. In the regression that includes the fragility index, reported in Table 5, I found $\overline{s} = 3$ maximizes the fitness of the regression. Thus, in the following analysis, I use $\overline{s} = 3$ for the fragility index.[20]

It is worth noting that the fragility index is constructed from the readily observable micro-level variables: the past investment history of large firms, most of which are listed and subject to financial reporting regulations. Therefore, the index can be measured in a timely manner and can contribute to predicting the near future of aggregate investment. This feature is starkly contrasted with the existing indices in the literature based on the joint distribution between capital stock and productivity that is not directly observable (Caballero and Engel, 1993; Bachmann et al., 2013; Baley and Blanco, 2021).

Figure 5 shows the time series of fragility index and large firms' spike ratio in the simulation (panel (a)) and the data (panel (b)), where each series is normalized by the standard deviation and demeaned. In both panels, the time series of the spike ratio leads the fragility index by two to three years. As the average inaction takes around six years, around three years after a surge of lumpy investment (spike ratio), a trough is expected to arrive. By the definition of the fragility index, during this trough of lumpy investment, the index will rise, indicating only a small fraction of large firms are willing to make a lumpy investment. Therefore, the growth rate of the spike ratio and the fragility index tend to co-move in the opposite direction. Figure 6 is the scatter plot of the simulated time series where the horizontal axis is the fragility index normalized by the standard deviation, and the vertical axis is the growth rate

[20]The main results are not significantly affected by the choice of $\overline{s}$.

Figure 5: Time series of fragility indices in simulation and data




(a) Simulation (b) Data

*Notes:* Using the histogram method in Young (2010), firms are simulated for 5,000 periods (years) based on the recursive competitive equilibrium allocations. Panel (a) plots a part of the simulated allocations, and panel (b) plots the time series in the data. The solid line plots the large firms' spike ratio normalized by the standard deviation. The dotted line plots the time series of the fragility index normalized by the standard deviation.

of the large firms' spike ratio.[21] By fitting the relationship between the fragility and the growth rate of spike ratio into linear regression, I find the following relationship:

$$g_t^{SpikeRatio}(\%) = -\,1.984 * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.824 \tag{38}$$

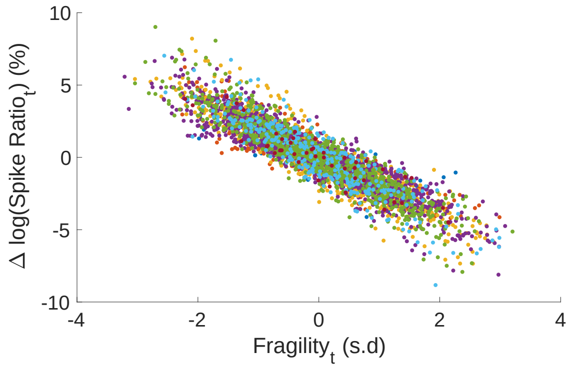
$$(0.012) \tag{39}$$

The relationship indicates that one standard deviation increase in fragility is negatively associated with the growth rate of the large firms' spike ratio by 1.984%. As can be seen from the high $R^2$ at 0.824, these two variables are tightly related along the business cycle. While the growth rate of the large firms' spike ratio is unknown ahead of period $t$, the fragility index is known before period $t$. Therefore, the fragility index has predictability for the one-period-ahead growth rate of the large firms' spike ratio.

I study how the fragility index fluctuations affect the sensitivity of aggregate investment growth to the output shock. Table 5 reports the regression result of the

[21]The past aggregate shock $A_{t-1}$ and the contemporaneous shock $A_t$ are controlled by taking out fixed effects. The different colors of the dots are for different combinations of $A_{t-1}$ and $A_t$.

Figure 6: Fragility index and the growth rate of the large firms' spike ratio




*Notes:* The vertical axis of the scatter plot is the spike ratio in percentage deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. Using the histogram method in Young (2010), firms are simulated for 5,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

following specification in both the model and the data separately for negative and positive output shocks:

$$g_t^I = \alpha + \beta^{Shock} OutputShock_t + \beta^{Fragility} OutputShock_t \times Fragility_t + \epsilon_t \tag{40}$$

where $g_t^I$ is the *aggregate* investment growth rate. $OutputShock_t$ is a shock in the log output, obtained from the residuals in the AR(1) fitting of the log output time series. The aggregate investment and output data are from National Income and Product Accounts data. In this specification, $OutputShock_t$ exogenously arrives at $t$, while the $Fragility_t$ is determined at $t-1$. Therefore, two variables are independent of each other.[22]

In Table 5, the coefficient estimates from the model and data are statistically indifferent, while each coefficient is statistically significant. When the fragility index increases by one standard deviation, the aggregate investment growth rate additionally decreases by around 1.5% and 2.4% for one-standard deviation negative output

[22]The measurement of output shock is subject to an endogeneity issue which will be discussed below.

Table 5: State-dependent sensitivity of the aggregate investment growth

| | Dependent variable: $g_t^I$ (p.p.) | | | |
|---|---|---|---|---|
| | (−) $OutputShock_t$ | | (+) $OutputShock_t$ | |
| | Model | Data | Model | Data |
| $OutputShock_t$ (*s.d.*) | 9.389 | 5.818 | 8.490 | 6.937 |
| | (0.066) | (1.338) | (0.064) | (1.221) |
| $OutputShock_t \times Fragility_t$ (*s.d.*) | 1.537 | 2.430 | -2.011 | -1.486 |
| | (0.042) | (1.311) | (0.045) | (0.495) |
| Constant | Yes | Yes | Yes | Yes |
| Observations | 2,296 | 16 | 2,705 | 18 |
| $R^2$ | 0.908 | 0.790 | 0.884 | 0.705 |
| Adjusted $R^2$ | 0.908 | 0.755 | 0.884 | 0.663 |

Notes: The dependent variable is the growth rate of aggregate investment. The independent variables are output shocks obtained from fitting output series into AR(1) process and the interaction between the output shock and the fragility index. The fragility index is based on the years from the last lumpy investment of large firms. The first two columns report the regression coefficients from the simulated data and Compustat data when the negative output shock hits. The third and fourth columns report the regression coefficients when the positive output shock hits. The numbers in the brackets are standard errors.

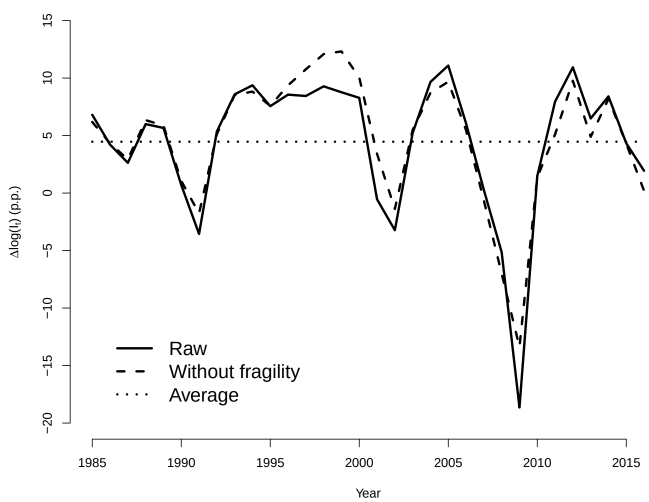
shock in the model and the data. In contrast, the aggregate investment growth rate increases less by around 2.0% and 1.5% for one-standard deviation positive output shock in the model and the data when the fragility index increases by one standard deviation. The amplifying effect of the negative output shock and the mitigating effect of the positive output shock under the high fragility state are all due to the missing lumpy investments of large firms. That is, after a surge of lumpy investments of large firms, the negative shock leads to a deeper drop in the aggregate investment, and the positive shock leads to only a mitigated increase in the aggregate investment.

In Table C.6, I report the additional regression results under different specifications. When the output shock is the only independent variable in the regression, around 73% and 52% of the investment growth rate variations are explained, respectively, for negative and positive shocks in the data. Once the fragility fluctuation is considered, $R^2$'s improve to 79% and 71%.

Using the estimate from the data in Table 5, I quantify the portion of the investment growth rate that is accounted for by the interaction between the output shock and the fragility index. Specifically, the fragility-adjusted investment growth rate $g_t^{adj,I}$ is obtained as follows:

$$g_t^{adj,I} = g_t^I - \widehat{\beta}^{Fragility} \cdot OutputShock_t \times Fragility_t. \tag{41}$$

Figure 7: Fragility-adjusted investment growth




Notes: The solid line is the aggregate investment growth rate from NIPA. The dashed line is the fragility-adjusted investment growth. The dotted line is the average level of the aggregate investment growth rate.

Figure 7 plots the time series of the raw aggregate investment growth rate (solid line) and the fragility-adjusted investment growth rate (dashed line). After the adjustment, the investment drops during the three recessions are mitigated. Table 6 compares the deviations from the average level for the raw and the fragility-adjusted investment growth rates in the recent three recessions of the sample period. Around 23% of the deviation from the average level is accounted for by the fragility effect during recessions. When the standard deviations of each time series are compared, around 30% of aggregate investment volatility can be explained by the interaction effect ($0.30 \approxeq 0.018/0.060$).

Table 6: Investment growth rates during the recessions

| | Distance between inv. growth rate and average: $\Delta g_t^I$ (*p.p.*) | | |
|---|---|---|---|
| | Raw data (NIPA) | Without fragility | Adjusted portion (%) |
| Recession-1991 | -8.019 | -6.239 | 22.197 |
| Recession-2001 | -7.695 | -5.852 | 23.951 |
| Recession-2009 | -23.112 | -17.847 | 22.780 |

*Notes:* The first column reports the investment growth rate (%) at recession years of 1991, 2001, and 2009 minus the average investment growth ($\approxeq 4.5\%$). The second column reports the adjusted investment growth rate after removing the predicted component from the fragility indices using the coefficients of Table 5. The third column reports the adjusted portion (%).

However, the results above are subject to an endogeneity issue. Specifically, the measured output shock is not fully exogenous because the fragility dynamics affects the future output realization. For example, a high fragility lowers the future capital stock, leading to a lower output. However, the current measurement of the output shock makes the fragility-driven output drop loaded on the shock magnitude.[23] This problem is hard to solve in a reduced-form approach due to the nonlinear dynamics of the fragility index.

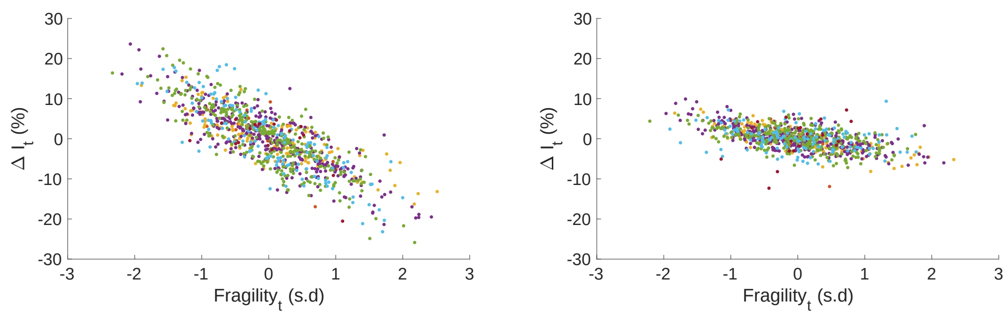
To sharply quantify the extra variation of aggregate investment driven by the fragility fluctuations without the endogeneity problem, I utilize the simulated path of the equilibrium allocations. Specifically, I collect only the periods where a negative one-standard-deviation TFP shock hits and compare the responsiveness of the aggregate investments across the different fragility index levels. Each period on the simulated path features a different fragility index level while the TFP shock's magnitude is fixed. Therefore, this experiment provides a setup to sharply quantify the relationship between the investment response variations and the fragility index fluctuations.[24]

[23]Therefore, it is likely that the role of the fragility index is underestimated.

[24]If there are any responsiveness differences, they are from the endogenous aggregate state, the

Figure 8 is the scatter plots of the state-dependent contemporaneous responses of the aggregate investment (vertical axis) along with the fragility variation (horizontal axis) for baseline model (panel (a)) and for a model with convex and constant fixed adjustment costs (panel (b)). The fragility indices are normalized by the standard deviation. The responses of the aggregate investments are demeaned and normalized in percentage deviation from the steady-state level. The prior and contemporaneous aggregate TFP levels $(A_{t-1}, A_t)$ are controlled by teasing out the pair-specific fixed effect.[25]

Figure 8: State-dependent responses of aggregate investment




(a) Baseline (b) Constant fixed adj. cost

*Notes:* The vertical axis of the scatter plot is the instantaneous response of the aggregate investment to a negative one-standard-deviation TFP shock in percentage for baseline model (panel (a)) and a model with convex and constant fixed adjustment costs (panel (b)), and the horizontal axis is the fragility index measured in the unit of standard deviation from the average. In each responses, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 5,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

As can be seen from the figure, there is a significant negative relationship between the contemporaneous response of the aggregate investment $\Delta I_t$ and the fragility index in the baseline model. The negative association is significantly reduced for the model with convex and constant fixed adjustment costs. By fitting the negative relationship

distribution of firms, as the exogenous states are identical. The fragility index is used as a sufficient statistic for the endogenous aggregate state in this experiment.

[25]The different colors of dots represent the different fixed-effect groups.

in the baseline model into linear regression, I obtain the following result:

$$\varDelta I_t \text{ (\% w.r.t. s.s. response)} = -\underset{(0.173)}{7.875} * Fragility_t \ (s.d.) + \epsilon_t, \quad R^2 = 0.677 \tag{42}$$

When the fragility index increases by one standard deviation, a contemporaneous response of the aggregate investment to the negative one-standard-deviation shock is amplified by 7.875% compared to the steady-state response. On the other hand, in the model with convex and constant fixed adjustment cost, the coefficient is -2.193, of which the absolute magnitude is significantly lower than the baseline level. This shows that the baseline model, where large firms' inelastic adjustment carries the nonlinearity, leads to a greater endogenous fragility than the canonical model with convex and constant fixed adjustment costs.

Lastly, I study how the fragility affects output dynamics through the firm-level investment channel. When a negative aggregate TFP shock hits, a high fragility index additionally reduces the future capital stock due to the amplified aggregate investment response. The reduced capital stock leads to the extra drop in the output in the following period. Taking the same steps as above, I analyze how the future output changes along with the fragility variations when a negative one-standard-deviation TFP shock hits:

$$\varDelta Y_{t+1} \text{ (p.p. w.r.t. s.s.)} = -\underset{(0.008)}{0.322} * Fragility_t \ (s.d.) + \epsilon_t, \quad R^2 = 0.631 \tag{43}$$

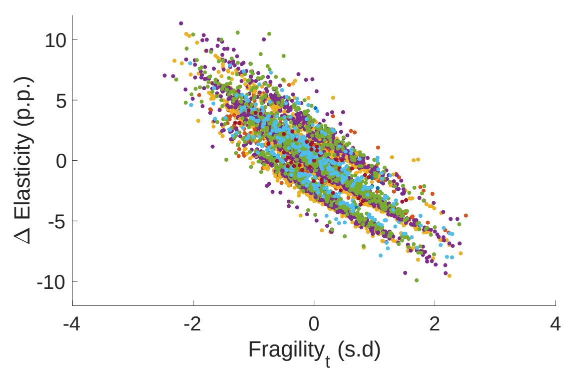
If the fragility increases by one standard deviation, the future output decreases by 0.322 percentage points through the amplified aggregate investment response to the negative aggregate shock.

## 4.5 Policy implication: State-dependent interest elasticity of aggregate investment

In this section, I discuss the policy implications of the fluctuations of the fragility index over the business cycle. In the baseline model economy, the aggregate investment features a strong history dependence.[26] This history dependence not only affects the aggregate investment's response to the TFP shock but affects its elasticity to the interest rate change.

To study how the aggregate investment responds differently to the same interest shock depending on the fragility state, I hit the economy at each period on the simulated path with an unexpected interest rate shock and compute the contemporaneous response under the partial equilibrium. I compute the elasticity by taking an average of the elasticities from positive and negative 1 % interest rate shocks to account for the asymmetric responses.

Figure 9: State-dependent semi-elasticities of aggregate investment




*Notes:* The vertical axis of the scatter plot is the semi-elasticity of aggregate investment in percentage point deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. For each elasticity, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 5,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

[26] Given that the aggregate states include all the relevant information from history, the state dependence and the history dependence are interchangeable in the model.

Figure 9 is the scatter plot of the interest elasticities of the aggregate investment in relation to the fragility state. The horizontal axis is the fragility index normalized by the standard deviation; the vertical axis is the interest elasticity in percentage deviation from the steady-state level.[27] According to the figure, there is a significant negative relationship between the fragility and the interest elasticity of aggregate investment. By fitting the relationship into linear regression, I obtain the following result:

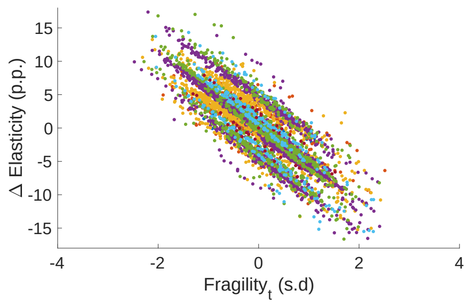
$$\Delta Elasticity_t \text{ (\% w.r.t. s.s.)} = -\underset{(0.032)}{3.350} * Fragility_t \ (s.d) + \epsilon_t, \quad R^2 = 0.689 \tag{44}$$

One standard deviation increase in the fragility index decreases the interest elasticity of aggregate investment by around 3.022% compared to the steady-state level. The intuitive explanation for the result is that when the fragility index is high, there are not many large firms that can flexibly participate in and out of large-scale investment. Therefore, the aggregate investments' responsiveness to the interest rate change decreases in a high-fragility state.

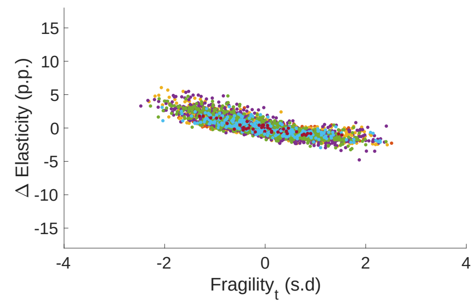
To verify that large firms drive interest elasticity fluctuations in aggregate investment, I compute the elasticity variations separately for large and small firms. Figure 10 is the scatter plot of interest elasticities along with the fragility variation for large (panel (a)) and small firms (panel (b)). The negative relationship between the fragility index and the elasticity is significantly stronger in large firms. When two different elasticities are fitted into linear regression, the following relationship is obtained:

[27]The prior and contemporaneous aggregate TFP levels ($A_{t-1}, A_t$) are controlled by teasing out the pair-specific fixed effect, and the different colors of dots represent the different fixed-effect groups.

Figure 10: State-dependent semi-elasticities of investments: Decomposition




(a) Large firms




(b) Small firms

*Notes:* The vertical axis of the scatter plots is the semi-elasticity of large (panel (a)) and small (panel (b)) firms' investment in percentage point deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. For each elasticity, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 5,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

$$\Delta Elasticity_t^{Large} \text{ (\% w.r.t. s.s.)} = \underset{(0.054)}{-5.257} * Fragility_t \; (s.d) + \epsilon_t, \quad R^2 = 0.655 \tag{45}$$

$$\Delta Elasticity_t^{Small} \text{ (\% w.r.t. s.s.)} = \underset{(0.013)}{-1.244} * Fragility_t \; (s.d) + \epsilon_t, \quad R^2 = 0.639 \tag{46}$$

When the fragility index increases by one standard deviation, large firms' investment elasticity decreases by around 5.257%. On the other hand, the same variation in the fragility index decreases small firms' elasticity by 1.244%, and the difference is statistically significant. This result shows that large firms dominantly drive the stark negative relationship between the interest elasticities of the aggregate investments and the fragility index.

The analysis above implicitly shows that if the fragility index is high, the monetary policy would not effectively operate through the firm-level investment channel.

Given there were recessions in the recent periods that happened in the time of high fragility, the policy implication echoes Tenreyro and Thwaites (2016) that conventional monetary policies have been less powerful during recessions especially through the business investment channels. Moreover, my paper adds to the related literature by providing an endogenous mechanism for the state dependence of monetary policy effectiveness. Importantly, the fragility index is a forward-looking variable and can be easily measured using readily observable large firms' data. Therefore, the fragility index can potentially contribute to the optimal monetary policy design in practice.

# 5 Concluding remarks

This paper analyzes the endogenous state dependence in the aggregate investment dynamics driven by the synchronized lumpy investments of large firms. An economy becomes substantially more fragile to a negative aggregate shock after a surge of large firms' lumpy investments than it would otherwise be. I show this is due to the interest inelasticity of the large firms' investments, which generates persistently synchronized investment timings even under the general equilibrium. The economic significance of this channel is quantified in a heterogeneous-firm real business cycle model in which the cross-section of the semi-elasticities of firm-level investment is matched with the empirical estimates. In the model, the aggregate investment features a significant state dependence in the interest elasticities driven by fragility index fluctuations. This implies that after a surge of large firms' lumpy investments, the effectiveness of monetary policy can substantially fall due to the lowered interest elasticity of the aggregate investment.

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