Striking While the Iron Is Cold: Fragility after a Surge of Lumpy Investments[†]

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Abstract

This paper studies the endogenous state dependence of the aggregate investment dynamics stemming from synchronized lumpy investments at the firm level. I develop a heterogeneous-firm real business cycle model where the semi-elasticities of large and small firms' investments are matched with the empirical estimates. In the model, following a negative TFP shock, the timings of large firms' lumpy investments are persistently synchronized due to the low sensitivity to the general equilibrium effect, leading to a surge of lumpy investments. After the surge, TFP-induced recessions are especially severe, and the semi-elasticity of the aggregate investment drops significantly.

Keywords: Business cycle, state dependence, lumpy investment, interest elasticity.

JEL codes: E32, E22, D25.

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1 Introduction

1980, 1998, and 2007 were the three years with the large surges in the fraction of large firms making large-scale investments. These three years were followed by recessions within two years.[1] Is it merely a coincidence that investment surges of large firms precede recessions? This paper studies an endogenous pre-condition of an economy that makes aggregate allocations respond differently to shocks of the same magnitude. In particular, I study a mechanism that makes an economy more fragile to a negative TFP shock after synchronized large-scale investments of large firms.

The large-scale investment of large firms is distinguished from the others as they are highly interest-inelastic.[2] Then, if a negative TFP shock hits after a surge of large firms' lumpy investments, the lowered real interest rate due to the shrunk investment demand does not motivate the interest-inelastic large firms to make another round of lumpy investments. Therefore, due to missing large firms' investments, the economy suffers from a deeper recession and slower recovery than it would otherwise do despite a moderate magnitude of the negative aggregate shock.

To investigate this channel, I develop and analyze a business cycle model with heterogeneous firms where the semi-elasticities of large and small firms' investments are matched with the empirical estimates. In the existing models in the literature, the cross-sectional ranking of the interest-elasticities of investment between large and small firms are counterfactually flipped: the large

[1] Following Cooper and Haltiwanger (2006), I define an investment beyond 20% of existing capital stock as a large-scale capital adjustment. Firms that hold capital stocks greater than the 90th percentile of the capital distribution in each industry based on the two-digit NAICS code are defined as large firms.

[2] I use the terms *lumpy investment* and *large-scale investment* interchangeably in this paper.

becomes more interest-elastic than the small. I fix this problem by introducing and calibrating a parameter that governs how the size affects the inaction bands through the fixed adjustment cost. Using the model, I qualitatively and quantitatively analyze the amplification of productivity-driven aggregate fluctuations. Due to the low interest-elasticity, the large firms' nonlinear investment patterns are not washed out by the general equilibrium effect, leaving the lumpy investment timings synchronized persistently after a negative aggregate TFP shock. These synchronized investments of large firms generate macro-level state dependence.

Large firms are a particular focus of this paper for three reasons. First, large firms are insensitive to fluctuations in macroeconomic conditions, including the general equilibrium effect. Therefore, their firm-level nonlinearity generates a significant macro-level nonlinearity in the aggregate investment dynamics, while small firms do not. Second, large firms are the most observable group of firms as most of them are listed and subject to financial disclosure regulations mandated by the U.S. Securities and Exchange Commission (SEC). Therefore, any forward-looking information contained in the large firms' investment dynamics can be traced in a timely manner and be conducive to designing contemporaneous policies. I show that fragility indices constructed based on both Compustat data and the simulated data of only large firms consistently display a significant predictive power on the aggregate investment sensitivity to aggregate TFP shocks. Lastly, large firms account for a substantial portion of the aggregate investment. Therefore, the large firms' investment fluctuations significantly impact the aggregate investment dynamics.

The state dependence in the business cycle induced by the firm-level heterogeneity has been underexplored in the related literature due to the computational difficulty: the true nonlinear law of motion for the distribution cannot be properly specified. Using a new methodology concurrently developed in Lee (2023), I globally and accurately solve the nonlinear dynamic stochastic general equilibrium without a perfect foresight assumption. By doing so, the endogenous state dependence in the aggregate investment fluctuation is sharply quantified in this paper.

Especially, I develop a fragility index based on the large firms' recent capital adjustment history. This index has predictive power on the one-period-ahead investment growth and serves as a sufficient statistic on the post-shock dynamics of the aggregate investment after a TFP shock. In practice, this index is relatively easy to trace contemporaneously compared to other indices in the literature, as the index is based on large firms' readily observable data (e.g., 10-K/Q reports or Compustat data). Using the fragility index, I show that the economy becomes significantly more fragile to a negative aggregate shock after a surge of lumpy investment of large firms, and I validate the model implication with the data.

Lastly, I show that aggregate investment's interest elasticity depends on the level of the fragility index over the business cycle. This result implies that the monetary policy's effectiveness can be low after a surge of large firms' lumpy investments.[3] Also, this provides a solid explanation of why monetary policy has not been effective during the recessions, especially through the business investment channel (Tenreyro and Thwaites, 2016).

Related literature This paper is related to the literature that studies how firm-level lumpy investments affect the business cycle. Abel and Eberly (2002)

[3]The policy implication is limited to a positive implication, as the model does not include a monetary policy block.

empirically showed that firm-level investments feature statistically and economically significant nonlinearity. They point out that tracking the cross-sectional distribution of firm-level investments is necessary to account for aggregate investment. Cooper et al. (1999) and Gourio and Kashyap (2007) found that aggregate investment is largely driven by establishment-level capital adjustment in the extensive margin. Especially, Cooper et al. (1999) found that synchronized lumpy investments can generate an echo effect of aggregate shocks in partial equilibrium. Gourio and Kashyap (2007) pointed out that if a fixed cost is drawn from a highly concentrated non-uniform distribution, aggregated lumpy investments show different impulse responses than frictionless models. In contrast, Khan and Thomas (2008) found that lumpiness in investment at the establishment level is washed out after aggregation due to a strong general equilibrium effect. Towards this point, my paper shows that the lumpiness in the firm-level investment survives the aggregation if the interest-elasticity is disciplined at the empirically observed range.

The fragility index of my paper is closely related to several papers measuring the responsiveness of an economy to exogenous aggregate shocks. Caballero et al. (1995) develops a micro-level adjustment-hazard function that captures heterogeneous price adjustment probability. The dynamics in the cross-section of the hazard rates generate substantial nonlinearity in the economy's aggregate dynamics. Bachmann et al. (2013) defines a responsiveness index as a function of aggregate productivity and sufficient statistics of the joint distribution of capital stocks and idiosyncratic productivities. They show that the responsiveness index is significantly driven by the fraction of capital-adjusting firms. Baley and Blanco (2021) shows that two sufficient statistics can characterize aggregate investment dynamics: 1) the capital-to-productivity ratio's dispersion and 2) its covariance with the duration of inaction. Compared to these papers, my paper highlights the role of the marginal distribution of large firms' inaction duration over the business cycle, which is readily observable in the data in a timely manner due to their mandated financial disclosure.

Also, this paper is related to the literature studying the state-dependent effectiveness of monetary policy. The most closely related paper is Tenreyro and Thwaites (2016), which shows that business investment and durables expenditure are less responsive to monetary policies during recessions. I document that the rising fragility index substantially accounts for the investment drop during the recession of the dot-com bubble crash. At the same time, I show that the interest-elasticity of aggregate investment significantly decreases in the fragility index. According to this result, monetary policy could not have functioned effectively during the dot-com bubble crash. Likewise, my paper gives a micro-founded explanation of why monetary policy is not effective during a recession. Going one step further, it provides a testable implication: monetary policy in a recession not preceded by a surge of large firms' lumpy investments might be as effective as in normal years.

Lastly, this paper contributes to the nonlinear business cycle literature. A large body of research has focused on the nonlinearity in aggregate fluctuations that arise when heterogeneous agents are subject to micro frictions. Berger and Vavra (2015) concludes that lumpiness in households' durable adjustment results in pro-cyclical responsiveness of aggregate durable expenditures to an aggregate shock. Fernandez-Villaverde et al. (2022) found that financial frictions can generate endogenous aggregate risk under the heterogeneous household model. In this setup, the aggregate allocations display state-dependent responsiveness to an aggregate TFP shock. Volatility shocks to real interest rates studied in Fernandez-Villaverde et al. (2011) and uncertainty shocks in Bloom et al. (2018) are also highlighted as an important source of the nonlinearity in the business cycle. To this literature, this paper contributes by analyzing interest-inelastic large firms' lumpy investments as a significant source of nonlinearity in the aggregate investment dynamics.

Roadmap Section 2 shows motivating facts about surges of large firms' lumpy investments before and after the recessions. Section 3 develops a heterogeneous-firm business cycle model where the cross-section of the interest-elasticities is matched with the empirical estimates. Section 4 analyzes the macroeconomic implications of the calibrated model. Section 5 concludes. Proofs and other detailed figures and tables are included in appendices.

2 Motivating fact

In this section, I empirically analyze the cyclical pattern of the lumpy investments of large firms. I use U.S. Compustat data for the firm-level empirical analysis. While Compustat data covers only public firms, its coverage is relatively less of an issue in this analysis because the focus is on large firms. Throughout the empirical analysis, large firms are defined as firms that hold capital stocks greater than the 40th percentile of the capital distribution in each industry of the two-digit NAICS code. The choice of the 40th percentile is to define large firms in the Compustat space consistent with large firms in Zwick and Mahon (2017), which estimated the interest-elasticities of firm-level investments.[4] The sample period covers from 1980 to 2016. Firms with negative assets and zero employment are excluded from the sample. All the firm-level variables except capital stock and investment are deflated by the

[4] In Zwick and Mahon (2017), large and small firms are defined as the top 30% and bottom 30% of sales distribution. From the size cutoffs (15.4M, 48.8M) in terms of sales in the years 1998 through 2000 and 2005 through 2007 (Table B.1, panel (d)), I compute the corresponding capital size cutoffs in Compustat.

GDP deflator. Investment is deflated by non-residential fixed investment deflator available from National Income and Product Accounts data (NIPA Table 1.1.9, line 9). The firm-level real capital stock is obtained by applying the perpetual inventory method to net real investment. The industry is categorized by the first two-digit NAICS code.[5]

2.1 Surges of large firms' lumpy investments and recessions

In the following analysis, I empirically analyze the relationship between large firms' lumpy investments and the timing of recessions. I define an investment spike as a firm-specific event where a firm makes a large-scale investment greater than 20% of the firm's existing capital stock.[6] I refer to this investment spike as a lumpy investment or capital adjustment in the extensive margin interchangeably. Then, I define spike ratio as follows:

$$\text{Spike ratio}_{j,t} := \frac{\sum_{i \in j} \mathbb{I}\{i_{it}/k_{it} > 0.2\}}{\#\text{ of } j\text{-type firms at } t}, \quad j \in \{small, large\}$$

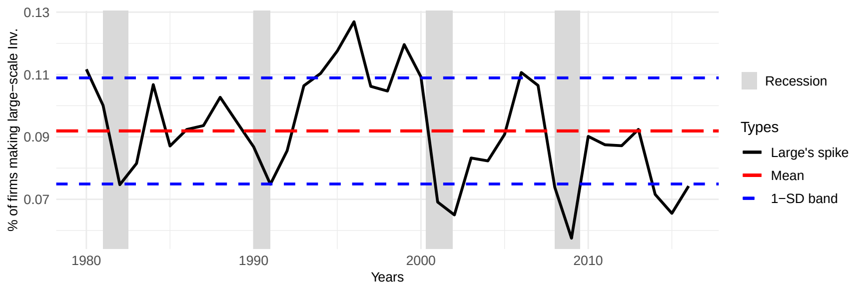
The numerator counts all the incidences of investment spikes from firm type $j \in \{small, large\}$ at time t, and it is normalized by the total number of j-type firms.

Figure 1 plots the time series of the spike ratio of large firms. On average,

[5]If only SIC code is available for a firm, I imputed the NAICS code following online appendix D.2 of Autor et al. (2020). If both NAICS and SIC are missing, I filled in the next available industry code for the firm.

[6]20% cutoff is from the non-convex adjustment cost literature (Cooper and Haltiwanger, 2006; Gourio and Kashyap, 2007; Khan and Thomas, 2008). If a firm's acquired capital stock is greater than 5% of existing capital stock in a certain year, I rule out the observation from the sample due to possible noise in the reported items in the balance sheet during the acquisition year.

Figure 1: Three surges of large firms' lumpy investments before recessions



Notes: The firm-level large-scale investment is defined as an investment greater than 20% of the existing capital stock. The solid line plots the time series of the fraction of large firms making large-scale investments. The grey areas highlight the NBER recession periods.

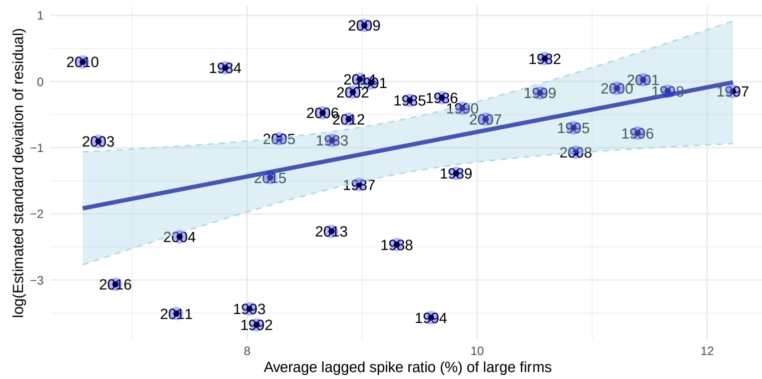
9.2% of large firms adjust their existing capital stocks in the extensive margin in a year. As can be seen from Figure 1, since 1980, there have been only four periods (1980, 1996, 1998, and 2007) where the fraction of large firms making spiky investments surged beyond one-standard deviation. Three out of the four events were followed by recessions within two years.

Conversely, there were four recessions in the U.S. over the same period, and three out of four recessions were preceded by the surge of large firms' lumpy investments. The exception was the recession in 1990, and it was the mildest recession among the four recessions.

In the following analysis, I show aggregate investment rate is conditionally heteroskedastic on the average lagged spike ratio of large firms. That is, the residualized volatility of aggregate investment rate is high if a great fraction of large firms have made lumpy investments in recent years.

For this analysis, I use aggregate data on non-residential investment (NIPA Table 1.1.5, line 9) and aggregate capital (Fixed Asset Accounts Table 1.1, line 4) from BEA. The thick line in Figure 2 plots the estimates of the log standard deviation of residuals from the autoregression of aggregate investment rates as

Figure 2: Conditional heteroskedasticity of aggregate investment



Notes: The estimated standard deviation of the residual (y-axis) is obtained from fitting the aggregate investment-to-capital ratio (%) into an autoregressive process with four lags. The average lagged spike ratio of large firms (%) is obtained by averaging the most recent past two spike ratios for each observation of residualized investments. The years overlaid on the dots are the observation year of the residualized investment-to-capital ratios.

a function of the recent average of large firms' spike ratio.[7] The recent average is based on the average spike ratio of the past three years. As can be seen from this figure, aggregate investment rates are heteroskedastic conditional on the lagged average spike ratio. Table E.9 reports the regression coefficients for the fitted line. According to the regression result, a one-standard-deviation increase (1.47%) in the large firms' past spike ratio is associated with a one-standard-deviation increase (0.50%) in the aggregate investment's residualized volatility. Consistent with the patterns in Figure 1, the three recession years of interest are located at the top-right corner in Figure 2.

3 Model

I develop and analyze a heterogeneous-firm real business cycle model in which the cross-section of the semi-elasticities of firm-level investment is matched

[7]This empirical analysis is motivated from the conditional heteroskedasticity analysis in Figure 1 of Bachmann et al. (2013).

with the empirical estimates. In the model, time is discrete and lasts forever. There is a continuum of measure one of firms that own capital, produce business outputs, and make investments. The business output can be reinvested as capital after a firm pays adjustment costs.

3.1 Technology

A firm owns capital. It produces a unit of goods that can be converted to a unit of capital after paying an adjustment cost. The production technology is a Cobb-Douglas function with decreasing returns to scale:

$$z_{it}A_t f(k_{it}, l_{it}) = z_{it}A_t k_{it}^{\alpha} l_{it}^{\gamma}, \quad \alpha + \gamma < 1$$

where k_{it} is firm i's capital stock at the beginning of period t; l_{it} is labor input; z_{it} is idiosyncratic productivity; A_t is aggregate TFP. Idiosyncratic productivity, z_{it}, and aggregate TFP, A_t, follow the stochastic processes as specified below:

$$ln(z_{it+1}) = \rho_z ln(z_{it}) + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim_{iid} N(0, \sigma_z)$$
$$ln(A_{t+1}) = \rho_A ln(A_t) + \epsilon_{A,t+1}, \quad \epsilon_{A,t+1} \sim_{iid} N(0, \sigma_A)$$

where ρ_s and σ_s are persistence and standard deviation of $i.i.d$ innovation in each process $s \in \{z, A\}$, respectively. Both stochastic processes are discretized using the Tauchen method in computation.

3.1.1 Investment and adjustment cost

I assume a firm-level large-scale investment could be made only after paying a total adjustment cost, C_{it}, which varies over firm-level allocations. The total adjustment cost is a function of capital stock, k_{it}, investment size I_{it}, and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \overline{\xi}]$ as in Winberry (2021). And this total adjustment cost is composed of two additively separable parts: a convex adjustment cost and a fixed adjustment cost. The convex adjustment cost is a function of the current capital stock, k_{it}, and the investment I_{it} as assumed in the literature. The fixed adjustment cost, F_{it}, is a function of the current capital stock k_{it} and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \overline{\xi}]$. The fixed cost does not incur if a firm adjusts capital within a moderate range ($I_{it} \in \Omega(k_{it}) := [-\nu k_{it}, \nu k_{it}]$). A firm needs to pay a fixed cost for investment beyond this range. The fixed cost is assumed to be overhead labor cost, so it varies over the business cycle due to wage fluctuations.[8]

To summarize, I assume the following total adjustment cost structures:

$$
\begin{aligned}
C_{it} &= C(k_{it}, I_{it}, \xi_{it}; w_t) \\
&= \mu \left(\frac{I_{it}}{k_{it}} \right)^2 k_{it} + F(k_{it}, \xi_{it}) w_t
\end{aligned}
$$

$$
F(k_{it}, \xi_{it}) = \begin{cases} \xi_{it} k^{\zeta} & \text{if } I_{it} \notin \Omega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \\ 0 & \text{if } I_{it} \in \Omega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \end{cases}
$$

This model's difference from the existing literature is the size-dependent fixed cost parametrized by the extensive-margin elasticity dispersion parameter, ζ. As ζ increases, the extensive-margin elasticity gap between small and large firms widens, leaving the cross-section of the interest-elasticity consistent with the empirical level in Zwick and Mahon (2017) and Koby and Wolf (2020). In Section 4, I quantitatively investigate how the ζ parameter affects the dispersion of interest-elasticity.

[8] This setup is following Khan and Thomas (2008) and Winberry (2021).

3.1.2 Size-dependent fixed cost: A theoretical explanation

In this section, I provide a theoretical ground for size-dependent fixed cost. The presence of a fixed cost for the firm-level investment has been widely accepted in the literature. However, it has been relatively less investigated whether the fixed cost occurs at the establishment or firm levels. Depending on the model specification and the granularity of the data, each paper flexibly defines the fixed cost.

In this paper, the fixed cost is modeled at the firm level, but its functional form is grounded on the establishment-level fixed cost. I argue that if a firm decides to make a large-scale investment by expanding establishments, fixed cost occurs at each existing establishment due to interdependence across the establishments. For example, if a new establishment is constructed, the production lines in the existing establishments have to be adjusted to coordinate with the new one, and managers have to be reallocated across the different production units. Therefore, intuitively, firm-level fixed cost increases in the number of establishments and the degree of interdependence across the establishments.

To sharpen the theoretical points, let's assume a firm has n establishments and plans to expand a new factory. Then, if establishments are coordinated pairwise, and if the fixed cost of each coordinated pair is ξ, the total firm-level fixed cost F is as follows:

$$F_2 = \binom{n}{2} \times \xi = \frac{n(n-1)}{2}\xi$$

which features quadratic growth in the number of establishments. This was when each establishment is interdependent pairwise. Then, if an establishment's operation is dependent on $\zeta - 1$ number of other establishments on average, the firm-level fixed cost becomes as follows:

$$F_\zeta = \binom{n}{\zeta} \times \xi = \frac{n(n-1)(n-2)\dots(n-\zeta+1)}{\zeta!}\xi$$

The firm-level fixed cost F_ζ exponentially increases in the number of establishments to the power of ζ. For a higher interdependence across the establishments, the fixed cost increases faster. Even if the source of the fixed cost is not at the establishment level, the intuitive explanation is that the interdependence across the basic operation unit (e.g., department or team) convexly raises the complexity inside the firm. And this increases the firm-level fixed cost when the firm makes a large-scale capital adjustment.

In this paper, the number of establishments (or basic production units) is proxied by the total capital stock k_{it}. This is consistent with Cao et al. (2019). Using the US administrative data, Cao et al. (2019) points out that the firm growth is substantially driven by the expansion in the number of establishment. Therefore, the number of establishments is well-proxied by the size of the capital stock k_{it}.

3.2 Household

A stand-in household is considered. The household consumes, supplies labor, and saves in a complete market. In the beginning of a period, the household is given with an equity portfolio a, information on the contemporaneous distribution of firms Φ, and the aggregate TFP level A. The household problem is as follows:

$$
\begin{aligned}
V(a;S) = \max_{c,a',l_H} \quad & log(c) - \eta l_H + \beta \mathbb{E} V(a';S') \\
\text{s.t. } & c + \int \mathit{\Gamma}_{A,A'} q(S,S') a(S') dS' = w(S) l_H + a(S) \\
& G_{\mathit{\Phi}}(S) = \mathit{\Phi}', \quad \mathbb{P}(A'|A) = \mathit{\Gamma}_{A,A'}, \quad S = \{\mathit{\Phi}, A\}
\end{aligned}
$$

where V is the value function of the household; $\mathit{\Phi}$ is a distribution of firms; A is an aggregate productivity; $\mathit{\Gamma}_{A,A'}$ is the state transition probability; c is consumption; a' is a state-contingent future saving portfolio; l_H is labor supply; w is wage, and r is real interest rate. Household is holding the equity of firms as their asset.

From the household's first-order condition and the envelope condition, I obtain the following characterization of the stochastic discount factor $q(S,S')$:

$$
q(S,S') = \beta \frac{C(S)}{C(S')}
$$

I define $p(S) := \frac{1}{C(S)}$. In the recursive formulation of a firms' problem in the next section, I use $p(S)$ to normalize the firm's value function following Khan and Thomas (2008).

3.3 A firm's problem: Recursive formulation

In this section, I formulate a firm's problem in the recursive form. A firm is given with capital k, an idiosyncratic productivity z, in the beginning of a period. Also, they are given with the knowledge on the contemporaneous distribution of firms $\mathit{\Phi}$ and the aggregate TFP level A. For each period, firm determines investment level I and labor demand n_d. A firm's problem is formulated in the following recursive form:

$$
\begin{aligned}
J(k,z;S) = \quad & \pi(k,z;S) + (1-\delta)k \\
& + \int_0^{\overline{\xi}} \max\left\{R^*(k,z;S) - F(k,\xi)w(S), R^c(k,z;S)\right\} dG_\xi(\xi) \qquad (1) \\
R^*(k,z;S) = & \max_{k'\geq 0} \quad -k' - c(k,k') + \mathbb{E}q(S,S')J(k',z';S') \\
R^c(k,z;S) = & \max_{k^c\in\Omega(k)} \quad -k^c - c(k,k^c) + \mathbb{E}q(S,S')J(k^c,z';S')
\end{aligned}
$$

The following lines explain the details of each component in the value functions.

$$
\begin{aligned}
\text{(Operating profit)} \quad & \pi(z,k;S) := \max_{n_d} zAk^\alpha n_d^\gamma - w(S)n_d \ (n_d\text{: labor demand}) \\
\text{(Convex adjustment cost)} \quad & c(k,k') := \left(\mu^I/2\right)\left((k'-(1-\delta)k)/k\right)^2 k \\
\text{(Size-dependent fixed cost)} \quad & F(k,\xi) := \xi k^\zeta \\
\text{(Constrained investment)} \quad & k^c \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta) \\
\text{(Idiosyncratic productivity)} \quad & z' = G_z(z) \text{ (AR(1) process)} \\
\text{(Stochastic discount factor)} \quad & q(S,S') = \beta\left(C(S)/C(S')\right) \\
\text{(Aggregate states)} \quad & S = \{A, \Phi\} \\
\text{(Aggregate law of motion)} \quad & \Phi' := H(S),\ A' = G_A(A) \text{ (AR(1) process)},
\end{aligned}
$$

Then, I multiply $p(S) = 1/C(S)$ on the both sides of line (1) to obtain

$$
\begin{aligned}
p(S)J(k,z;S) = \quad & p(S)(\pi(k,z;S) + (1-\delta)k) \\
& + \int_0^{\overline{\xi}} \max\left\{p(S)R^*(k,z;S) - p(S)w(S)F(k,\xi), p(S)R^c(k,z;S)\right\} dG_\xi(\xi)
\end{aligned}
$$

I define the normalized value functions as follows:

$$\widetilde{J}(k, z; S) := p(S) J(k, z; S)$$
$$\widetilde{R}^*(k, z; S) := p(S) R^*(k, z; S)$$
$$\widetilde{R}^c(k, z; S) := p(S) R^c(k, z; S)$$

It is necessary to check whether the recursive formulation naturally follows for the normalized value functions. Using $p(S)q(S, S') = \beta p(S')$,

$$\begin{aligned}
\widetilde{R}^* &= \max_{k' \geq 0} \; (-k' - c(k, k'))p(S) + \mathbb{E} p(S) q(S, S') J(k', z'; S') \\
&= \max_{k' \geq 0} \; (-k' - c(k, k'))p(S) + \mathbb{E} \beta p(S') J(k', z'; S') \\
&= \max_{k' \geq 0} \; (-k' - c(k, k'))p(S) + \beta \mathbb{E} \widetilde{J}(k', z'; S')
\end{aligned}$$

Similarly,

$$\widetilde{R}^c = \max_{k^c \in \Omega(k)} \; (-k^c - c(k, k^c))p(S) + \beta \mathbb{E} \widetilde{J}(k^c, z'; S').$$

Therefore, the recursive form is preserved for the normalized value functions. As in Khan and Thomas (2008), the recursive form based on the normalized value function eases computation of the dynamic stochastic general equilibrium because the price, p, depends only on the current aggregate state variable, S.

A firm makes a large scale investment only if $R^*(k, z; S) > R^c(k, z; s)$. Therefore, a firm-level extensive-margin investment decision can be characterized by the threshold rule, g_{ξ^*}, as follows:

$$g_{\xi^*}(k, z; S) = \min \left\{ \frac{\widetilde{R}^*(k, z; S) - \widetilde{R}^c(k, z; S)}{w(S) p(S) k^{\zeta}}, \bar{\xi} \right\}.$$

This threshold rule is distinguished from the threshold rules in other existing models in that the threshold weakly decreases in the size of a firm. In other words, the required marginal benefit of large-scale investment is greater for large firms to make the extensive-margin investment than for small firms. This generates an empirically-supported cross-section of interest-elasticities. I quantitatively show this in Section 4.

I denote g_{k^*} as the optimal future capital stock conditional on the extensive-margin investment, g_{k^c} as the optimal future capital stock conditional on the small-scale investment, and g_k as the unconditional optimal investment.

Then, the following relationship holds:

$$g_k(k, z; S) = \begin{cases} g_{k^*}(k, z; S) & \text{if } \xi < g_{\xi^*}(k, z; S) \\ g_{k^c}(k, z; S) & \text{if } \xi \geq g_{\xi^*}(k, z; S). \end{cases}$$

That is, if a fixed cost shock ξ is less than the threshold, a firm makes a large-scale investment.

3.4 Recursive competitive equilibrium

In this section, I define the recursive competitive equilibrium in the economy.

$(g_c, g_a, g_{l_H}, g_{k^*}, g_{k^c}, g_{\xi^*}, g_{n_d}, \widetilde{V}, \widetilde{J}, \widetilde{R}^*, \widetilde{R}^c, p, w, G, H)$ is a recursive competitive equilibrium if the following conditions are satisfied.

1. $g_c, g_{lH}, \widetilde{V}$ and g_a, solves the household's problem.

2. $g_{k^*}, g_{k^c}, g_{\xi^*}, g_{n_d}, \widetilde{J}, \widetilde{R}^*$, and $\widetilde{R}^c$ solve a firm's problem.

3. Market Clearing:

(Labor Market)

$$g_{lH}(\Phi; S) = \int \left(g_{n_d}(k, z; S) + \left(\frac{g_{\xi^*}(k, z; S)}{\bar{\xi}} \right) \left(\frac{g_{\xi^*}(k, z; S)}{2} \right) k^{\zeta} \right) d\Phi$$

(Product Market)

$$\begin{aligned} g_c(\Phi; S) = \int \Bigg(& zAk^{\alpha} g_{n_d}(k, z; S)^{\gamma} \\ & - \Big((g_{k^*}(k, z; S) - (1-\delta)k) + c(k, g_{k^*}(k, z; S)) \Big) \times \frac{g_{\xi^*}(k, z; S)}{\bar{\xi}} \\ & - \Big((g_{k^c}(k, z; S) - (1-\delta)k) + c(k, g_{k^c}(k, z; S)) \Big) \times \frac{1 - g_{\xi^*}(k, z; S)}{\bar{\xi}} \Bigg) d\Phi \end{aligned}$$

4. Consistency Condition:[9]

$$\text{(Consistency)} \quad G_{\Phi}(\Phi) = H(\Phi) = \Phi', \text{ where for } \forall K' \subseteq \mathbb{K} \text{ and } z' \in \mathbb{Z},$$

$$\begin{aligned} \Phi'(K', z') = \int \Gamma_{z,z'} \Bigg(& \mathbb{I}\{g_{k^*}(k, z; S) \in K'\} \frac{g_{\xi^*}(k, z; S)}{\bar{\xi}} \\ & + \mathbb{I}\{g_{k^c}(k, z; S) \in K'\} \frac{1 - g_{\xi^*}(k, z; S)}{\bar{\xi}} \Bigg) d\Phi \end{aligned}$$

4 Quantitative analysis

This section quantitatively analyzes the macroeconomic implications of large firms' lumpy investments. First, I discipline the baseline model to fit the data moments by calibration. Especially, the different interest elasticities between small and large firms are the key moments to be fitted, which are hardly captured in alternative models. Second, I study the nonlinear dynamics of lumpy investments using impulse response analysis. The nonlinear dynamics arise from the synchronization of large-scale investment timing. Lastly, I

[9] $\mathbb{K}$ and $\mathbb{Z}$ are the supports of the marginal distributions of capital and productivity induced from Φ.

quantitatively analyze how the large firms' synchronization pattern affects the aggregate investment dynamics after a one-standard-deviation TFP shock and the aggregate interest-elasticity.

4.1 Calibration

In this section, I elaborate on how the model is fitted to the data and compare the fitness with alternative models. Table 1 reports the target and untargeted moments from the data and the simulated moments in the model. Table 2 reports the calibrated parameters given the fixed parameters reported in Table E10. In the simulation step, I use the non-stochastic method in Young (2010).

Table 1: Fitted Moments

Moments	Data	Model	Reference
Targeted moments			
Semi-elasticity of investment (%)	7.20	6.63	Zwick and Mahon (2017)
Cross-sectional semi-elasticity ratio (%)	1.95	2.13	Zwick and Mahon (2017)
Cross-sectional average of i_t/k_t ratio	0.10	0.10	Zwick and Mahon (2017)
Cross-sectional dispersion of i_t/k_t ($s.d.$)	0.16	0.16	Zwick and Mahon (2017)
Cross-sectional average spike ratio	0.14	0.14	Zwick and Mahon (2017)
Positive investment rate	0.86	0.86	Winberry (2021)
Time-series volatility of $log(Y_t)$	0.06	0.07	NIPA data (Annual)
Untargeted moments (all in yrs.)			
Average inaction periods	6.38	7.72	Compustat data
Dispersion of inaction periods	4.87	5.50	Compustat data
Average of lag diff. of inaction periods	0.27	0.67	Compustat data
Dispersion of lag diff. of inaction periods	6.47	8.36	Compustat data

Notes: The data moments are from the sources specified in the reference column. The same sample restriction as in the empirical analysis applies to Compustat data. I use linearly detrended real GDP from the National Income and Product Accounts at the annual frequency for the aggregate output volatility.

The target semi-elasticity of average investment is from Zwick and Mahon (2017). The cross-sectional semi-elasticity ratio is also from the same paper, which documents that small firms' investments are around twice elastic as large firms towards the interest rate change. In the paper, large and small firms are defined as the top 30% and bottom 30% firms in terms of size, respectively. I define large and small firms consistently in the model with their definition. The cross-sectional average and dispersion of the investment-to-capital ratio and the average spike ratio are targeted to match the levels in Zwick and Mahon (2017) as in Winberry (2021). Consistent with the literature, I define the spike ratio as the fraction of firms investing greater than 20% of the existing capital stock. The target of positive investment rate is from Winberry (2021). The positive investment rate is defined as the fraction of firms with an investment that is greater than 1% but smaller than 20% of existing capital stock. Only a negligible fraction of firms make a negative investment in both data and the model. To discipline the aggregate TFP-driven fluctuations in the model, I target the output volatility calculated from annual National Income and Product Accounts (NIPA) data.

Table 2: Calibrated Parameters

Parameters	Description	Value
Internally calibrated parameters		
ζ	Fixed cost curvature	3.500
$\bar{\xi}$	Fixed cost upperbound	0.440
μ^I	Capital adjustment cost	0.780
ν	Small investment range	0.041
σ	Standard deviation of idiosyncratic TFP	0.130
σ_A	Standard deviation of aggregate TFP shock	0.025
Externally estimated parameters		
ρ	Persistence of idiosyncratic TFP	0.750

Notes: Parameters in the upper part of the table are calibrated to match the moments in Table 1. The persistence of idiosyncratic TFP is directly computed from fitting the estimated firm-level TFP (Compustat) into AR(1) process. The firm-level TFP is estimated following Ackerberg et al. (2015) using US Compustat data.

In the model, variations in the fixed cost parameter and convex adjustment cost parameter lead to a sharply divergent effect on the dispersion of the investment rate (investment-to-capital ratio), while both lowers the average investment rate. For a higher fixed cost parameter, the dispersion of investment rate is higher as the difference in the investment rate between extensive-margin adjusters and non-adjusters increases.[10] On the other hand, a higher convex adjustment cost uniformly mutes down the investment rate, leading to a lower dispersion in the investment rate. These two divergent effects, together with the average investment rate, identify the fixed and convex adjustment cost parameters.

The fixed cost curvature parameter ζ is identified from the cross-sectional semi-elasticity ratio between small and large firms. As ζ increases beyond unity, the large firms' interest-elasticity decreases due to the lengthened (S, s) band.[11] The calibrated level of ζ is 3.5, which I interpret as 3.5 establishments are involved per production line on average.

As can be seen from Table 1, the baseline model (column 1) can correctly capture the cross-sectional elasticity ratio between small and large firms. Therefore, the baseline model provides an appropriate framework for analyzing the role of large firms' investment in the dynamic stochastic general equilibrium. This is one of this paper's contributions, as the interest-elasticity cross-section is not well-captured in the existing model framework.[12]

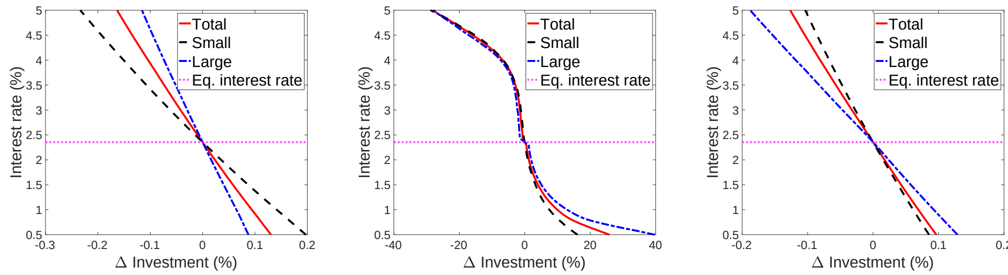
Figure 3 visualizes the large and small firms' interest-elasticities for the

[10] If a fixed cost is too high, the fraction of adjusters become too small to have meaningful contribution to the investment rate dispersion.

[11] Gnewuch and Zhang (2022) studies how monetary policy shock affects the distribution of investment rates, and they document that young firms are more sensitive to the shock than old firms. In this elasticity difference, they conclude that the extensive-margin sensitivity plays a crucial role, consistent with the results in my paper.

[12] I theoretically and quantitatively point out that the cross-sectional ranking of the interest-elasticities of investment between large and small firms is counterfactually flipped in existing model frameworks in Appendix A.

Figure 3: Semi-elasticities of investments across different models



(a) Baseline (b) Fixed cost only (c) Convex + Fixed

Notes: The figure plots the deviation of investment from the steady-state level when the interest rate changes for each different model. The vertical axis is the interest rate in per cent, and the horizontal axis is the percentage deviation from the steady-state investment. The horizontal dotted line indicates the equilibrium interest rate.

baseline model (panel (a)), for a model with fixed cost only (panel (b)), and for a model with convex and fixed cost (panel (c)).[13] In each panel, the vertical axis is the interest rate in per cent, and the horizontal axis is the percentage deviation from the steady-state investment. The horizontal dotted line indicates the equilibrium interest rate. As the interest rate decreases, all models' average deviation of investment from the steady-state increases. In the baseline model (panel (a)), the ranking of the interest elasticity across the firm-size group is consistent with the empirical patterns, as can be seen from the steeper curve of the large firms. However, in the model with convex and fixed adjustment cost (panel (c)), the large firms' average deviation of investment from the steady-state increases faster than small firms as the interest rate decreases. In the model with a fixed cost only (panel (b)), the interest-elasticities of all groups are significantly higher than the ones in the other two models, as can be checked from the large-scale variation along the horizontal axis.

[13]The model with convex and fixed adjustment cost is a prototype of the models in Winberry (2021) and Koby and Wolf (2020).

Finally, I compare the business cycle statistics implied in the baseline model with the aggregate-level data. The aggregate-level data at the annual frequency is from National Income and Product Accounts (NIPA) data, and the sample period starts from 1955. All the variables are in log and linearly detrended. Table E.11 reports the business cycle statistics from the data and the model. Among the statistics, the time-series volatility of the log output is the targeted moment.

The correlations across the aggregate variables in the baseline model are well-matched with the observed level in the data. Especially, the autocorrelation of aggregate investment and the cross-correlation between the aggregate investment and output are sharply matched even if they are not the targeted moment. For the relative volatilities of consumption and investment, the model's moments are slightly lower than the observed level.

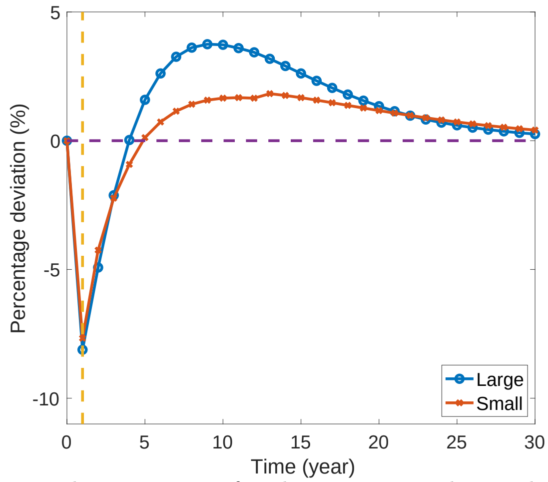
4.2 Synchronization

In this section, I analyze how the large and small firms differently respond to the same productivity shock using the impulse response analysis. Figure 4 plots the impulse responses of the spike ratios of large and small firms to the negative one-standard-deviation aggregate TFP shock.[14] The impulse response is obtained from the method that computes the transition path to the stationary allocation after an unexpected negative one-standard deviation TFP shock. All the responses are expressed in percentage deviation from the steady-state level.

Upon the arrival of the negative aggregate TFP shock, the extensive-margin investment timings are synchronized for a large group of firms regardless of

[14] The shock is assumed to be as persistent as the calibrated aggregate TFP shock.

Figure 4: Impulse response of spike ratio



Notes: The impulse response of spike ratios are obtained from the transition dynamics to the stationary equilibrium allocations after an unexpected negative one-standard-deviation TFP shock.

their size. It is because the firms realize it is not a good idea to install new large-scale capital as the business prospect is not promising in the near future. So, firms that are ready for the extensive-margin investment tend to delay the plan, leading to synchronized timings of large-scale investments.[15] The dynamics of the investment timings after this initial synchronization are starkly different across the different firm size groups.

For large firms, initial synchronization leads to a surge in spiky investments. This is because the large firms are interest-inelastic in the model and thus are strictly less affected by the general equilibrium effect. Therefore, the timings of large firms' lumpy investments are persistently synchronized.

On the other hand, the synchronized investment timings of small firms are spread out over the post-shock period. This is because the general equilibrium effect makes the small firms deviate from the concentrated period for

[15] In other words, it is an exogenous aggregate shock that initially synchronizes the investment timings of the firms.

large-scale investment. In other words, the general equilibrium effect strongly smoothens their investment timings.

4.3 Fragility after a surge of lumpy investments

I solve the model with the aggregate uncertainty using a new methodology called the repeated transition method. Due to the highly nonlinear aggregate dynamics, the existing solution algorithms fail to accurately compute the solution. So, I have contemporaneously developed the new methodology in Lee (2023), which can solve nonlinear dynamic stochastic general equilibrium globally and accurately without specifying the aggregate law of motion. The method is described in Appendix D. Using the equilibrium allocations obtained from the new methodology, I study how the synchronized investment timings of large firms affect the aggregate investment dynamics over the business cycle. First, I define a fragility index that captures the portion of large firms that have just finished large-scale investments as follows:

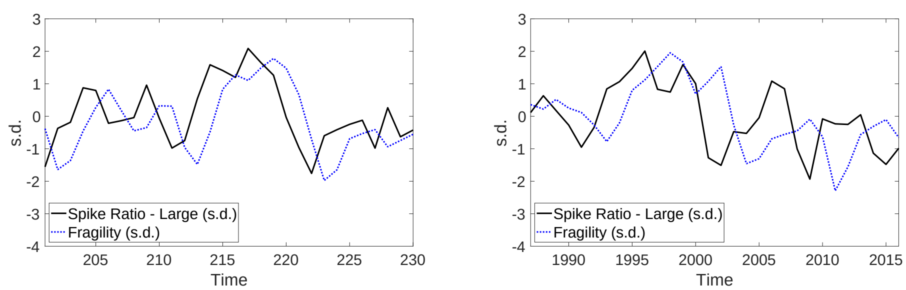
$$Fragility_t := \frac{\sum \mathbb{I}\{s_{it} \leq \overline{s}\}\mathbb{I}\{k_{it} > \overline{k}\}}{\sum \mathbb{I}\{k_{it} > \overline{k}\}}$$

where s_{it} is the time from the last lumpy investment; $\overline{s}$ is the threshold where any firm i with s_{it} below the level has recently adjusted its capital in the extensive margin; $\overline{k}$ is the size threshold of large firms. If a great fraction of large firms have just finished a large-scale investment, a relatively small fraction of large firms are willing to make a large-scale investment due to the presence of the fixed adjustment cost. Over the business cycle, the fluctuations in this index interplay with the exogenous TFP fluctuations, as the following analyses will conclude.

The median duration between two lumpy investments is 6 years in both the model and the data. In the regression that includes the fragility index, reported in Table 3, I found $\overline{s} = 3$ maximizes the fitness of the regression. The size cutoff $\overline{k}$ is set consistent with Section 2.

It is worth noting that the fragility index is constructed from the readily observable micro-level variables. Especially, the measure is based on the past investment history of large firms, which are mostly listed and subject to financial reporting regulations. Therefore, the index can be measured in a timely manner and can contribute to predicting the near future of aggregate investment. This feature is starkly contrasted with the existing indices in the literature based on the joint distribution between capital stock and productivity that is not directly observable (Caballero and Engel, 1993; Bachmann et al., 2013; Baley and Blanco, 2021).

Figure 5: Time series of fragility indices in simulation and data



(a) Simulation (b) Data

Notes: Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. Panel (a) plots a part of the simulated allocations. The solid line plots the aggregate investment growth rate (%). The dotted line plots the fragility indices normalized by the standard deviation. The fragility indices are calculated based on the distribution of large firms.

Figure 5 shows the time series of fragility index and spike ratio in the simulation (panel (a)) and the data (panel (b)), where each series is normalized by the standard deviation around the average. In both panels, the time series of the spike ratio leads the fragility index by two to three years. As the average inaction takes around six years, around three years after a surge of lumpy investment (spike ratio), a trough is expected to arrive. By the definition of the fragility index, during this trough of lumpy investment, the index will rise, indicating only a small fraction of firms are willing to make a lumpy investment. Therefore, the growth rate of the spike ratio and the fragility index tend to co-move in the opposite direction. Figure 6 is the scatter plot of the simulated time series where the horizontal axis is the fragility index normalized by the standard deviation, and the vertical axis is the growth rate of the large firms' spike ratio.[16] By fitting the relationship between the fragility and the growth rate of spike ratio into linear regression, I find the following relationship:

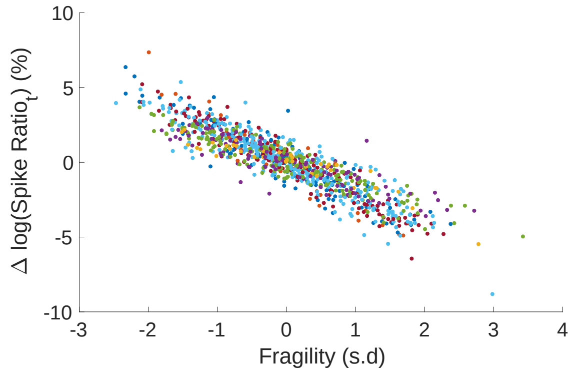
$$\Delta log(SpikeRatio_t)(\%) = \underset{(0.0274)}{-\,1.8936} * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.828$$

The relationship indicates that one standard deviation increase in fragility is negatively associated with the growth rate of the large firms' spike ratio by 1.89%. As can be seen from the high R^2, these two variables are tightly related along the business cycle. While the growth rate of the large firms' spike ratio is not known before period t, the fragility index is known ahead of period t. Therefore, the fragility index has predictability for the one-period-ahead growth rate of the large firms' spike ratio.

Then, I study how the fragility index fluctuations affect the sensitivity of aggregate investment growth to the output shock. Table 3 reports the regression result of the following specification in both the model and the data separately for negative and positive output shocks:

[16] The past aggregate shock A_{t-1} and the contemporaneous shock A_t are controlled by taking out fixed effects. The different colors of the dots are for different combinations of A_{t-1} and A_t.

Figure 6: Fragility index and the growth rate of the large firms' spike ratio



Notes: The vertical axis of the scatter plot is the spike ratio in percentage deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

$$\Delta log(I_t) = \alpha + \beta^{Shock} OutputShock_t + \beta^{Fragility} OutputShock_t \times Fragility_t + \epsilon_t$$

where $\Delta log(I_t)$ is the *aggregate* investment growth rate. $OutputShock_t$ is a shock in the log output, obtained from the residuals in the AR(1) fitting of the log output time series. The aggregate investment and output data are from National Income and Product Accounts data. In this specification, $OutputShock_t$ exogenously arrives at t, while the $Fragility_t$ is determined at $t-1$. Therefore, two variables are independent of each other.[17]

In Table 3, the coefficient estimates from the model and data are statistically indifferent, while each coefficient itself is statistically significant. When the fragility index increases by one standard deviation, the aggregate investment growth rate additionally decreases by 1.8% and 2.4% for one-standard

[17]The measurement of output shock is subject to an endogeneity issue which will be discussed below. The independence holds only when the exogenous output shock is properly measured.

Table 3: State-dependent sensitivity of the aggregate investment growth

	Dependent variable: $\Delta log(I_t)$ (p.p.)			
	(−) $OutputShock_t$		(+) $OutputShock_t$	
	Model	Data	Model	Data
$OutputShock_t$ (*s.d.*)	9.218	5.818	8.928	6.937
	(0.145)	(1.338)	(0.141)	(1.221)
$OutputShock_t \times Fragility_t$ (*s.d.*)	1.753	2.430	-1.861	-1.486
	(0.094)	(1.311)	(0.103)	(0.495)
Constant	Yes	Yes	Yes	Yes
Observations	507	16	494	18
R^2	0.904	0.790	0.900	0.705
Adjusted R^2	0.903	0.755	0.900	0.663

Notes: The dependent variable is the growth rate of aggregate investment. The independent variables are output shocks obtained from fitting output series into AR(1) process and the interaction between the output shock and the fragility index. The fragility index is based on the years from the last lumpy investment of large firms. The first two columns report the regression coefficients from the simulated data and Compustat data when the negative output shock hits. The third and fourth columns report the regression coefficients when the positive output shock hits. The numbers in the brackets are standard errors.

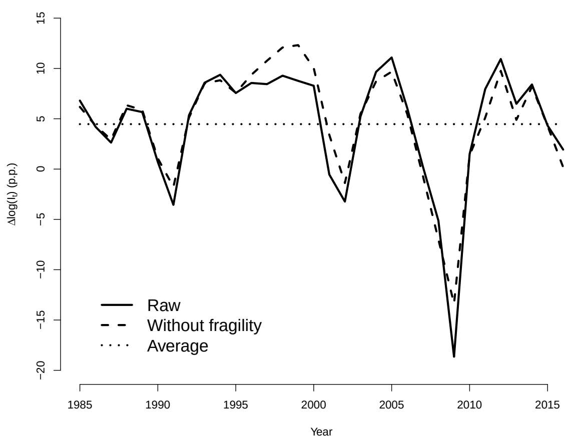
deviation negative output shock in the model and the data. In contrast, the aggregate investment growth rate increases less by 1.9% and 1.5% for one-standard deviation positive output shock in the model and the data when the fragility index increases by one standard deviation. The amplifying effect of the negative output shock and the mitigating effect of the positive output shock under the high fragility state are all due to the missing lumpy investments of large firms. That is, after a surge of lumpy investments of large firms, the negative shock leads to a deeper drop in the aggregate investment, and the positive shock leads to only a mitigated increase in the aggregate investment.

In Table C.6, I report the full regression results under different specifications. When the output shock is the only independent variable in the regression, around 73% and 52% of the investment growth rate variations are explained, respectively, for negative and positive shocks in the data. Once the fragility fluctuation is considered, R^2's improve to 79% and 71%.

Using the estimate from the data in Table 3, I quantify the portion of the investment growth rate that is accounted for by the interaction between the output shock and the fragility index. Specifically, the fragility-adjusted investment growth rate $g_t^{adj,I}$ is obtained as follows:

$$g_t^{adj,I} = \Delta log(I_t) - \widehat{\beta}^{Fragility} OutputShock_t \times Fragility_t.$$

Figure 7: Fragility-adjusted investment growth



Notes: The solid line is the aggregate investment growth rate from NIPA. The dashed line is the fragility-adjusted investment growth. The dotted line is the average level of the aggregate investment growth rate.

Figure 7 plots the time series of the raw aggregate investment growth rate (solid line) and the fragility-adjusted investment growth rate (dashed line). After the adjustment, the investment drops during the three recessions are mitigated. Table 4 compares the deviations from the average level for the raw and the fragility-adjusted investment growth rates in the recent three recessions of the sample period. Around 23% of the deviation from the average level is accounted for by the fragility effect during the recession. When the standard deviations of each time series are compared, around 30% of aggregate investment volatility can be explained by the interaction effect ($0.30 \approxeq 0.018/0.060$).

Table 4: Investment growth rates during the recessions

	Distance between investment growth rate and average: $\Delta log(I_t)$ (p.p)		
	Raw data (NIPA)	Without fragility	Adjusted portion (%)
Recession-1991	-8.019	-6.239	22.197
Recession-2001	-7.695	-5.852	23.951
Recession-2009	-23.112	-17.847	22.780

Notes: The first column reports the investment growth rate (%) at recession years of 1991, 2001, and 2009 minus the average investment growth ($\approxeq$ 4.5%). The second column reports the adjusted investment growth rate after removing the predicted component from the fragility indices using the coefficients of Table 3. The third column reports the adjusted portion (%).

However, the results above are only partially satisfactory due to an endogeneity issue. Specifically, the measured output shock is not fully exogenous because the fragility dynamics affects the future output realization. For example, a high fragility lowers the future capital stock, leading to a lower output. However, the current measurement of the output shock makes the fragility-driven output drop loaded on the shock magnitude.[18] This problem is hard to solve in a reduced-form approach due to the nonlinear dynamics of the fragility index.

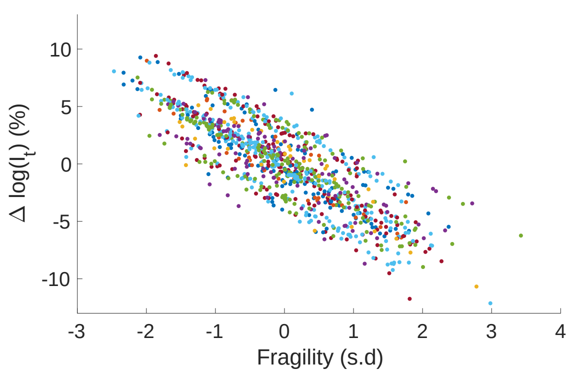
To sharply quantify the extra variation of aggregate investment driven by the fragility fluctuations without the endogeneity problem, I utilize the simulated path of the equilibrium allocations. Specifically, I hit the economy at each period on the simulated path with an unexpected one-standard-deviation TFP shock and compute the contemporaneous response under the general

[18]Therefore, it is likely that the role of the fragility index is underestimated.

equilibrium. Each period on the simulated path features a different fragility index level while the TFP shock's magnitude is fixed. Therefore, this experiment provides a setup to investigate the relationship between the investment response variations and the fragility index fluctuations.

Figure 8 shows the state-dependent contemporaneous responses of aggregate investment growth rates.[19] The horizontal axis is the fragility index normalized by the standard deviation. The vertical axis is the deviation of the aggregate investment growth from the average in percentage point. The prior aggregate shock A_{t-1} is controlled by teasing out the fixed effect.[20]

Figure 8: State-dependent responses of aggregate investment growth



Notes: The vertical axis of the scatter plot is the instantaneous response of the aggregate investment growth to a negative one-standard-deviation TFP shock in percentage point, and the horizontal axis is the fragility index measured in the unit of standard deviation from the average. In each responses, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

As can be seen from the figure, there is a significant negative relationship between the contemporaneous response of the aggregate investment growth

[19] Figure 2 can be understood as a data counterpart of this figure, as the residualized investment variation increases in the average of the recent spike ratio of large firms.

[20] The different colors of dots represent the different fixed-effect groups.

rate $\Delta log(I_t)$ and the fragility index. By fitting the relationship into linear regression, I obtain the following result:

$$\Delta log(I_t)\ (p.p.) = -\underset{(0.0665)}{3.2022} * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.699$$

When the fragility index increases by one standard deviation, a contemporaneous response of the aggregate investment growth to the negative one-standard-deviation shock is mitigated by 3.2 percentage points. During the recessions in 1991 and 2001, the aggregate investment growth rate was around -3.5% and -3.2%. If the magnitude of the negative output shocks during these periods was beyond a single standard deviation, the drop in the investment growth would not have been sub-zero during these periods if it had not been for the fragility effect.

Lastly, I study how the fragility effect affects the output through the firm-level investment channel. Taking the same steps as above, I analyze how the instantaneous response of the output changes along with the fragility variation:

$$log(Y_t)\ (p.p.) = -\underset{(0.0142)}{0.6111} * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.651$$

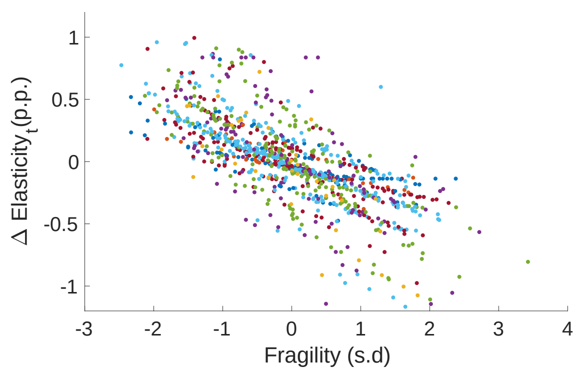
When the fragility increases by one standard deviation, the output drops by 0.6 percentage points further to the same negative aggregate TFP shock of a one-standard-deviation magnitude.[21]

[21] The scatter plot of the output responses and the fragility variations is available in Appendix E.

4.4 Policy implication: State-dependent interest-elasticity of aggregate investment

In this section, I discuss the policy implications of the fluctuations of the fragility index over the business cycle. In the economy captured in the baseline model, the aggregate investment features a strong history dependence.[22] This history dependence not only affects the aggregate investment's response to the TFP shock but affects its elasticity to the interest rate change.

Figure 9: State-dependent semi-elasticities of aggregate investment



Notes: The vertical axis of the scatter plot is the semi-elasticity of aggregate investment in percentage point deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. For each elasticity, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

To study how the aggregate investment responds differently to the same interest shock depending on the fragility state, I hit the economy at each period on the simulated path with an unexpected interest rate shock and compute the contemporaneous response under the partial equilibrium.[23] In

[22]Given that the aggregate state includes all the relevant information from history, the state dependence and the history dependence are interchangeable in the model.

[23]Therefore, the analysis is measuring the semi-elasticity of investment at each timing on

particular, I compare the contemporaneous average change in the investment when the interest rate unexpectedly changes and returns immediately in the subsequent period to the level where the interest is supposed to be without the exogenous shock. And the benchmark investment level is the contemporaneous investment when the interest rate is assumed to be staying at the same level. I calculate the average between the elasticity measured when the interest rate increases by 1% and the one measured when the interest rate drops by 1% to address the asymmetry in the responses to the positive and negative interest rate shocks.[24]

Figure 9 is the scatter plot of the interest-elasticities of the aggregate investment in relation to the fragility state. The horizontal axis is the fragility index normalized by the standard deviation; the vertical axis is the interest-elasticity in percentage point deviation from the steady-state.[25] According to the figure, there is a significant negative relationship between the fragility and the interest-elasticity of aggregate investment. By fitting the relationship into linear regression, I obtain the following result:

$$\mathit{\Delta Elasticity}_t \ (p.p) = -\underset{(0.0086)}{0.2689} * \mathit{Fragility}_t \ (s.d) + \epsilon_t, \quad R^2 = 0.497$$

One standard deviation increase in the fragility index decreases the interest elasticity of aggregate investment by around 0.27 percentage points. The in-

the business cycle.

[24]For example, if the interest is 0.03 at period t, I first compute the firm-level investment in the following three cases: i) when the interest rate jumps up to 0.04 only in period t and then stays in 0.03; ii) when the interest rate drops down to 0.02 only in period t and then stays in 0.03; iii) when the interest rate stays at 0.03 forever. Then, I obtain the average between the investment difference between case iii) and case i) and the investment difference between case iii) and case ii).

[25]The prior aggregate shock A_{t-1} is controlled by teasing out the fixed effect, and the different colors of dots represent the different fixed-effect groups.

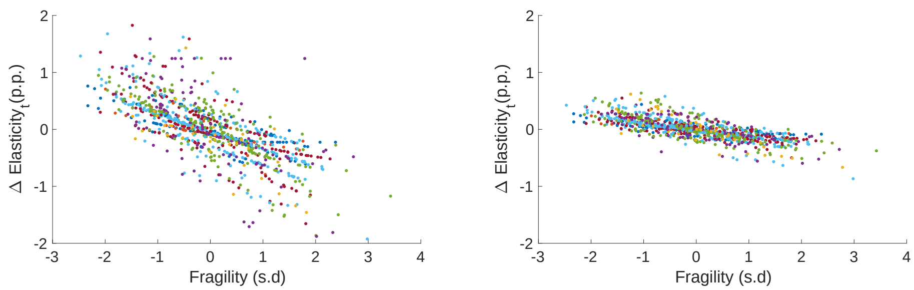
tuitive explanation for the result is that when the fragility index is high, there are not many large firms that can flexibly participate in and out of large-scale investment. This decreases the interest-elasticity of aggregate investment in a high-fragility state.

To verify that large firms drive interest elasticity fluctuations in aggregate investment, I compute the interest elasticity variations separately for the investment of large and small firms. Figure 10 is the scatter plot of interest elasticities along with the fragility variation for large (panel (a)) and small firms (panel (b)). The negative relationship between the fragility index and the elasticity is significantly stronger in large firms. When two different elasticities are fitted into linear regression, the following relationship is obtained:

$$\begin{aligned}
\mathit{\Delta Elasticity}_t^{Large}\ (p.p) &= -\underset{(0.0130)}{0.3992} * Fragility_t\ (s.d) + \epsilon_t, \quad R^2 = 0.484 \\
\mathit{\Delta Elasticity}_t^{Small}\ (p.p) &= -\underset{(0.0039)}{0.1403} * Fragility_t\ (s.d) + \epsilon_t, \quad R^2 = 0.569
\end{aligned}$$

When the fragility index increases by one standard deviation, large firms' investment elasticity decreases by around 0.40 percentage points. On the other hand, the same variation in the fragility index decreases small firms' elasticity by 0.14 percentage points, and the difference is statistically significant. The time-series correlation between the elasticities of the aggregate investment and the large firms' elasticities is 0.99. This result shows that large firms dominantly drive the stark negative relationship between the average interest elasticities (of all firms) and the fragility index. Although large firms are interest-inelastic, the time-series variation in their interest elasticities is

Figure 10: State-dependent semi-elasticities of investments: Decomposition



(a) Large firms (b) Small firms

Notes: The vertical axis of the scatter plots is the semi-elasticity of large (panel (a)) and small (panel (b)) firms' investment in percentage point deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. For each elasticity, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

greater than those of small firms. This is because large firms' responses are highly state-dependent, while small firms are flexible to adjust at all times due to their small fixed adjustment cost.

The analysis above implicitly shows that if the fragility index is high, the monetary policy would not effectively operate through the firm-level investment channel. Given there were recessions in the recent periods that happened in the time of high fragility, the policy implication echoes Tenreyro and Thwaites (2016) that conventional monetary policies are less powerful during recessions especially through the business investment channels. Moreover, this paper adds to the findings by providing an endogenous mechanism of state dependence in monetary policy effectiveness. Importantly, the fragility index is a forward-looking variable and can be easily measured using readily observable large firms' data. Therefore, the fragility index can potentially contribute to the optimal monetary policy design in practice.

5 Concluding remarks

This paper analyzes the endogenous state dependence in the aggregate investment dynamics driven by synchronized firm-level lumpy investments. An economy becomes substantially more fragile to a negative aggregate shock after a surge of large firms' lumpy investments than it would otherwise be. I show this is due to the interest inelasticity of the large firms' investments, which generates persistently synchronized investment timings even under the general equilibrium. The economic significance of this channel is quantified in a heterogeneous-firm real business cycle model in which the cross-section of the semi-elasticities of firm-level investment is matched with the empirical estimates. In the model, the aggregate investment features a significant state dependence in the interest elasticities driven by fragility index fluctuations. This implies that after a surge of large firms' lumpy investments, the effectiveness of monetary policy can substantially fall due to the lowered interest elasticity of the aggregate investment.

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