Appendix: For Online Publication

Contents

A Firm-level interest-elasticity in the existing model and the data

In this section, I theoretically and quantitatively investigate how the existing models predict the interest-elasticities of large and small firms. Especially, I study the role of convex and fixed adjustment costs on the cross-section of the interest-elasticities.

A.1 A two-period model with a convex adjustment cost

Consider a firm that is given capital stock k and productivity z. For simplicity, I assume a firm lives only for two periods. A firm's investment is subject to a standard convex adjustment cost. A firm produces business output using a concave production function, $f(z,k) = zk^{\alpha}$.[1] The idiosyncratic productivity follows a Markov chain, $z'|z \sim \mathit{\Gamma}$. Then, the problem of firm-level investment can be summarized as follows:

$$\max_{I} \quad -I - \frac{\mu}{2}\left(\frac{I}{k}\right)^2 k + q\mathbb{E}_z z'((1-\delta)k + I)^{\alpha}$$

where I is the investment; μ is the convex adjustment cost parameter; z' is the future productivity; $\alpha \in (0,1)$ is the span of control parameter; q is the discount factor. A variation in q is equivalent to the change in the interest rate. The first-order condition with respect to investment I leads to the following inter-temporal optimality condition:

$$1 + \mu\left(\frac{I^*}{k}\right) = q\mathbb{E}z'\alpha((1-\delta)k + I^*)^{\alpha-1} \tag{1}$$

Taking a log on both sides and using an approximation of $log(1+x) \approxeq x$ for small

[1] For simplicity, I assume the optimal labor demand is implicitly considered in the production function.

x, Equation (1) can be reduced into the following form:[2]

$$\mu\left(\frac{I^*}{k}\right) \approxeq log(q) + log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) + (\alpha-1)\left(\frac{I^*}{k} - \delta\right)$$

Then, I re-arrange the terms to obtain the following equation:

$$\frac{I^*}{k} \approxeq A(\mu)log(q) + B(\mu, k) \tag{2}$$

where $A(\mu) = \frac{1}{\mu+(1-\alpha)}$ and $B(\mu, k) = A(\mu)(log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) - (\alpha-1)\delta)$. It is worth noting that the second term on the right-hand side, $B(\mu, k)$ does not play any role in the response of investment to the change in q. Equation (2) provides rich implications about the response of investment to the change in q.

First, Equation (2) implies that the investment-to-capital ratio positively (negatively) responds to an increase in q (decrease in r). This is because an increase in q makes future production more profitable to a firm, leaving greater investment motivation for the firm. This is formally proven without an approximation in Lemma 3 in Appendix F.1.

Second, Equation (2) implies that the firm-level interest-elasticity increases in size. To clearly present the implication, I multiply k on both sides of Equation (2), and I take partial derivatives with respect to k and q on both sides to get

$$\frac{\partial^2}{\partial q \partial k} I^* \approxeq A(\mu)\frac{\partial^2}{\partial q \partial k} k log(q) = \underbrace{A(\mu)}_{>0} \underbrace{\frac{1}{q}}_{>0} > 0.$$

The inequality above holds for any $\mu > 0$. In a model without the convex adjustment cost, the same equation could be derived with $\mu = 0$. In the following statement, I formally show that the interest sensitivity of investment, $\frac{\partial log I^*}{\partial q}$, increases in size k.

[2] I use the following sub-step: $log(k(1-\delta) + I^*) = log\left(k\left((1-\delta) + \frac{I^*}{k}\right)\right) = log(k) + log\left((1-\delta) + \frac{I^*}{k}\right) \approxeq log(k) - \delta + \frac{I^*}{k}$

Proposition 1 (Size-monotonicity in the interest-elasticity).
Given $\mu > 0$*, the following inequalities holds:*

$$
\begin{aligned}
(i) \quad & \frac{\partial}{\partial k}\left(\frac{\partial k^*}{\partial q}\right) > 0 \text{ for } \forall k > 0 \\
(ii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial log k^*}{\partial q}\right) > 0 \text{ for } \forall k > 0 \\
(iii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) > 0 \text{ for } \forall k > 0 \\
(iv) \quad & \frac{\partial}{\partial k}\left(\frac{\partial log I^*}{\partial q}\right) > 0 \text{ if } I^* > 0.
\end{aligned}
$$

Proof. See Appendix F.1 ■

The result of Proposition 4 is contradictory to the empirical findings in Zwick and Mahon (2017). According to the paper, the large firms' interest-elasticities are significantly smaller than those of the small firms.[3] From the fact that $A(\mu)$ decreases in μ, a large μ can mitigate the counterfactually diverged elasticity ranking, but it cannot flip the order. Therefore, a model with convex adjustment cost only cannot be a proper model to study the role of large firms' investments on the business cycle. This theoretical prediction will be quantitatively verified in the following section's comparison of the elasticities in the infinite period problem.

Third, Equation (2) implies that the elasticity of firm-level investment decreases in μ, as $A(\mu)$ decreases in μ. This is a consistent theoretical prediction with the computational outcomes in Winberry (2021) and Koby and Wolf (2020), which argues that the convex adjustment cost helps the elasticity of the average investment be lowered to the empirical estimate. Intuitively, the higher the convex adjustment cost parameter, the higher the marginal cost of adjustment, leaving the marginal response to a change in q costlier. This prediction is formally proved in the following proposition without the approximation:

[3] Zwick and Mahon (2017) defines large firms as the top 30% firms in the sales distribution and the small firms as bottom 30% in the sales distribution. Under this definition, the elasticity ratio between small and large firms is around 2.

Proposition 2 (Elasticity dampening effect)**.**

Given $\mu > 0$, if $I^* > 0$, the following statements hold:

$$
\begin{aligned}
(i) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial k^*}{\partial q}\right) < 0 \\
(ii) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial log k^*}{\partial q}\right) < 0 \\
(iii) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial I^*}{\partial q}\right) < 0 \\
(iv) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial log I^*}{\partial q}\right) \begin{cases} \leq 0 & \text{if } \frac{1}{1-\delta} \geq \mu \\ > 0 & \text{if } \frac{1}{1-\delta} < \mu \end{cases}.
\end{aligned}
$$

Proof. See Appendix F.1 ■

In the last statement of Proposition 5, the response of investment to q in per cent can increase in μ if μ is sufficiently large. This is due to the convex adjustment parameter's dominant shrinking force on the level of the denominator in $\frac{1}{I^*}\frac{\partial I^*}{\partial q} = \frac{\partial log I^*}{\partial q}$.

To sum up, the convex adjustment cost is helpful for controlling the average elasticities of firm-level investment. However, it does not help flip the counterfactual ranking of elasticities between large and small firms.

A.2 A two-period model with a fixed adjustment cost

Now I consider a two-period model where a firm needs to pay a fixed adjustment cost $\xi \sim Unif[0, \overline{\xi}]$ to invest.[4] If a firm does not pay the fixed cost, the firm's capital stock simply depreciates at the rate of δ. Except that the convex adjustment cost is replaced by the fixed cost, the model is the same as the one in the previous section.

I define $\xi^*(k, q)$ as the threshold of adjustment with respect to the shock realiza-

[4] The random shock assumption is following Khan and Thomas (2008).

tion, ξ as follows:

$$\xi^*(k,q) := \underbrace{-I^* + q\mathbb{E}_z z'((1-\delta)k + I^*)^\alpha}_{\text{Net benefit of capital adjustment}} - \underbrace{q\mathbb{E}_z z'((1-\delta)k)^\alpha}_{\text{Net benefit of inaction}}.$$

Thus, a firm invests if $\xi^*(k,q) > \xi$. Then, I define $\psi(k,q)$ as a probability of adjustment as follows:

$$\psi(k,q) := \frac{\min\{\xi^*(k,q), \overline{\xi}\}}{\overline{\xi}}.$$

The ex-ante investment, $\widehat{I}$ can be characterized in the following form:

$$\widehat{I} = \psi(k,q)I^*$$

where I^* is unconstrained optimal level of investment that satisfies the first-order condition (1) under $\mu = 0$.[5] The interest-elasticity of the ex-ante investment $\widehat{I}$ depends on how both $\psi(k,q)$ and I^* respond to a change in q. Then, I define a cutoff $\widehat{k}(q) = \frac{k^*}{1-\delta}$, where a firm with k greater than this threshold makes a negative investment. In Lemma 5 of Appendix F.3, I formally show that such $\widehat{k}$ uniquely exists given q. The following decomposition holds for $\forall k \in (0, \widehat{k}(q))$:[6]

$$\begin{aligned} \frac{\partial}{\partial q} log(\widehat{I}) &= \frac{\partial}{\partial q} log(\psi(k,q)) && \text{[Extensive-margin responsiveness]} \\ &+ \frac{\partial}{\partial q} log(I^*) && \text{[Intensive-margin responsiveness]} \end{aligned}$$

The average response of firm-level investment per cent is additively separable into extensive and intensive-margin responsiveness. As the intensive margin has been

[5] For the simplicity of the proofs, I assume $\mu = 0$ for the model with a fixed cost.

[6] We focus only on firms that make positive investments as in the empirical specification in Zwick and Mahon (2017). Therefore, the extensive-margin transition from non-adjuster to adjuster is ignored. However, the transition in the opposite direction is counted.

studied in the previous section, I focus on the extensive-margin responsiveness in this section.

In Lemma 6, I show that $\psi(k, q)$ increases in q. This is because a higher discount factor leads to a greater discounted future profit, leaving the marginal benefit of investment greater. Therefore, firms respond to an interest rate change in both extensive and intensive margin in the same direction. However, when it comes to the rankings of extensive-margin elasticity over the size, the theoretical prediction in the extensive margin diverges from the one in the intensive margin.

To understand the cross-sectional ranking of the extensive-margin interest-elasticities, I decompose the partial derivative of $log(\psi(k, q))$ with respect to q and k for $\forall k \in (0, \widehat{k}(q))$ as follows:

$$\begin{aligned}
\frac{\partial}{\partial k}\frac{\partial}{\partial q} log(\psi(k,q)) &= \frac{\partial}{\partial k}\frac{\partial}{\partial q} log(\xi^*(k,q)) \\
&= \frac{\partial}{\partial k}\frac{\frac{\partial}{\partial q}\xi^*(k,q)}{\xi^*(k,q)} \qquad (3) \\
&= \underbrace{-\frac{\frac{\partial \xi^*(k,q)}{\partial k}\frac{\partial \xi^*(k,q)}{\partial q}}{\xi^*(k,q)^2}}_{\text{Denominator effect } (>0)} + \underbrace{\frac{\frac{\partial^2 \xi^*(k,q)}{\partial q \partial k}}{\xi^*(k,q)}}_{\text{Direct effect } (<0)} \qquad (4)
\end{aligned}$$

In the following proposition, I determine the sign of each component in the decomposition.

Proposition 3 (The effect of the firm size and the price on the adjustment probability)**.**
For $\forall k$ *s.t.* $\xi^*(k, q) < \overline{\xi}(q)$*,*

$$\frac{\partial \xi^*(k,q)}{\partial k}\frac{\partial \xi^*(k,q)}{\partial q} < 0 \text{ and } \frac{\partial}{\partial k}\frac{\partial}{\partial q}\xi^*(k,q) < 0.$$

Proof. See Appendix F.3. ■

According to Proposition 6, the first term of the right-hand side in Equation (4) is positive while the second term is negative. In other words, as the size of a firm increases, the magnitude of the change in the adjustment probability (the numerator of (3)) decreases, but at the same time, the adjustment probability also decreases (the denominator of (3)). Therefore, the ranking of the investment response in the extensive margin in per cent across the firm size cannot be determined.

To sum up, the fixed cost affects the ex-ante investment response through the extensive margin. When measured in the absolute value, the ranking of the interest-elasticity in the extensive margin decreases in firm size. However, when measured in per cent, the ranking becomes unclear due to the countervailing force from the interest rate effect on the level of the adjustment probability. The ex-ante investment elasticity is determined by both the intensive and extensive margin responsiveness. From the previous section, the ranking of the intensive margin responsiveness is counterfactually flipped in the model. Therefore, to correct the counterfactual ranking by including the fixed cost, the extensive-margin elasticity needs to be substantially lowered for large firms in the model with both convex and fixed adjustment costs. In the next section, I quantitatively investigate the ranking of the large and small firms' interest-elasticities under the infinite-period models with different adjustment costs.

A.3 Comparison of the semi-elasticities across models

This section compares the semi-elasticities of firm-level investment across different models. I consider three different models: 1) a model with fixed cost (Khan and Thomas, 2008); 2) a model with convex adjustment cost; 3) a model with both fixed and convex adjustment cost (Winberry, 2021). As each model is based on the description of the reference paper, I abstract the detailed explanation of each model. The models are calibrated to match the cross-sectional average of the investment-to-

[6] The model with convex adjustment cost is a simpler version of the model with both fixed and convex adjustment cost, where the fixed cost is discarded. The models do not include the habit formation in the household utility differently from Winberry (2021).

capital ratio and the cross-sectional average spike ratio.[7] Additionally, for the model with both fixed and convex adjustment costs, I matched the cross-sectional dispersion of the investment-to-capital ratio.

Table A.1 reports the semi-elasticities of firm-level investments for different groups across different models. The elasticities are measured by the average contemporaneous change in the firm-level investment in per cent from the steady-state when the interest rate changes by 1%.[8] In particular, I calculate the average between the elasticity measured when the interest rate increases by 1% and the one measured when the interest rate decreases by 1% to address the asymmetry in the responses to the positive and negative interest rate shocks. The average interest-elasticity of group $j \in \{All, Small, Large\}$ is defined as follows:

$$Elasticity_{jt} = \frac{\int_{\{I_{ijt}>0\}} \Delta log(I_{ijt}\psi_{ijt} + I^{c}_{ijt}(1-\psi_{ijt}))d\Phi_j}{\Delta r_t}$$

where ψ_{ijt} is the extensive-margin adjustment probability; I_{ijt} is then investment after fixed cost is paid and I^{c}_{ijt} is the investment when the fixed cost is unpaid; Φ_j is the joint distribution of firms conditional on group j.

The elasticity of the spike ratio of group j is defined as the average contemporaneous change in the fraction of firms investing greater than 20% of the existing capital stock when the interest rate changes by 1%.

$$Elasticity^{SpikeRatio}_{jt} = \frac{\int_{\{I_{ijt}>0\}} \Delta\mathbb{I}\left\{\frac{I_{ijt}\psi_{ijt}+I^{c}_{ijt}(1-\psi_{ijt})}{k_{ijt}} > 0.2\right\} d\Phi_j}{\Delta r_t}$$

According to Table A.1, in any of the three models, the interest-elasticity of investment is greater in large firms than in small firms. By including both fixed and convex adjustment costs, the counterfactual elasticity divergence is slightly mitigated,

[7]The target moment is the same as in the baseline model calibration, which is reported in Table 1.

[8]The elasticity is measured in the partial equilibrium as in Winberry (2021) and Koby and Wolf (2020).

as can be seen from the lowest small-to-large ratio, 0.62. Still, the ratio is substantially lower than the empirical ratio of 1.95, as reported in the fourth column. This is due to a dominant intensive-margin impact that is proved in Proposition 4.

Consistent with findings from the literature, the average interest-elasticity is in the empirically-supported range when convex adjustment cost is included. When both fixed and convex adjustment costs are included, the average elasticity is around 5, satisfying an empirical upper bound of 7.2 from Zwick and Mahon (2017).

Table A.1: Semi-elasticity comparison across models

	Fixed	Convex only	Convex + Fixed	Data
Investment				
All	382.73	18.18	5.01	7.2
Small	313.76	14.8	4.32	
Large	481.93	21.79	6.99	
S/L ratio	0.65	0.68	0.62	1.95
Spike ratio				
All	25.61	1.97	1.04	
Small	37.97	0.74	1.24	
Large	16.39	1.35	1.14	
S/L ratio	2.32	0.55	1.09	

Notes: The semi-elasticities of investment variables are computed from contemporaneous investment response to an interest rate change in the partial equilibrium. To address the asymmetry between responses to the positive and negative interest rate shocks, I report the average responses to the positive 1% and negative 1% interest rate changes.

I also analyze the spike ratio's elasticity as this elasticity can be directly measured in the data and can guide us on the missing component in the model for capturing the cross-section of the empirically supported interest-elasticities. Based on the comparison of the model-implied elasticities and the data estimates in the next section, I discuss how the models need to be improved.

When a model includes only a fixed cost, the large firms' spike ratio elasticities are 2.6 times lower than those of small firms. In contrast, the convex adjustment cost flips the ranking of elasticity of the spike ratio, leaving large firms to become relatively more elastic than small firms. When both adjustment costs are considered, small and large firms' elasticities of the spike ratios are a similar level.

A.4 Firm-level interest-elasticities of investments in the data

In this section, I empirically estimate the elasticity of firm-level investment using firm-level balance sheet data and monetary policy shocks in the literature. Prior research papers in the literature have provided the well-identified interest-elasticities of firm-level investments, but those estimates are not informative enough to pin down the missing component in the existing model frameworks. For this, I estimate the elasticities of small and large firms' spike ratios to develop a model with realistic firm-level investment.

I estimate the following regression separately for large firms and small firms:

$$f(k_{it}, k_{it+1}) = \beta MP_t + \alpha_i + \alpha_{sy} + Controls_{it} + \epsilon_{it}$$

where MP_t is the monetary policy shock; α_i is firm fixed effect; α_{sy} is sector-year fixed effect. The control variables include lagged current account (ACT_{t-1}), lagged total debt (DT_{t-1}), and operating profit ($OIBDP_t$) normalized by lagged total asset (AT_{t-1}), log of lagged capital stock, and log of employment (EMP_t). The standard errors are two-way clustered across firms and years.

Table A.2 reports the coefficient of monetary policy shock (MP_t) for large and small firms across different choices of dependent variables.[9] As can be seen from the first two columns, the elasticity of the investment is significantly lower in large firms than in small firms. This is consistent with the empirical results in the literature and contradictory to the model-implied elasticities in the previous section. Also, the sensitivity of the spike ratio is significantly lower in large firms than small firms, as reported in the third and fourth columns.[10]

[9] I check the robustness of result using a different cutoff 10% than 20% in Table B.3.

[10] Two estimates are statistically different under the significance level of 0.05.

Table A.2: Investment sensitivities to the monetary policy shocks

	Dependent variables:			
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$	
	L	S	L	S
$MP_{Tight,t}$	-2.201	-7.025	-0.870	-2.072
	(0.606)	(2.41)	(0.366)	(0.676)
Obs.	29,400	7,903	29,400	7,903
R^2	0.929	0.791	0.603	0.558
Firm FE	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes

Notes: The independent variables include monetary policy shocks, fixed effects (firm and sector-year), and firm-level control variables (lagged current account (ACT_{t-1}), lagged total debt (DT_{t-1}), and operating profit ($OIBDP_t$) normalized by lagged total asset (AT_{t-1}), log of lagged capital stock, and log of employment (EMP_t)). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by firm and year.

The differences in the elasticities in Table A.1 and Table A.2 sharply indicate that the existing models with fixed and convex adjustment costs cannot correctly capture the ranking of interest-elasticities between large and small firms. Therefore, a new model is needed to study the role of large firms' investments over the business cycle. Then, a question still remains about which component of the existing model needs to be improved to capture the empirical relationship. There are broadly two options: lowering either intensive or extensive margin elasticities of large firms.

On this issue, the elasticity of spike ratio gives an answer. I set the model with both fixed and convex adjustment costs as a benchmark model. From the comparison of the interest-elasticities of spike ratios between the benchmark model and the data, the large firms' spike ratio needs to be less elastic, and small firms' spike ratio needs to be more elastic than in the benchmark model to match the empirical counterpart. Therefore, the extensive-margin elasticity needs to be improved from the benchmark model. In the following section, I develop a heterogeneous-firm real business cycle model where the elasticities of investments and spike ratios are at the empirically-supported level through the modification in the extensive-margin investment patterns of the benchmark model.

B Monetary policy shock

I construct an exogenous monetary policy shock following Ottonello and Winberry (2020) and Jeenas (2018). The monetary policy shock is obtained by time aggregating high-frequency monetary policy shock identified from the unexpected jump (drop) in the federal funds rate during a 30-minutes window around the FOMC announcement.[11] To capture the unexpected component in the federal funds rate, I use the change in the rate implied by the current-month federal funds futures contract. All the data on the timings of the FOMC announcement and the high-frequency surprise are from Gurkaynak et al. (2005) and Gorodnichenko and Weber (2016). The sample period covers from March 1990 until December 2009. I follow the convention that the positive monetary policy shock is an unexpected increase in the federal funds futures rate, so it implies the contractionary monetary policy.

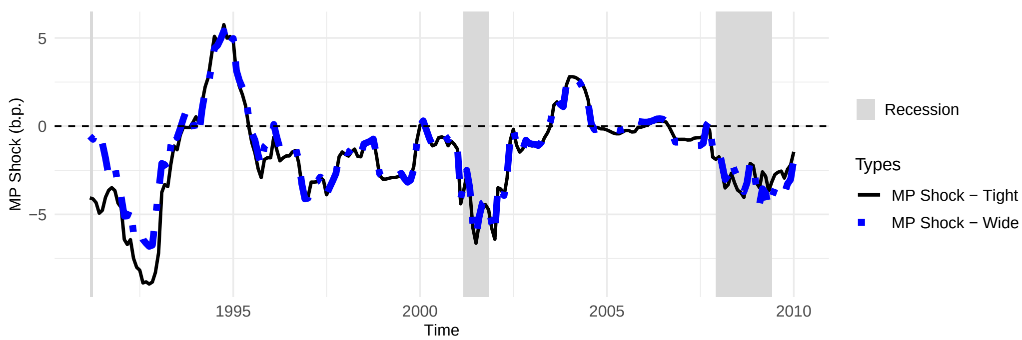
To match the data frequency between the firm-level data and the monetary policy shock, I time aggregate the monetary policy shocks. Specifically, I compute the one-year backward weighted average monetary policy shock at each firm's financial year end. The weight of each surprise is determined by the number of days between the corresponding FOMC announcement and the next FOMC announcement.[12] If the next FOMC announcement was made after the financial year end, the days are counted until the financial year end. This data joining process matches a firm's balance sheet information and the monetary policy shock at the same financial year. The weighted moving average monetary policy shock is plotted in Figure B.1.

In the regression analysis, large firms and small firms are defined as in Section 2. I consider log of firm-level investment, $log(I_{it})$ and binary indicator of investment greater than 20% of existing capital stock, $\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$ as dependent variables in the regression.

[11] The result is robust over the choice of a wider window (one-hour window) as reported in Table B.4.

[12] A higher weight is assigned for a monetary policy shock when there was greater amount of time for a firm to respond to the shock (Ottonello and Winberry, 2020).

Figure B.1: One-year moving average monetary policy shock: March 1990 ∼ December 2009



Notes: The monetary policy shocks are obtained by time aggregating high-frequency monetary policy shocks identified from the unexpected jump (drop) in the federal funds rate during 30-minutes (Tight) and one-hour (Wide) windows around the FOMC announcement. To capture the unexpected component in the federal funds rate, I use the change in the rate implied by the current-month federal funds futures contract. All the data on the timings of the FOMC announcement and the high-frequency surprise are from Gurkaynak et al. (2005) and Gorodnichenko and Weber (2016).

B.1 Investment elasticities to the monetary policy shocks: Full tables

Table B.3: Investment sensitivity to the monetary policy shocks with the narrow window

	Dependent variables:									
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.1\}$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$		$log(I_{it}) \mid_{\frac{I_{it}}{k_{it}}>0.1}$		$log(I_{it}) \mid_{\frac{I_{it}}{k_{it}}>0.2}$	
	L	S	L	S	L	S	L	S	L	S
$MP_{Tight,t}$	-2.201	-7.025	-0.656	-2.993	-0.870	-2.072	-0.936	-2.317	-0.246	-3.512
	(0.606)	(2.41)	(0.363)	(0.688)	(0.366)	(0.676)	(0.676)	(1.668)	(0.912)	(2.187)
Obs.	29,400	7,903	29,400	7,903	29,400	7,903	19,524	5,039	11,181	3,643
R^2	0.929	0.791	0.596	0.562	0.603	0.558	0.954	0.865	0.96	0.895
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The dependent variable of the probit regression is the indicator of lumpy investment. The independent variables include monetary policy shocks, fixed effects (sector, year, and sector-year), and firm-level control variables (lagged total debt (DT), lagged current account (ACT), lagged size (AT), and sales (Sale) growth). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by sector and year.

Table B.4: Investment sensitivity to the monetary policy shocks with the wide window

	Dependent variables:									
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.1\}$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$		$log(I_{it}) \big\|_{\frac{I_{it}}{k_{it}}>0.1}$		$log(I_{it}) \big\|_{\frac{I_{it}}{k_{it}}>0.2}$	
	L	S	L	S	L	S	L	S	L	S
$MP_{Wide,t}$	-2.178	-6.583	-0.643	-2.856	-0.762	-1.870	-0.850	-1.703	-0.333	-3.400
	(0.662)	(2.604)	(0.383)	(0.73)	(0.377)	(0.728)	(0.698)	(1.835)	(0.956)	(2.44)
Obs.	29,400	7,903	29,400	7,903	29,400	7,903	19,524	5,039	11,181	3,643
R^2	0.929	0.791	0.596	0.562	0.603	0.558	0.954	0.865	0.96	0.895
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The dependent variable of the probit regression is the indicator of lumpy investment. The independent variables include monetary policy shocks, fixed effects (sector, year, and sector-year), and firm-level control variables (lagged total debt (DT), lagged current account (ACT), lagged size (AT), and sales (Sale) growth). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by sector and year.

B.2 Semi-elasticity: The full table

Table B.5 compares the semi-elasticities of investment to the interest rate change for different models. The first column is the result from the calibrated baseline model; the second is from the model with fixed adjustment cost only; the third is from the model with convex adjustment cost only; the fourth is from the benchmark model with convex and fixed adjustment costs; the fifth is from the model with convex and linearly size-dependent fixed adjustment costs; the last is from the model with convex and quadratic size-dependent fixed adjustment costs.[13]

The first four rows of the table report the elasticities of investment conditional on $I_{ijt} > 0$ where i is a firm index, j is a size group indicator, and t is the time subscript; the next four rows report the extensive-margin elasticities; the following four rows report the intensive-margin elasticities; the last four rows report the spike ratio elasticities.[14] The elasticity of investment is as defined in section A.3. I calculate the average between the elasticity measured when the interest rate increases by 1% and the one measured when the interest rate decreases by 1% to address the asymmetry in the responses to the positive and negative interest rate shocks.

In the table, there are two additional interest-elasticities to be defined. The extensive-margin elasticity of group $j \in \{All, Small, Large\}$ is defined as the average contemporaneous change in the firm-level investment driven by extensive-margin probability changes in per cent from the steady-state when the interest rate changes by 1%. Therefore, the investment policy functions are fixed at the steady-state level, while the extensive-margin probabilities deviate from the steady-state:

$$Elasticity^{ext}_{jt} = \frac{\int_{\{I_{ijt}>0\}} \Delta log(I^{ss}_{ijt}\psi_{ijt} + I^{ss,c}_{ijt}(1-\psi_{ijt}))d\Phi_j}{\Delta r_t}$$

[13] Each model is calibrated to match the same moments as in the baseline calibration, except for the cross-sectional elasticity ratio. For the models with fixed adjustment cost only and with a convex adjustment costs only, I did not match the cross-sectional dispersion of i_{it}/k_{it} as these models have one less parameter than the others.

[14] Following Zwick and Mahon (2017), I define the elasticity conditional on $I_{ijt} > 0$ as investment elasticity.

where ψ_{ijt} is the extensive-margin adjustment probability; I_{ijt} is then investment after fixed cost is paid and I^c_{ijt} is the investment when the fixed cost is unpaid; Φ_j is the joint distribution of firms conditional on group j. The intensive-margin elasticity of group j is defined as the average contemporaneous change in the firm-level investment driven by investment magnitude changes in per cent from the steady-state when the interest rate changes by 1%. Therefore, the extensive-margin probability is fixed at the steady-state level, while the investment policy functions deviate from the steady-state.

$$Elasticity^{int}_{jt} = \frac{\int_{\{I_{ijt}>0\}} \Delta log(I_{ijt}\psi^{ss}_{ijt} + I^c_{ijt}(1-\psi^{ss}_{ijt}))d\Phi_j}{\Delta r_t}.$$

According to Table B.5, the aggregate investment elasticity of 6.63 is consistent with the empirical findings in Zwick and Mahon (2017); the small-to-large elasticity ratio of 2.13 is also close to the empirical level. The response of investment is further decomposed into the extensive and intensive margins. Each margin accounts for an almost identical portion of the total response in the baseline model. However, the small-to-large elasticity ratios are greater in the extensive margin response than the intensive margin.

As can be seen from the columns other than the second and the third in Table B.5, the aggregate investment elasticities are well-matched with the empirical level once we consider both convex and fixed adjustment costs. Especially, the inclusion of convex adjustment cost dramatically dampens the aggregate elasticity, as can be seen from the aggregate elasticity in the third column compared to that of the second column (Winberry, 2021; Koby and Wolf, 2020). Again, this is consistent with the theoretical prediction of Proposition 5.

The cross-sectional elasticity ratio between small and large in other models than the baseline cannot match the empirical estimate of 1.95 from Zwick and Mahon (2017). However, as the fixed cost becomes size-dependent and as the intra-firm interdependence across establishments rises, the cross-sectional elasticity ratio increases.

Table B.5: Semi-elasticity of investment across the models and the decomposition

	Baseline	Fixed	Convex	Benchmark	Linear-Fixed	Quad.-Fixed
Investment						
All	6.63	382.73	18.18	5.01	5.49	5.87
Small	9.85	313.76	14.8	4.32	5.41	7.06
Large	4.62	481.93	21.79	6.99	6.38	4.65
S/L ratio	2.13	0.65	0.68	0.62	0.85	1.52
Ext. margin						
All	3.34	84.63		2.58	2.94	3.37
Small	4.63	90.72		2.89	3.6	4.38
Large	1.99	74.28		2.27	2.29	2.35
S/L ratio	2.32	1.22		1.27	1.57	1.86
Int. margin						
All	3.28	152.6	18.18	2.43	2.54	2.5
Small	5.21	95.89	14.8	1.43	1.81	2.67
Large	2.62	244.77	21.79	4.71	4.09	2.29
S/L ratio	1.99	0.39	0.68	0.3	0.44	1.17
Spike ratio						
All	1.3	25.61	1.97	1.04	1.27	1.33
Small	2.36	37.97	0.74	1.24	1.67	2.42
Large	0.98	16.39	1.35	1.14	0.91	1.06
S/L ratio	2.4	2.32	0.55	1.09	1.84	2.29

Notes: The semi-elasticities of investment variables are computed from contemporaneous response to an interest rate change in the partial equilibrium. To address the asymmetry between responses to the positive and negative interest rate shocks, I report the average responses to the positive 1% and negative 1% interest rate changes.

From the middle and lower part of the table, the size-dependence and the intra-firm linkages increase not only the extensive-margin S/L ratio but the intensive-margin S/L ratio. This is due to the selection effect on those large firms that remain to adjust despite the higher fixed cost.

C State-dependent sensitivity of the aggregate investment growth: Full table

Table C.6: State-dependent sensitivity of the aggregate investment growth: full table

	Dependent variable: $\Delta log(I_t)$ (p.p.)							
	$(-)$ $OutputShock_t$				$(+)$ $OutputShock_t$			
	Model		Data		Model		Data	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Shock_t$	9.506	9.218	7.193	5.818	9.008	8.928	4.483	6.937
	(0.187)	(0.145)	(1.213)	(1.338)	(0.182)	(0.141)	(1.123)	(1.221)
$Shock_t \times Fragility_t$		1.753		2.430		-1.861		-1.486
		(0.094)		(1.311)		(0.103)		(0.495)
Constant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	507	507	16	16	495	494	18	18
R^2	0.837	0.904	0.730	0.790	0.730	0.900	0.515	0.705
Adjusted R^2	0.837	0.903	0.709	0.755	0.709	0.900	0.483	0.663

Notes: The dependent variable is the growth rate of aggregate investment. The independent variables are output shocks obtained from fitting output series into AR(1) process and the interaction between the output shock and the fragility index. The fragility index is based on the years from the last lumpy investment of large firms. The first two columns report the regression coefficients from the simulated data when the negative output shock hits. The third and fourth columns report the regression coefficients using Compustat data when the negative output shock hits. The fifth and sixths columns report the regression coefficients from the simulated data when the positive output shock hits. The last two columns report the regression coefficients using Compustat data when the positive output shock hits. The numbers in the brackets are standard errors.

D Solution method: The repeated transition method

This section explains the solution method I use to compute the recursive competitive equilibrium. I use the repeated transition method, which I concurrently developed for the computation of nonlinear aggregate dynamics under aggregate uncertainty in Lee (2023). As highlighted in Bachmann et al. (2013), the strong general equilibrium effect significantly contributes to the linearity in the dynamics of aggregate allocations. However, once the model captures realistic interest-elasticity, the general equilibrium effect is necessarily weakened, leaving the aggregate dynamics highly nonlinear. Due to this highly nonlinear aggregate dynamics in general equilibrium, there are two layers of difficulties in using the algorithm of Krusell and Smith (1998). The first is difficulty in choosing a sufficient statistics for the aggregate dynamics. The model's nonlinear aggregate dynamics might not be sufficiently explained by the moves in aggregate capital stocks, unlike Khan and Thomas (2008). The second difficulty is in setting the parametric form in the law of motion. This problem interacts with the former difficulty because even correctly chosen sufficient statistics would not give accurate computation results due to the wrong specification of the functional form of the law of motion. Therefore, it is almost impossible to jointly identify the correct sufficient statistics and functional form in the law of motion.

The repeated transition method departs from the state-space-based approach, so it does not require a researcher to specify the law of motion. The method is based on the ergodic theorem: if a simulated path is long enough, all the possible equilibrium allocations should be realized on the path. Then, by simply utilizing the realized allocations including the value functions, the method accurately constructs the rationally expected future value functions at each time on the simulated path.[15]

Using this method, I compute the predicted aggregate allocations, which the time series of the simulated aggregate allocations almost perfectly converges to. And this

[15] As the method relies on the dynamics over the simulated aggregate shock path, it is similar to Boppart et al. (2018). However, the repeated transition method departs from the perfect foresight and is a global solution method.

time series of the predicted aggregate allocations is not based on a parametric form of the law of motion in a state-space representation. Figure D.2 compares the time series of the predicted allocations and the simulated allocations.[16] In the figure, panel (a) shows the predicted aggregate dynamics and the simulated dynamics of the marginal utility, p_t. These two dynamics converged to each other with an extremely small error, as can be seen in the solid line in panel (c). However, if the dynamics of simulated marginal utility are fitted into the log-linear law of motion in the contemporaneous capital stock K_t, the prediction error can become substantially large as in the dashed line in panel (c). A similar pattern is observed in the aggregate dynamics of aggregate capital stock K_t in panel (b). The simulated and predicted paths for K_t are computed at extremely high accuracy with the repeated transition method, while the log-linear fitting leads to a significant prediction error as in panel (d).

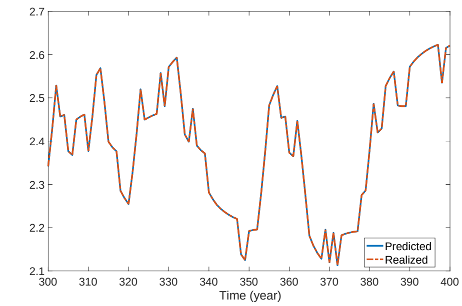
Then, I compare the fitness of different specifications of the law of motion by fitting the equilibrium dynamics into each of them.[17] Table D.7 and Table D.8 report the fitness of the different laws of motion of p_t and K_t, respectively. When the law of motion includes only a log of contemporaneous capital stock K_t (specification (1)), the prediction errors remain large, indicating the nonlinear nature of the equilibrium dynamics.[18] However, once the law of motion includes the fragility index in the law of motion (specification (2)), which I define in Section 4.3, the fitness significantly improves for the dynamics of p_t. However, it does not make a significant change in the fitness for the dynamics of K_t. Finally, if the law of motion includes contemporaneous and lagged capital stocks up to three lags in a non-parametric form (specification (3)), the fitness substantially improves from the basic log-linear specification for both p_t and K_t.

[16] This figure is the fundamental accuracy plot suggested in Den Haan (2010).

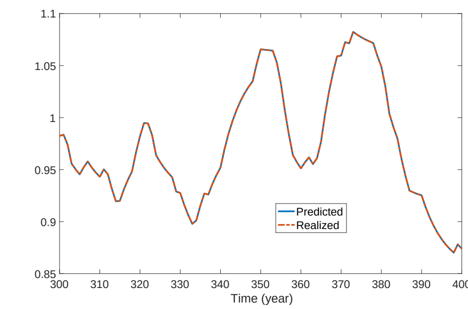
[17] I compare only the fitness of the law of motion to the converged dynamics of equilibrium allocations. Therefore, if the model is solved based on each of the laws of motion, the implied dynamics might display even greater prediction errors than the reported level.

[18] Den Haan (2010) points out that a slight deviation in R^2 from unity such as $R^2 = 0.995$ can imply a substantially large prediction error and significant nonlinearity.

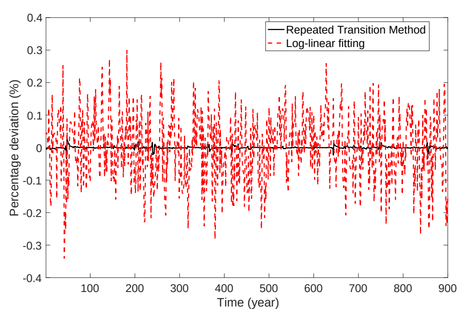
Figure D.2: Aggregate fluctuations in the marginal utility and the aggregate capital stock



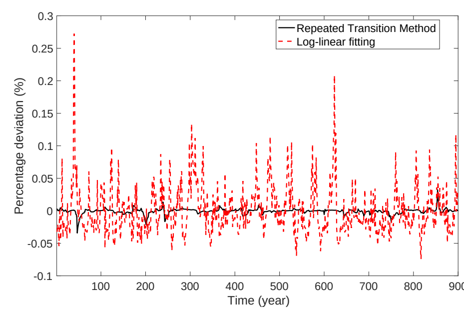
(a) Marginal utility, $p_t = \frac{1}{c_t}$



(b) Aggregate capital stock K_t



(c) Prediction error in marginal utility



(d) Prediction error in aggregate capital stock

Notes: Panel (a) plots the rationally expected path and the simulated path of the marginal utility. Panel (b) plots the rationally expected path and the simulated path of the aggregate capital stock. Panel (c) plots the prediction errors in the marginal utility path from the repeated transition method and the log-linear fitting. Panel (d) plots the prediction errors in the aggregate capital stock path from the repeated transition method and the log-linear fitting.

Table D.7: The fitness comparison across the different law of motions: p_t

	Dependent variables: $log(p_t)$								
	R^2			$max(\|error\|)(\%)$			$mean(\|error\|)(\%)$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
A_1	0.9965	0.9995	0.9999	0.1960	0.0722	0.0393	0.0619	0.0225	0.0098
A_2	0.9951	0.9994	0.9999	0.2613	0.0936	0.0423	0.0756	0.0235	0.0117
A_3	0.9958	0.9993	0.9999	0.2793	0.1394	0.0676	0.0662	0.0263	0.0128
A_4	0.9945	0.9994	0.9999	0.3261	0.0900	0.0468	0.0657	0.0248	0.0115
A_5	0.9966	0.9992	0.9999	0.1954	0.1146	0.0669	0.0532	0.0266	0.0084

Notes: The table reports R^2, the maximum absolute prediction error, and the mean absolute prediction error by different law of motion (columns) and aggregate states (rows). Specification (1) includes a constant and log of contemporaneous capital stock as a independent variable; Specification (2) includes a constant, log of contemporaneous capital stocks, and log of fragility index as independent variables; Specification (3) includes constant and contemporaneous and lagged capital stocks up to three lags in a non-parametric form as independent variables.

Table D.8: The fitness comparison across the different law of motions: K_{t+1}

	Dependent variables: $log(K_{t+1})$								
	R^2			$max(\|error\|)(\%)$			$mean(\|error\|)(\%)$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
A_1	1.0000	1.0000	1.0000	0.0793	0.0785	0.0233	0.0150	0.0141	0.0057
A_2	0.9999	0.9999	1.0000	0.1253	0.1295	0.0402	0.0230	0.0237	0.0082
A_3	0.9999	0.9999	1.0000	0.2286	0.2248	0.0481	0.0210	0.0207	0.0090
A_4	0.9999	0.9999	1.0000	0.2503	0.2508	0.0784	0.0254	0.0244	0.0095
A_5	0.9998	0.9998	1.0000	0.1994	0.1886	0.0409	0.0259	0.0227	0.0076

Notes: The table reports R^2, the maximum absolute prediction error, and the mean absolute prediction error by different law of motion (columns) and aggregate states (rows). Specification (1) includes a constant and log of contemporaneous capital stock as a independent variable; Specification (2) includes a constant, log of contemporaneous capital stocks, and log of fragility index as independent variables; Specification (3) includes constant and contemporaneous and lagged capital stocks up to three lags in a non-parametric form as independent variables.

E Additional tables and figures

E.1 Conditional heteroskedasticity: Regression result

Table E.9: Residual volatility of the aggregate investment and spike ratios

	Dependent variable: $log(\hat{\sigma}_t)$	
	Large	Non-large
$\overline{spike}_{t-1}$ (%)	0.337	0.077
	(0.138)	(0.074)
Constant	−4.131	−2.317
	(1.290)	(1.270)
Observations	35	35
R^2	0.154	0.032
Adjusted R^2	0.128	0.002

Notes: The dependent variable is the log absolute value of the residuals from fitting the aggregate investment to capital ratio into AR(4) process. The independent variables are the past average spike ratio, $\overline{spike}_{t-1}$, and the intercept.

$\overline{spike}_{t-1}$ is defined as follows:

$$\overline{spike}_{t-1} := \frac{1}{J}\sum_{j=0}^{J-1} SpikeRatio_{t-1-j}$$

$$SpikeRatio_t := \frac{\#\text{Extensive-margin adjustment}_t}{\#\text{Firms}_t}$$

where J is the number of past years to be included in the average. In the reported result, I use $J = 3$. The result is robust over $J = 1, 2, 4$.

E.2 Fixed parameters

Table E.10: Fixed Parameters

Parameters	Description	Value
Firm-side Fundamentals		
α	Capital share	0.2800
γ	Labor share	0.6400
δ	Depreciation rate	0.0900
Household		
β	Discount factor	0.9770
η	Labor disutility parameter	2.4000
Aggregate TFP Process		
ρ_A	Persistence of aggregate TFP	0.8145

Notes: The fixed parameters are chosen at the level widely used in the relevant literature. The household labor disutility parameter is set at the level where the total labor supply becomes around one-third in the equilibrium. The persistence of aggregate TFP is fixed at 0.8145 following Bachmann et al. (2013).

E.3 Business cycle statistics

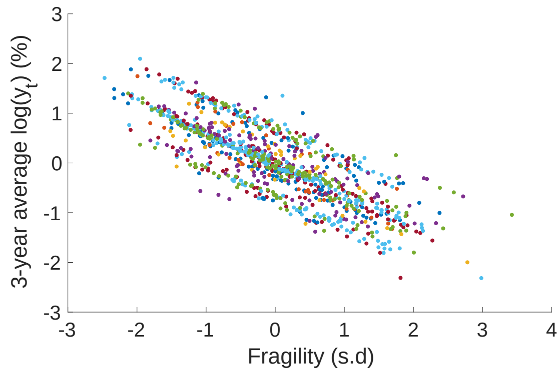
Table E.11: Business cycle statistics

	Data	Model
$corr(Y_t, Y_{t-1})$	0.941	0.843
$corr(I_t, I_{t-1})$	0.742	0.742
$corr(C_t, C_{t-1})$	0.954	0.903
$corr(I_t, Y_t)$	0.795	0.796
$corr(L_t, Y_t)$	0.898	0.771
$corr(C_t, Y_t)$	0.978	0.980
$sd(Y_t)$	0.060	0.065
$sd(I_t)/sd(Y_t)$	1.976	1.809
$sd(C_t)/sd(Y_t)$	0.945	0.823

Notes: The business cycle statistics are obtained from the simulated data using the dynamic stochastic general equilibrium allocations. 5,000 firms are simulated for 1,000 periods (years). All the variables are in log and linearly detrended. The data counterpart is from National Income and Product Accounts (NIPA) data.

E.4 State-dependent responses of aggregate output

Figure E.3: State-dependent responses of aggregate output



Notes: The vertical axis of the scatter plot is the instantaneous response of the aggregate output to a negative one-standard-deviation TFP shock in percentage point, and the horizontal axis is the fragility index measured in the unit of standard deviation from the average. In each responses, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

F A theory of the interest-elasticity and the firm size: Proofs

F.1 A model with convex adjustment cost : Propositions and proofs

Proposition 4 (Size-monotonicity in the interest-elasticity)**.**
Given $\mu > 0$, the following inequalities holds:

$$
\begin{aligned}
(i) \quad & \frac{\partial}{\partial k}\left(\frac{\partial k^*}{\partial q}\right) > 0 \; for \; \forall k > 0 \\
(ii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial log k^*}{\partial q}\right) > 0 \; for \; \forall k > 0 \\
(iii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) > 0 \; for \; \forall k > 0 \\
(iv) \quad & \frac{\partial}{\partial k}\left(\frac{\partial log I^*}{\partial q}\right) > 0 \; if \; I^* > 0.
\end{aligned}
$$

Proof.

$$log\left(1+\mu\left(\frac{I^*}{k}\right)\right) = log\left(q\mathbb{E}z'\alpha\right) + (\alpha-1)log\left((1-\delta)k + I^*\right)$$

The equation above holds for all possible k and q. I take a partial derivative with respect to q for both sides of the equation.

$$\left(\frac{\mu}{k+\mu I^*}\right)\frac{\partial I^*}{\partial q} = \frac{1}{q} + (\alpha-1)\frac{1}{(1-\delta)k+I^*}\frac{\partial I^*}{\partial q}$$

Rearranging the terms, I get the following equations:

$$\begin{aligned}\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{(1-\delta)k+I^*}\right)\frac{\partial I^*}{\partial q}&=\frac{1}{q}\\ \left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)\frac{\partial I^*}{\partial q}&=\frac{1}{q}\end{aligned} \tag{5}$$

where $k^*=(1-\delta)k+I^*$. Then, I take a log for both sides.

$$log\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)+log\left(\frac{\partial I^*}{\partial q}\right)=-log(q)$$

The equation above holds for all possible k and q. I take a partial derivative with respect to k for both sides of the equation.

$$\frac{1}{\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}}\left(-\frac{\mu}{(k+\mu I^*)^2}\left(1+\mu\frac{\partial I^*}{\partial k}\right)-\frac{1-\alpha}{k^{*2}}\frac{\partial k^*}{\partial k}\right)+\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)=0$$

Therefore,

$$\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)=\frac{1}{\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}}\left(\frac{\mu}{(k+\mu I^*)^2}\left(1+\mu\frac{\partial I^*}{\partial k}\right)+\frac{1-\alpha}{k^{*2}}\frac{\partial k^*}{\partial k}\right).$$

Due to Lemma 1, all the terms on the right-hand side are positive except for $\left(1+\mu\frac{\partial I^*}{\partial k}\right)$. Thus, the following statement holds:

$$\left(1+\mu\frac{\partial I^*}{\partial k}\right)>0 \implies \frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)>0.$$

Going back to the inter-temporal optimality condition, I multiply k in the both sides to have

$$k+\mu I^*=q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k.$$

Then, I take a log and a partial derivative with respective to k. It leads to

$$\begin{aligned}\frac{1+\mu\frac{\partial I^*}{\partial k}}{k+\mu I^*} &= \frac{(\alpha-1)}{k^*}\frac{\partial k^*}{\partial k}+\frac{1}{k}\\ &= \frac{1}{k}\left((\alpha-1)\frac{k}{k^*}\frac{\partial k^*}{\partial k}+1\right)\\ &= \frac{1}{k}\left(1-(1-\alpha)\frac{\partial logk^*}{\partial logk}\right).\end{aligned}$$

From Lemma 2, $\frac{\partial logk^*}{\partial logk} < 1$. Also I assume $\alpha < 1$. Therefore, the right-hand side is positive. The denominator on the left-hand side is also positive because $k + \mu I^* = q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k > 0$. Therefore, $\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right) > 0$. Then,

$$(iii) \quad \frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) = \left(\frac{\partial I^*}{\partial q}\right)\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right) > 0.$$

The right-hand side is positive because $\frac{\partial I^*}{\partial q} > 0$ from equation (5). This result is formally stated in Lemma 3. As $\frac{\partial I^*}{\partial q} = \frac{\partial k^*}{\partial q}$, I conclude (i) $\frac{\partial}{\partial k}\left(\frac{\partial k^*}{\partial q}\right) > 0$ for $\forall k > 0$.

Now I will prove (ii) $\frac{\partial}{\partial k\partial q}log(k^*) > 0$ and (iv) $\frac{\partial}{\partial k\partial q}log(I^*) > 0$.

From Equation (5), the following is true:

$$\left(\frac{\mu}{k+\mu I^*}k^*+\frac{1-\alpha}{k^*}k^*\right)\frac{1}{k^*}\frac{\partial k^*}{\partial q} = \frac{1}{q}$$

As $\frac{\partial}{\partial k}\left(\frac{\partial log(k^*)}{\partial q}\right) = \frac{\partial}{\partial k}\frac{1}{k^*}\left(\frac{\partial k^*}{\partial q}\right)$,

$$\left(\frac{\mu}{\frac{k}{k^*}+\mu\frac{I^*}{k^*}}+1-\alpha\right)\frac{\partial log(k^*)}{\partial q} = \frac{1}{q}.$$

From $I^* = k^* - (1-\delta)k$,

$$\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)\frac{\partial log(k^*)}{\partial q} = \frac{1}{q}.$$

I take the partial derivatives with respect to k on both sides.

$$\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)\frac{\partial log(k^*)}{\partial q}+\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)\frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q}=0.$$

By rearranging the terms, I obtain

$$\underbrace{\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)}_{>0}\frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q}=-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)\underbrace{\frac{\partial log(k^*)}{\partial q}}_{>0}.$$

From Lemma 3, $\frac{\partial log(k^*)}{\partial q}=\frac{1}{k^*}\frac{\partial k^*}{\partial q}=\frac{1}{k^*}\frac{\partial I^*}{\partial q}>0$. Also, $\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)>0$. Therefore, the sign of $\frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q}$ is equal to the sign of $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)$. Then, I investigate the sign of $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)$ as follows:

$$\begin{aligned}-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right)&=\left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\left(\frac{1}{k^*}-\frac{(k-\mu(1-\delta))\frac{\partial k^*}{\partial k}}{(k^*)^2}\right)\\&=\left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\frac{1}{k^*}\left(1-\frac{(k-\mu(1-\delta))\frac{\partial k^*}{\partial k}}{k^*}\right)\\&=\left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\frac{1}{k^*}\left(1-\left(1-\mu(1-\delta)\frac{1}{k}\right)\frac{k}{k^*}\frac{\partial k^*}{\partial k}\right)\\&=\underbrace{\left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)}_{>0}\frac{1}{k^*}\underbrace{\left(1-\overbrace{\left(1-\mu(1-\delta)\frac{1}{k}\right)}^{<1}\overbrace{\frac{\partial logk^*}{\partial logk}}^{>0,<1}\right)}_{>0}>0.\end{aligned}$$

From Lemma 1 and Lemma 2, $0 < \frac{\partial logk^*}{\partial logk} < 1$. Thus,

$$(ii) \quad \frac{\partial}{\partial k} \frac{\partial log(k^*)}{\partial q} > 0.$$

Similarly, we can derive the following equation from Equation (5),

$$\underbrace{\left(\frac{\mu}{\frac{k}{I^*} + \mu} + (1-\alpha)\frac{I^*}{k^*} \right)}_{>0} \frac{\partial}{\partial k} \frac{\partial log(I^*)}{\partial q} = -\frac{\partial}{\partial k} \left(\frac{\mu}{\frac{k}{I^*} + \mu} + (1-\alpha)\frac{I^*}{k^*} \right) \underbrace{\frac{\partial log(I^*)}{\partial q}}_{>0}.$$

As $I^* > 0$, $\frac{\partial log(I^*)}{\partial q} = \frac{1}{I^*}\frac{\partial I^*}{\partial q} > 0$ from Lemma 3. And $\left(\frac{\mu}{\frac{k}{I^*}+\mu} + (1-\alpha)\frac{I^*}{k^*} \right) > 0$, as $I^* > 0$. Thus, the sign of $\frac{\partial}{\partial k}\frac{\partial log(I^*)}{\partial q}$ is equal to the sign of $-\frac{\partial}{\partial k} \left(\frac{\mu}{\frac{k}{I^*}+\mu} + (1-\alpha)\frac{I^*}{k^*} \right)$.

$$\begin{aligned}
-\frac{\partial}{\partial k} \left(\frac{\mu}{\frac{k}{I^*} + \mu} + (1-\alpha)\frac{I^*}{k^*} \right) &= -\frac{\partial}{\partial k} \left(\frac{\mu}{\frac{k}{I^*} + \mu} + \frac{(1-\alpha)}{1 + \frac{(1-\delta)k}{I^*}} \right) \\
&= \left(\frac{\mu}{\left(\frac{k}{I^*} + \mu\right)^2} \right) \left(\frac{\partial}{\partial k} \frac{k}{I^*} \right) + \frac{1-\alpha}{\left(1 + \frac{(1-\delta)k}{I^*}\right)^2}(1-\delta) \left(\frac{\partial}{\partial k} \frac{k}{I^*} \right) \\
&= \underbrace{\left(\left(\frac{\mu}{\left(\frac{k}{I^*} + \mu\right)^2} \right) + \frac{1-\alpha}{\left(1 + \frac{(1-\delta)k}{I^*}\right)^2}(1-\delta) \right)}_{>0} \left(\frac{\partial}{\partial k} \frac{k}{I^*} \right)
\end{aligned}$$

And we can drive the sign of $\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right)$ as follows:

$$\begin{aligned}
\left(\frac{\partial}{\partial k} \frac{k}{I^*} \right) &= \frac{1}{I^*} \left(1 - \frac{k}{I^*} \frac{\partial I^*}{\partial k} \right) \\
&= \frac{1}{I^*} \left(1 - \frac{k}{I^*} \left(\frac{\partial k^*}{\partial k} - (1-\delta) \right) \right) \\
&> \frac{1}{I^*} \left(1 - \frac{k}{I^*} \left(\frac{k^*}{k} - (1-\delta) \right) \right) \quad \left(\because \frac{\partial logk^*}{\partial logk} < 1, \text{ Lemma } 2 \right) \\
&= \frac{1}{I^*} \left(1 - \frac{k}{I^*} \left(\frac{I^*}{k} \right) \right) = 0
\end{aligned}$$

Thus, $\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right) > 0$, so $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k}{I^*}+\mu} + (1-\alpha)\frac{I^*}{k^*}\right) > 0$. Therefore,

$$(iv) \quad \frac{\partial}{\partial k}\left(\frac{\partial logI^*}{\partial q}\right) > 0 \text{ if } I^* > 0.$$

■

Proposition 5 (Elasticity dampening effect)**.**

Given $\mu > 0$, if $I^* > 0$, the following statements hold:

$$
\begin{aligned}
(i) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial k^*}{\partial q}\right) < 0 \\
(ii) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial logk^*}{\partial q}\right) < 0 \\
(iii) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial I^*}{\partial q}\right) < 0 \\
(iv) \quad & \frac{\partial}{\partial \mu}\left(\frac{\partial logI^*}{\partial q}\right) \begin{cases} \leq 0 & \textit{if } \frac{1}{1-\delta} \geq \mu \\ > 0 & \textit{if } \frac{1}{1-\delta} < \mu \end{cases}.
\end{aligned}
$$

Proof.

Taking partial derivative with respect to μ on Equation (5), I obtain

$$\underbrace{\left(\frac{\mu}{k+\mu I^*} + \frac{1-\alpha}{k^*}\right)}_{>0}\frac{\partial}{\partial \mu}\frac{\partial I^*}{\partial q} = -\frac{\partial}{\partial \mu}\left(\frac{\mu}{k+\mu I^*} + \frac{1-\alpha}{k^*}\right)\underbrace{\frac{\partial I^*}{\partial q}}_{>0}.$$

From Lemma 3, $\frac{\partial I^*}{\partial q} > 0$. And $\left(\frac{\mu}{k+\mu I^*} + \frac{1-\alpha}{k^*}\right) > 0$, as $k + \mu I^* = q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k > 0$. Thus, the sign of $\frac{\partial}{\partial \mu}\frac{\partial I^*}{\partial q}$ is equal to the sign of $-\frac{\partial}{\partial \mu}\left(\frac{\mu}{k+\mu I^*} + \frac{1-\alpha}{k^*}\right)$.

$$
\begin{aligned}
-\frac{\partial}{\partial \mu}\left(\frac{\mu}{k+\mu I^*} + \frac{1-\alpha}{k^*}\right) &= -\left(\frac{k+\mu I^* - \mu\left(I^* + \mu\frac{\partial I^*}{\partial \mu}\right)}{(k+\mu I^*)^2} + (1-\alpha)\frac{-\frac{\partial k^*}{\partial \mu}}{(k^*)^2}\right) \\
&= -\underbrace{\frac{k - \mu\frac{\partial I^*}{\partial \mu}}{(k+\mu I^*)^2}}_{>0} + \underbrace{\frac{(1-\alpha)}{(k^*)^2}\frac{\partial k^*}{\partial \mu}}_{<0} < 0
\end{aligned}
$$

From Lemma 4, $\frac{\partial I^*}{\partial \mu} = \frac{\partial k^*}{\partial \mu} < 0$. Thus the first term is positive and the second term is negative. Thus, the sign of the left-hand side is negative. Therefore, (i) and (iii) are proved.

$$(i) \quad \frac{\partial}{\partial \mu}\left(\frac{\partial k^*}{\partial q}\right) < 0$$
$$(iii) \quad \frac{\partial}{\partial \mu}\left(\frac{\partial I^*}{\partial q}\right) < 0$$

From the similar logic, the sign of $\frac{\partial}{\partial \mu}\left(\frac{\partial log k^*}{\partial q}\right)$ is equivalent to the sign of $-\frac{\partial}{\partial \mu}\left(\frac{\mu k^*}{k+\mu I^*}\right)$.

$$\begin{aligned}
-\frac{\partial}{\partial \mu}\left(\frac{\mu k^*}{k+\mu I^*}\right) &= -\left(\frac{(\mu\frac{\partial k^*}{\partial \mu} + k^*)(k+\mu I^*) - \mu k^*\left(\mu\frac{\partial k^*}{\partial \mu} + I^*\right)}{(k+\mu I^*)^2}\right) \\
&= -\left(\frac{k^* k + k\mu\frac{\partial k^*}{\partial \mu} + \mu^2\frac{\partial k^*}{\partial \mu}(I^* - k^*)}{(k+\mu I^*)^2}\right) \\
&= -\left(\frac{k^2\left((1-\delta) + \frac{I^*}{k} + \frac{\mu}{k}\frac{\partial I^*}{\partial \mu}\right) - \mu^2 \overbrace{\frac{\partial k^*}{\partial \mu}}^{<0\ (\because \text{Lemma } 4)} (1-\delta)k}{(k+\mu I^*)^2}\right) \\
&< -\left(\frac{k^2\left(\frac{I^*}{k} + \frac{\mu}{k}\frac{\partial I^*}{\partial \mu}\right)}{(k+\mu I^*)^2}\right) < 0
\end{aligned}$$

The last inequality holds because $\frac{I^*}{k} + \frac{\mu}{k}\frac{\partial I^*}{\partial \mu} = \alpha(\alpha-1)q\mathbb{E}z'(k^*)^{\alpha-2}\frac{\partial k^*}{\partial \mu} > 0$, which is obtained from taking a partial derivative with respect to μ on the first-order optimality condition. Therefore, (ii) is proved.

$$(ii) \quad \frac{\partial}{\partial \mu}\left(\frac{\partial log k^*}{\partial q}\right) < 0$$

From the similar logic, the sign of $\frac{\partial}{\partial \mu}\left(\frac{\partial log I^*}{\partial q}\right)$ is equal to the sign of $-\frac{\partial}{\partial \mu}\left(\frac{\mu I^*}{k+\mu I^*}+(1-\alpha)\frac{I^*}{k^*}\right)$.

$$
\begin{aligned}
&-\frac{\partial}{\partial \mu}\left(\frac{\mu I^*}{k+\mu I^*}+(1-\alpha)\frac{I^*}{k^*}\right)\\
&=-\left(\frac{(\mu\frac{\partial I^*}{\partial \mu}+I^*)(k+\mu I^*)-\mu I^*\left(\mu\frac{\partial k^*}{\partial \mu}+I^*\right)}{(k+\mu I^*)^2}+\frac{1-\alpha}{(k^*)^2}\left(k^*\frac{\partial I^*}{\partial \mu}-I^*\frac{\partial k^*}{\partial \mu}\right)\right)\\
&=-\left(\frac{(\frac{\mu}{k}\frac{\partial I^*}{\partial \mu}+\frac{I^*}{k})k^2}{(k+\mu I^*)^2}+\frac{1-\alpha}{(k^*)^2}\left(\frac{\partial k^*}{\partial \mu}\right)(1-\delta)k\right)\\
&=-\left(\frac{(\alpha(\alpha-1)q\mathbb{E}z'(k^*)^{\alpha-2}\frac{\partial k^*}{\partial \mu})k^2}{(k+\mu I^*)^2}+\frac{1-\alpha}{(k^*)^2}\left(\frac{\partial k^*}{\partial \mu}\right)(1-\delta)k\right)\\
&=-\frac{1-\alpha}{(k^*)^2}\left(-\frac{(\alpha q\mathbb{E}z'(k^*)^{\alpha})k^2}{(k+\mu I^*)^2}+(1-\delta)k\right)\left(\frac{\partial k^*}{\partial \mu}\right)
\end{aligned}
$$

From the first-order condition $\alpha q\mathbb{E}z'(k^*)^{\alpha-1}=1+\mu\left(\frac{I^*}{k}\right)$. Substituting this into the equation above, I obtain

$$
\begin{aligned}
-\frac{\partial}{\partial \mu}\left(\frac{\mu I^*}{k+\mu I^*}+(1-\alpha)\frac{I^*}{k^*}\right)&=-\frac{1-\alpha}{(k^*)^2}\left(-\frac{(1+\mu\left(\frac{I^*}{k}\right))k^*k^2}{(k+\mu I^*)^2}+(1-\delta)k\right)\left(\frac{\partial k^*}{\partial \mu}\right)\\
&=-\frac{1-\alpha}{(k^*)^2}k\left(-\frac{(k+\mu I^*)k^*}{(k+\mu I^*)^2}+(1-\delta)\right)\left(\frac{\partial k^*}{\partial \mu}\right)\\
&=\frac{1-\alpha}{(k^*)^2}k\left(\frac{k^*}{k+\mu I^*}-(1-\delta)\right)\left(\frac{\partial k^*}{\partial \mu}\right)\\
&=\frac{1-\alpha}{(k^*)^2}k(k+\mu I^*)\left(k^*-(1-\delta)(k+\mu I^*)\right)\left(\frac{\partial k^*}{\partial \mu}\right)\\
&=\frac{1-\alpha}{(k^*)^2}k(k+\mu I^*)I^*\underbrace{(1-(1-\delta)\mu)}_{(*)}\underbrace{\left(\frac{\partial k^*}{\partial \mu}\right)}_{<0}.
\end{aligned}
$$

Therefore, depending on the sign of the term $(*)$ above, the sign of $\frac{\partial}{\partial \mu}\left(\frac{\partial log I^*}{\partial q}\right)$ is determined.

$$(iv) \quad \frac{\partial}{\partial \mu}\left(\frac{\partial logI^*}{\partial q}\right)\begin{cases} \leq 0 & \text{if } \frac{1}{1-\delta} \geq \mu \\ > 0 & \text{if } \frac{1}{1-\delta} < \mu \end{cases}$$

■

F.2 A model with convex adjustment cost: Lemmas and proofs

Lemma 1 (Size-monotonicity in future capital stock)**.**

$$For\ \forall k > 0,\ \frac{\partial k^*}{\partial k} > 0$$

Proof.

From the inter-temporal optimality condition,

$$1 + \mu \left(\frac{k^*}{k} - (1-\delta) \right) = q\mathbb{E}z'\alpha(k^*)^{\alpha-1}.$$

I take a partial derivative with respect to k:

$$\mu \frac{1}{k} \frac{\partial k^*}{\partial k} - \mu \frac{k^*}{k} = q\mathbb{E}z'\alpha(\alpha-1)((1-\delta)k + I^*)^{\alpha-2} \frac{\partial k^*}{\partial k}.$$

By rearranging the terms,

$$\frac{\partial k^*}{\partial k} = \frac{\mu \frac{k^*}{k}}{\left(\mu \frac{1}{k} - q\mathbb{E}z'\alpha(\alpha-1)((1-\delta)k + I^*)^{\alpha-2}\right)} > 0.$$

The last line is from $q\mathbb{E}z'\alpha(\alpha-1)((1-\delta)k + I^*)^{\alpha-2} < 0$, as $\alpha - 1 < 0$. ■

Lemma 2 (Size-elasticity of future capital stock)**.**

$$For\ \forall k > 0,\ \frac{\partial log(k^*)}{\partial log(k)} < 1$$

Proof.

By taking log in the both sides of the inter-temporal optimality condition,

$$log\left(1 + \mu \left(\frac{k^*}{k} - (1-\delta) \right)\right) = log(q\mathbb{E}z'\alpha(k^*)^{\alpha-1}).$$

Then, I take a partial derivative with respect to $log(k)$ to obtain

$$\frac{\mu \frac{\partial}{\partial logk}\left(\frac{k^*}{k}\right)}{1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)} = (\alpha-1)\frac{\partial logk^*}{\partial logk}.$$

Thus,

$$\mu\frac{\frac{\partial logk^*}{\partial logk}\frac{k^*}{k}-\frac{k^*}{k}}{1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)} = (\alpha-1)\frac{\partial logk^*}{\partial logk}.$$

By rearranging terms, I get

$$\left(\frac{\mu}{1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)}\frac{k^*}{k}-(\alpha-1)\right)\frac{\partial logk^*}{\partial logk} = \frac{\mu\frac{k^*}{k}}{1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)}.$$

By multiplying $1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)$, I get

$$\left(\mu\frac{k^*}{k}-(\alpha-1)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right)\right)\frac{\partial logk^*}{\partial logk} = \mu\frac{k^*}{k}.$$

The, it leads to

$$\frac{\partial logk^*}{\partial logk} = \frac{\mu\frac{k^*}{k}}{\mu\frac{k^*}{k}-(\alpha-1)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right)}.$$

From Lemma 1, $\frac{\partial logk^*}{\partial logk} > 0$ and $\frac{k^*}{k} > 0$. Thus the denominator on the right-hand side is also positive. Therefore, I have the following equivalence:

$$\begin{aligned}
\frac{\partial logk^*}{\partial logk} < 1 &\iff \mu\frac{k^*}{k} < \mu\frac{k^*}{k}-(\alpha-1)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right) \\
&\iff 0 < (1-\alpha)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right) \\
&\iff 0 < 1+\mu\left(\frac{k^*}{k}-(1-\delta)\right) \\
&\iff 0 < q\mathbb{E}z'\alpha(k^*)^{\alpha-1}.
\end{aligned}$$

Because the last inequality is true, I conclude $\frac{\partial log k^*}{\partial log k} < 1$. ∎

Lemma 3 (Investment monotonicity in discount factor).

$$\frac{\partial I^*}{\partial q} > 0$$

Proof. From Equation (5), I have

$$\left(\frac{\mu}{k + \mu I^*} + \frac{1-\alpha}{k^*}\right)\frac{\partial I^*}{\partial q} = \frac{1}{q}$$

By rearranging terms, I get

$$\frac{\partial I^*}{\partial q} = \frac{\frac{1}{q}}{\left(\frac{\mu}{k+\mu I^*} + \frac{1-\alpha}{k^*}\right)}.$$

Therefore, the following statement holds:

$$k + \mu I^* > 0 \implies \frac{\partial I^*}{\partial q} > 0.$$

Going back to the inter-temporal optimality condition, I multiply k in the both sides to have

$$k + \mu I^* = q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k > 0.$$

Therefore, $\frac{\partial I^*}{\partial q} > 0$. ∎

Lemma 4 (Investment and convex adjustment parameter).

For $\mu > 0$,

$$\frac{\partial I^*}{\partial \mu} = \frac{\partial k^*}{\partial \mu} < 0 \ \textit{if}\ I^* > 0.$$

Proof. From the first-order condition,

$$1 + \mu\left(\frac{I^*}{k}\right) = \alpha q \mathbb{E} z'(k^*)^{\alpha-1}$$

Taking a partial derivative w.r.t μ, I obtain

$$\frac{I^*}{k} + \frac{\mu}{k}\frac{\partial I^*}{\partial \mu} = \alpha(1-\alpha) q \mathbb{E} z'(k^*)^{\alpha-2}\frac{\partial k^*}{\partial \mu}.$$

From $I^* = k^* - (1-\delta)k$, $\frac{\partial I^*}{\partial \mu} = \frac{\partial k^*}{\partial \mu}$. Then, by rearranging terms, I get

$$\frac{\partial I^*}{\partial \mu} = \frac{\frac{I^*}{k}}{\left(\underbrace{\alpha(1-\alpha) q \mathbb{E} z'(k^*)^{\alpha-2}}_{<0} - \frac{\mu}{k}\right)} < 0.$$

■

F.3 A model with fixed adjustment cost: Proposition and proofs

Proposition 6 (The effect of the firm size and the price on the adjustment probability)**.**
For $\forall k$ *s.t.* $\xi^*(k,q) < \overline{\xi}(q)$*,*

$$\frac{\partial \psi(k,q)}{\partial k}\frac{\partial \psi(k,q)}{\partial q} < 0 \textit{ and } \frac{\partial}{\partial k}\frac{\partial}{\partial q}\psi(k,q) < 0.$$

Proof.
As $\xi^*(k,q) < \overline{\xi}$, $\psi(k,q) = \xi^*(k,q)/\xi$. By taking the cross-derivative with respect to q and k on $\xi^*(k,q)$, I obtain

$$\frac{\partial^2 \xi^*(k,q)}{\partial q \partial k} = -\alpha \mathbb{E}_z z'((1-\delta)k)^{\alpha-1}(1-\delta) < 0.$$

Thus, $\frac{\partial}{\partial k}\frac{\partial}{\partial q}\psi(k,q) < 0$.

From Proposition 5, $\frac{\partial \xi^*(k,q)}{\partial k} < 0$ for $\forall k < \widehat{k}$, and $\frac{\partial \xi^*(k,q)}{\partial k} > 0$ for $\forall k > \widehat{k}$.

By taking a partial derivative with respect to q on F, I obtain

$$\frac{\partial \xi^*(k,q)}{\partial q} = \mathbb{E}z'(k^*)^\alpha - \mathbb{E}z'((1-\delta)k)^\alpha.$$

Thus, $\frac{\partial \xi^*(k,q)}{\partial q} > 0$ for $\forall k < \frac{k^*}{(1-\delta)} = \widehat{k}$, and $\frac{\partial \xi^*(k,q)}{\partial q} < 0$ for $\forall k > \frac{k^*}{(1-\delta)} = \widehat{k}$.

Therefore, $\frac{\partial \xi^*(k,q)}{\partial k}$ and $\frac{\partial \xi^*(k,q)}{\partial q}$ always take the opposite sign: $\frac{\partial \xi^*(k,q)}{\partial k}\frac{\partial \xi^*(k,q)}{\partial q} < 0$. And the equality holds when $k = \widehat{k}$. ■

F.4 A model with fixed adjustment cost: Lemmas and proofs

Lemma 5 (U-shaped probability of the extensive-margin investment)**.**
Given $q > 0$, there uniquely exist $\widehat{k}$ and $\overline{k}$ such that

$$F(\overline{k}, q) = \overline{\xi},\ \xi^*(k,q) > \overline{\xi} \text{ for } \forall k > \overline{k},\ \text{and}$$
$$\frac{\partial F}{\partial k}\Big|_{k=\widehat{k}} = 0$$

Proof.

$$\xi^*(k,q) := -I^* + q\mathbb{E}_z z'((1-\delta)k + I^*)^\alpha - q\mathbb{E}_z z'((1-\delta)k)^\alpha$$

After taking a partial derivative with respect to k, I get the following equation:[19]

$$\frac{\partial \xi^*(k,q)}{\partial k} = (1-\delta) - \alpha q \mathbb{E}_z z'((1-\delta)k)^{\alpha-1}(1-\delta).$$

Then, at $k = \widehat{k} := \frac{(\alpha q \mathbb{E}_z z')^{\frac{1}{1-\alpha}}}{1-\delta}$, $\frac{\partial \xi^*(k,q)}{\partial k}\Big|_{k=\widehat{k}} = 0$. From the first order condition, we can check $(\alpha q \mathbb{E}_z z')^{\frac{1}{1-\alpha}} = k^*$. Therefore, $\widehat{k} = \frac{k^*}{1-\delta}$.

Taking another partial derivative with respect to k, I obtain

$$\frac{\partial^2 \xi^*(k,q)}{\partial k^2} = \alpha(1-\alpha) q \mathbb{E}_z z'((1-\delta)k)^{\alpha-2}(1-\delta)^2 > 0.$$

Thus, for $\forall k < \widehat{k}$, $\frac{\partial \xi^*(k,q)}{\partial k} < 0$, and for $\forall k > \widehat{k}$, $\frac{\partial \xi^*(k,q)}{\partial k} > 0$. Therefore, $\xi^*(k,q) > F(\widehat{k}, q)$, for $\forall k > \widehat{k}$.

Then, I consider a limit case where $k \to \infty$.

$$\begin{aligned}\lim_{k\to\infty} \xi^*(k,q) &= \lim_{k\to\infty} -k^* + (1-\delta)k + q\mathbb{E}_z z'(k^*)^\alpha - q\mathbb{E}_z z'((1-\delta)k)^\alpha \\ &= \lim_{k\to\infty} \left(\frac{1}{\alpha} - 1\right) k^* + \underbrace{(1-\delta)k - q\mathbb{E}_z z'((1-\delta)k)^\alpha}_{\to\infty} \to \infty.\end{aligned}$$

[19]The first order condition is applied after taking the partial derivative.

$\xi^*(k,q)$ is continuous. Thus, if $\overline{\xi} \geq F(\widehat{k},q)$, from the intermediate value theorem, there exists $\overline{k}$ such that $F(\overline{k}) = \overline{\xi} \geq F(\widehat{k},q)$. Then, $\xi^*(k,q) > \overline{\xi}$ for $\forall k > \overline{k}$. If $\overline{\xi} < F(\widehat{k},q)$, then, $\xi^*(k,q) > \overline{\xi}$ for $\forall k > 0$. ■

Lemma 6 (The extensive-margin response to interest rate change)**.**
For $\forall k \in (0, \widehat{k}(q))$

$$\frac{\partial}{\partial q}\xi^*(k,q) > 0$$

Proof.

By taking a partial derivative with respect to q on F, I obtain

$$\frac{\partial \xi^*(k,q)}{\partial q} = \mathbb{E}z'(k^*)^\alpha - \mathbb{E}z'((1-\delta)k)^\alpha.$$

Thus, $\frac{\partial \xi^*(k,q)}{\partial q} > 0$ for $\forall k < \frac{k^*}{(1-\delta)} = \widehat{k}$. ■

Lemma 7 (Size-monotonicity of the interest-elasticity in a fixed-cost model)**.**

$$\frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) = 0 \; for \; \forall k > 0$$

If $I^* > 0$*, then*

$$\frac{\partial}{\partial k}\left(\frac{\partial log I^*}{\partial q}\right) > 0 \; for \; \forall k > 0$$

Proof.

From the first order condition,

$$1 = \alpha q \mathbb{E}_z z'(k^*)^{\alpha-1}.$$

This implies that the future capital stock does not depend on the current size of the firm.

$$\frac{\partial k^*}{\partial k} = 0$$

From $I^* = k^* - (1-\delta)k$, the following equations hold

$$\frac{\partial I^*}{\partial k} = -(1-\delta),$$
$$\frac{\partial logI^*}{\partial k} = -\frac{(1-\delta)}{I^*} \text{ for } I^* > 0.$$

Taking a partial derivative with respect to q,

$$\frac{\partial^2 I^*}{\partial q \partial k} = 0,$$
$$\frac{\partial^2 logI^*}{\partial q \partial k} = \frac{(1-\delta)}{I^{*2}} \frac{\partial I^*}{\partial q} \text{ for } I^* > 0.$$

Going back to the first order condition, the following equation holds after taking the partial derivative with respect to q.

$$0 = \alpha \mathbb{E} z'(k^*)^{\alpha-1} + \alpha(\alpha-1) q \mathbb{E} z'(k^*)^{\alpha-2} \frac{\partial I^*}{\partial q}$$

Thus,

$$\frac{\partial I^*}{\partial q} = \frac{\alpha \mathbb{E} z'(k^*)^{\alpha-1}}{\alpha(1-\alpha) q \mathbb{E} z'(k^*)^{\alpha-2}} > 0.$$

Therefore,

$$\frac{\partial^2 logI^*}{\partial q \partial k} = \frac{(1-\delta)}{I^{*2}} \frac{\partial I^*}{\partial q} > 0 \text{ for } I^* > 0.$$

■

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