The Convex Origin of Fixed Costs

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June 1, 2026

TWO TRADITIONS OF ADJUSTMENT COSTS

The central puzzle: micro lumpiness vs. macro smoothness.

- **Fixed cost tradition:** threshold rules, infrequent adjustment — price, capital.
- **Convex cost tradition:** smooth continuous actions — rational inattention.
- **Sharply different macro predictions:** monetary/inaction non-neutrality, Phillips curve slope, welfare costs.

Are these genuinely different economics, or two views of the same object?

THIS PAPER

Micro lumpiness vs. Macro smoothness

Fixed-cost and convex probability-cost models are ***Fenchel-dual****: a distribution* G *of fixed costs generates a convex cost* C *whose curvature reflects selection over heterogeneous adjusters — the deterministic fixed cost is the* ***knife-edge limit****, convexity is generic.*

(i) Closed-form $G \mapsto C \Rightarrow$ smooth FOC. Environment-free construction; value-function differentiability inherited from C replaces threshold machinery with $S = C'(\varphi)$.

(ii) Microfoundation of the adjustment hazard. Power-law G gives $\Lambda(x) = G(Bx^2/w)$ with local elasticity $\nu = \frac{d\log S}{d\log|x|} \cdot \gamma = 2 \cdot \gamma = \boxed{2\gamma}$ — surplus curvature × cost-concentration; the same γ decomposes the Phillips slope $\kappa = (1+2\gamma)\bar{\Lambda}$ into frequency and selection.

(iii) Rosetta stone: one G, five literatures; extends to GE frictions.

$$\boxed{\underbrace{\text{menu cost, lumpy investment}}_{G\text{-side}} \xleftrightarrow{\text{Fenchel}} \underbrace{\text{RI, discrete choice, control costs}}_{C\text{-side}}}$$

Pricing benchmark. Alvarez-Lippi-Oskolkov (2022)'s hazard exponent $\nu \approx 2$ implies $\hat{\gamma} = 1$ and a selection share $\approx \frac{2}{3}$ of the Phillips slope.

THE DUALITY

LUMPY ADJUSTMENT PROBLEM

Start from the simplest concrete instance — a Uniform menu cost. Consider an extensive-margin decision where adjustment entails a fixed cost $\xi \sim \mathit{Unif}(0, \bar{\xi})$:

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The optimal solution is a stochastic threshold rule: if ξ is low enough, adjust; otherwise, do not adjust. The threshold equates the cost of the marginal adjuster to the gain $v^a - v^n$:

$$\text{Adjust} \iff \xi \le \xi^* := \min\{v^a - v^n,\, \bar{\xi}\}.$$

The probability of adjustment is then $\mathbb{P}(\xi \le \xi^*) = \xi^*/\bar{\xi} = \min\left\{\dfrac{v^a - v^n}{\bar{\xi}},\, 1\right\}$.

Two objects emerge: the threshold ξ^* and the realised probability of adjustment.

PROBABILITY CHOICE PROBLEM

Now re-cast the same decision in a different language: instead of *drawing* a cost, directly *choose* the adjustment probability p and pay a convex penalty $c(p) = \frac{1}{2}\bar{\xi}\, p^2$:

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The optimality condition equates the marginal benefit of raising p to the marginal cost $\bar{\xi}\, p$:

$$v^a - v^n \;=\; \bar{\xi}\, p^*, \quad p^* \in [0,1].$$

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$$\boxed{\textit{[Fenchel duality]} \quad p^* \;=\; \mathbb{P}(\xi \le \xi^*)}$$

Same value, same probability. Two faces of one problem.

THEOREM 1: ANALYTICAL CONVEX DUALITY

The two formulations are equivalent under the closed-form analytical link:

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- **Payoff equivalence:** $\Pi^{FC}(S) = \Pi^{PC}(S)$ for all $S \geq 0$
- **Policy equivalence:** $\varphi^* = G(S/w)$ in both formulations
- **Fenchel duality structure:** Π^{PC} and C are Fenchel conjugates of each other
- **Path-by-path identity:** realised binary adjustment indicators coincide under natural coupling

The quantile integral gives the static, environment-free analytical link between G and C.

PROOF IN THREE LINES

1. **Expanded payoff:** substitute threshold $\xi^* = S/w$ into fixed-cost payoff

$$\Pi^{FC}(S) = S \cdot G(S/w) - w \int_0^{S/w} \xi \, dG(\xi)$$

2. **Change variables:** let $\varphi = G(S/w)$ so $S/w = G^{-1}(\varphi)$

$$\Pi^{FC}(S) = \varphi \cdot S - w \int_0^{G^{-1}(\varphi)} \xi \, dG(\xi) = \varphi \cdot S - C(\varphi)$$

3. **Convexity check:** differentiating C yields $C'(\varphi) = wG^{-1}(\varphi)$ (non-decreasing)

$$C''(\varphi) = \frac{w}{g(G^{-1}(\varphi))} > 0 \quad \Rightarrow \quad C \text{ strictly convex}$$

FOC $S = C'(\varphi)$ is sufficient $\Rightarrow \max_{\varphi}\{\varphi S - C(\varphi)\} = \Pi^{PC}(S)$

Economic intuition: Next slide.

CONVEXITY IS SELECTION (NOT ASSUMPTION)

The convex-cost tradition — Rotemberg pricing, quadratic capital adjustment, RI's quadratic attention penalty — *assumes* convexity as a modelling convenience, without a micro-story for why.

Here convexity is derived, with concrete economic content:

- **Marginal cost:** $C'(\varphi) = w\,G^{-1}(\varphi)$ — the cost faced by the *marginal adjuster*, the agent indifferent at the current threshold:
- Raising φ requires inducing a *more expensive* agent to act ⇒ marginal cost rises ⇒ **convexity is selection**
- **Curvature:** $C''(\varphi) = w/g(G^{-1}(\varphi))$ — the *inverse density of marginal types*; bunched g ⇒ gentle rise; sparse g ⇒ sharp rise

<u>*Economics*</u>: selection makes the marginal non-adjuster more expensive than the last adjuster
→ increasing MC → convexity.

Recasts an assumed object as a derived one:

- Quadratic adjustment cost ($C \propto \varphi^2$) is what uniform G induces (Khan-Thomas)
- Power-law adjustment cost ($C \propto \varphi^{(\gamma+1)/\gamma}$) is what power-law G induces (details will follow)
- Each "functional form" for C is secretly a distributional assumption on G

The micro-foundation the convex-cost literature has carried as an open question.

UNIVERSALITY: LUMPINESS IS KNIFE-EDGE

Proposition (Universality). The deterministic fixed cost is a singular limit.

- **Degenerate:** $G = \delta_{\bar{\xi}}$ gives $C(\varphi) = w\bar{\xi}\,\varphi$ (linear, no convexity); only $\varphi \in \{0, 1\}$.
- **Any ε-perturbation:** $G_\varepsilon = U[\bar{\xi} - \varepsilon,\ \bar{\xi} + \varepsilon]$ gives

$$C_\varepsilon(\varphi) = w(\bar{\xi} - \varepsilon)\varphi + w\varepsilon\,\varphi^2, \qquad C''_\varepsilon(\varphi) = 2w\varepsilon > 0.$$

- **As $\varepsilon \to 0^+$:** $C_\varepsilon \to w\bar{\xi}\,\varphi$ pointwise, $C''_\varepsilon \to 0$, and $\varphi^*_\varepsilon(S) \to \mathbf{1}[S \geq w\bar{\xi}]$.

Reframing: the convex model is generic; the lumpy model is the singular boundary case.

DIFFERENTIABILITY THEOREM (THEOREM 2)

For dynamic lumpy adjustment problems with absolute-continuity assumptions:

- **Value function:** $v(x; X) \in C^1$ in state x
- **Intensive policy:** $x'^*(x; X)$ is C^1 everywhere
- **Extensive policy:** $\Lambda^*(x; X)$ is C^1 where $\Lambda^* \in (0, 1)$
- **Envelope:** v_x given by Benveniste-Scheinkman

Why this matters: standard menu-cost methods need root-finding and kink handling. The duality delivers a smooth FOC $S = C'(\Lambda)$ with no threshold machinery — the model slots directly into FOC-based heterogeneous-agent solvers (Reiter 2009, Auclert et al. 2020).

Relation to prior work. The C^1 regularity itself is standard, following from general dynamic-programming arguments (Santos 1991) and obtained in lumpy settings via smooth-pasting (Baley-Blanco 2021). The cost object coincides with that of ALO (2022), recovered there from the hazard. *The contribution is the route: the map from the cost distribution to the convex cost is environment-free, and it yields a closed-form FOC $S = C'(\varphi)$ in any environment admitting a scalar adjustment surplus.*

KHAN AND THOMAS (2008) FOUNDATION MADE TRANSPARENT

- Khan-Thomas (2008): firms draw heterogeneous fixed costs uniform on $[0, \bar{\xi}]$
- Duality reveals *why* firm-level investment is smooth: uniform G induces $C(\varphi) = \frac{w\bar{\xi}}{2}\varphi^2$, strictly convex with smooth FOC $\varphi^* = S/(w\bar{\xi})$
- *The individual firm's problem is not lumpy* — smoothness is a property of the cost distribution, not of aggregation
- Structural reading: any G with continuous positive density on its support produces strictly convex C and smooth firm-level behaviour
- GE channels (demand aggregation, factor-price feedback) are a separate matter

G AS ROSETTA STONE

C–SIDE: COST SPECIFICATION AS IMPLICIT G

The reverse direction

- *Forward* (G-side): Pick a distribution G of fixed costs $\Rightarrow$ solve for cost function C via the quantile integral

$$C(\varphi) = w \int_0^{G^{-1}(\varphi)} \xi \, dG(\xi)$$

- *Reverse* (C-side): Given a cost function $C(\varphi)$, what is the implied distribution G?
- The link: $C'(\varphi) = wG^{-1}(\varphi)$ pins down G uniquely (up to scaling)
- *Implication*: Each literature's cost specification is an *implicit assumption* on the underlying distribution of fixed costs
  - Not a free modelling choice — an assumption that can be made explicit, tested, and recovered

THE ROSETTA STONE: FIVE LITERATURES, ONE PRIMITIVE

Distribution G	**Cost $C(\varphi)$**	**Form**	**Entropy**	**Literature**
Exponential	$(1-\varphi)\ln(1-\varphi)+\varphi$	Log. MC	KL divergence	Rational inattention
Power law (γ)	$\varphi^{1+1/\gamma}$	Iso-elastic	Tsallis q-div.	Generalised choice
Uniform $(\gamma=1)$	φ^2	Quadratic	L^2	Sparsity / KT2008
Degenerate	$\bar{\xi}\,\varphi$ on $\{0,1\}$	Linear	—	Lumpy adjustment
Type I EV (mv)	$\sum_j \varphi_j \ln \varphi_j$	Neg. entropy	Shannon	Logit discrete choice

- Each row: a distribution G on the left, its induced cost C via the quantile integral, and the literature where that cost function appears
- *Headline*: **Five literatures, one unifying parameter family — the distribution G of physical fixed costs**
- The choice of C in your model is secretly a choice of G.

WELFARE: PHYSICAL COST INTERPRETATION

From fixed to convex: closed-form welfare for menu costs

- Tractable welfare measure for fixed-cost models:

$$\text{Welfare cost} = C(\varphi^*) = w \int_0^{G^{-1}(\varphi^*)} \xi \, dG(\xi)$$

- The *expected physical cost paid by adjusters* — a resource-cost reading
- For power law G: $C(\varphi^*)$ closed-form alongside $\Lambda(x)$ and κ

From convex to fixed: welfare reading of information costs

- RI with exponential G: KL cost $\lambda \sum \varphi_j \ln(1/\varphi_j)$ (in nats) — λ is a shadow price of attention, no resource reading
- Fixed-cost primitive: $C(\varphi^*)$ is expected physical cost; λ becomes the *rate of the underlying cost distribution*, recoverable from micro data
- Same $C(\varphi)$, two structural interpretations; G selects which applies

Extended duality via externalities

FRICTION-EXTENDED DUALITY: $\widetilde{G}$ AND $\widetilde{C}$

Let an aggregate friction $F = F(X) \in (0,1]$ multiply the adjustment case (match / success probability). An agent enters iff

$$F \cdot V^a + (1-F)\cdot V^n - w\xi_i \;\geq\; V^n \quad\Longleftrightarrow\quad F \cdot S \;\geq\; w\,\xi_i.$$

Aggregate participation $\varphi = G(F\,S/w)$. Define the *friction-extended duality*:

$$\widetilde{G}(z) \equiv G(F\,z) \quad \text{on } [0, \bar{\xi}/F], \qquad \widetilde{C}(\varphi) \;=\; \frac{1}{F}\,C(\varphi).$$

The FOC under friction:

$$S \;=\; \widetilde{C}'(\varphi) \;=\; \frac{C'(\varphi)}{F} \quad\Longleftrightarrow\quad \boxed{F \cdot S = C'(\varphi)}$$

Power-law specialisation. $\widetilde{G}$ is the *same* power-law family with shape γ unchanged and support stretched to $\bar{\xi}/F$; $\widetilde{C}(\varphi) = (1/F)\,(w\bar{\xi}\gamma/(\gamma+1))\,\varphi^{(\gamma+1)/\gamma}$. **Cost-shape γ is orthogonal to the friction level F.**

SEARCH-AND-MATCHING AS THE CANONICAL INSTANTIATION

In equilibrium, F is endogenous: $F = F(\theta)$ where market tightness $\theta = v/u$ depends on aggregate participation φ. Each agent takes F *parametrically*; in equilibrium it is determined jointly.

Canonical specification: Diamond-Mortensen-Pissarides matching, $M(u, v) = Au^{\alpha} v^{1-\alpha}$, $F(\theta) = A\theta^{1-\alpha}$.

- The matching technology supplies F as a single scalar; the duality absorbs it via $\widetilde{G}$, $\widetilde{C}$
- F depends on aggregate φ through tightness: the duality is fully compatible with general-equilibrium feedback
- Cost-shape γ and matching elasticity α remain separable margins (shape-invariance of $\widetilde{G}$)

What this demonstrates: the framework reaches GE matching frictions without re-derivation. The well-known search-efficiency results — congestion externalities and the Hosios (1990) condition — follow as special cases of the construction, recovered on the next slide.

RECOVERING HOSIOS (1990) AS A SPECIAL CASE

When the friction $F = F(\theta)$ is supplied by an SaM matching technology, the friction-extended duality reproduces the canonical efficiency structure of the search literature:

$$\underbrace{F \cdot S}_{\text{private FOC}} + \underbrace{\frac{\partial F}{\partial \varphi} S\,\varphi}_{\text{congestion (Hosios)}} = \underbrace{C'(\varphi)}_{\text{planner FOC}} .$$

The wedge is the canonical congestion externality; the **Pigouvian entry tax** $\tau^{\star}(\varphi) = -(\partial F/\partial \varphi)\,S\,\varphi$ decentralises the planner — pinning down Hosios (1990), generalised by Mangin–Julien (2021).

What this demonstrates:

- The dual machinery *absorbs* GE matching frictions without re-derivation
- Hosios wedge and Pigouvian tax follow as a special case of the construction
- Cost-shape γ and matching elasticity α stay orthogonal: selection and search separable

Contribution claim. *Representational*: the unification reaches GE matching frictions; the efficiency results recovered here are Hosios (1990) and Mangin–Julien (2021), not new claims of this paper.

PRICING APPLICATION

A GENERIC MODEL

The aggregate state X: $X = (f, \text{exo. shocks})$

where $f(x)$ is the cross-sectional distribution of price gaps (following each firm's drift under inflation: $x \to x - \pi'$), and exogenous aggregates (demand, monetary policy).

The firm's value function in convex-cost form:

$$V(x;X) = \max_{\Lambda \in [0,1]} \Big\{ \Lambda\, V^a(X) + (1-\Lambda)\, V^n(x;X) - C(\Lambda) \Big\},$$

$$V^a(X) = \max_{x'} \Big\{ R(x';X) + \beta\, \mathbb{E}\big[\, m(X,X')\, V(x'-\pi';X')\,\big] \Big\},$$

$$V^n(x;X) = R(x;X) + \beta\, \mathbb{E}\big[\, m(X,X')\, V(x-\pi';X')\,\big].$$

Three key quantities:

- $V^a(X)$: *value of repricing*—choose optimal reset gap x', clear today's mispricing
- $V^n(x;X)$: *value of not repricing*—gap drifts by inflation $-\pi'$, future mispricing remains
- *Repricing surplus*: $S(x;X) := V^a(X) - V^n(x;X)$ — *the gain from closing the gap*

The Fenchel duality FOC $S = C'(\Lambda)$ pins the repricing hazard state-by-state.

REPRICING WITH STATIC APPROXIMATION

Why a static approximation? The full dynamic surplus $S(x;X)$ is hard to compute analytically. But near the optimum, flow mispricing losses are *approximately quadratic* in the gap. This quadratic structure is also stable across the parameter space—allowing a closed-form decomposition of the Phillips slope into frequency and selection that survives recalibration.

Static repricing rule: approximate $S(x;X) \approx Bx^2$ (Alvarez-Le Bihan-Lippi 2016; ALO 2022), so repricing occurs when

$$w\xi \leq Bx^2.$$

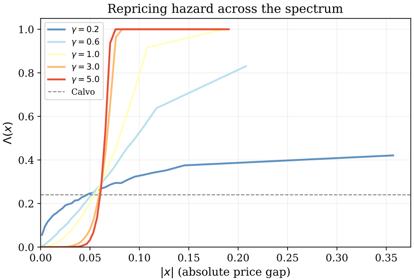
Apply the duality (Theorem 1): By Theorem 1, this fixed-cost problem maps exactly onto its convex-cost dual. The extensive-margin FOC $S = C'(\Lambda)$ becomes

$$\Lambda(x) = G\left(\frac{Bx^2}{w}\right),$$

the state-dependent *repricing hazard*. For power-law $G(\xi) = (\xi/\bar{\xi})^\gamma$, this simplifies to $\Lambda(x) = (Bx^2/(w\bar{\xi}))^\gamma$.

This hazard is the single primitive from which all aggregates follow: the Phillips slope, the selection wedge, monetary non-neutrality.

HAZARD SHAPES ACROSS THE SPECTRUM



- **One knob, two poles:** as γ rises the hazard morphs continuously from flat (Calvo) to step (Golosov–Lucas).
- Local slope $d\log\Lambda/d\log x = 2\gamma$: the chain rule reads off the selection elasticity directly from the curve.
- Intermediate γ (e.g. ALO's $\gamma = 1$) sits squarely inside the spectrum — neither extreme is a knife-edge.

The Calvo–Golosov–Lucas spectrum

Two canonical pricing models correspond to polar extremes of the hazard:

- Calvo (1983): Repricing probability $\Lambda(x) = \lambda$ is constant, independent of the gap. Cannot arise from any non-degenerate absolutely continuous G.
- Golosov–Lucas (2007): Fixed cost $\xi = \bar{\xi}$ is deterministic. Hazard is a step function: $\Lambda(x) = \mathbf{1}[|x| \geq \bar{x}]$ with inaction band $\bar{x} = \sqrt{w\bar{\xi}/B}$. Produces the most concentrated repricing.

Power-law family $G(\xi) = (\xi/\bar{\xi})^{\gamma}$ spans the spectrum:

- Low γ: Dispersed costs, gentle hazard → Calvo-like
- High γ: Concentrated costs, steep hazard → Golosov-Lucas-like
- Single parameter γ = degree of cost concentration; hazard: $\Lambda(x) = \min\{(Bx^2/(w\bar{\xi}))^{\gamma}, 1\}$

Micro-foundation of ALO's generalised hazard elasticity

ALO, in a continuous-time sticky-price model with Brownian gap and Poisson cost arrivals, show that a generalised hazard $\Lambda(x)$ admits two foundations: random menu cost G (their Section 2.1) and convex flow cost $c(\ell)$ on Poisson intensity (their Section 2.2), with c recovered from Λ by solving a 2nd-order linear ODE.

Where C and ALO's c coincide: on $\varphi \in [0, 1]$, Theorem 1's $C(\varphi)$ equals $w \cdot c(\varphi)$: both equal $w \int_0^{\varphi} G^{-1}(s)\, ds$.

In pricing: duality microfounds the hazard elasticity.

- The duality generates the entire hazard $\Lambda(x) = G(Bx^2/w)$ from $(G, \text{surplus})$
- ALO posit a reduced-form generalised hazard with local power law $\Lambda(x) - \bar{\Lambda} \sim c\,|x|^{\nu}$ as $x \to 0$; the observed slope ν is their hazard elasticity

$$\underbrace{\nu}_{\substack{\text{how fast adjustment}\\ \text{probability rises with the gap}}} = \underbrace{\frac{d\log S}{d\log|x|}}_{\substack{\text{how fast the incentive}\\ \text{rises with the gap}}} \times \underbrace{\gamma}_{\substack{\text{how fast the adjuster pool}\\ \text{fills with the incentive}}} = 2\,\gamma$$

Identification. The benefit-side "2" is pinned by theory, so the *observed* hazard exponent ν identifies the *unobserved* cost-shape γ: ALO's $\nu \approx 2 \Rightarrow \hat{\gamma} \approx 1$. A structural primitive read off a reduced-form moment.

Scope: the "2" is the local benefit curvature under the standard quadratic-surplus approximation (AL/ALO); portable, static, and environment-free across applications.

PHILLIPS CURVE DECOMPOSITION: FREQUENCY VS. SELECTION

Let $\kappa \equiv \partial\pi/\partial\mu$ be the **Phillips slope**: the elasticity of inflation to a monetary nudge μ that shifts every firm's desired price. To first order in μ, the price-level response is linear in firm-level changes, so two margins enter *additively*.

Frequency (extensive)

Repricers pass the nudge through.

$$+\bar{\Lambda}$$

Calvo benchmark

Selection (intensive)

Marginal large-gap firms tip over threshold.

$$+2\gamma\,\bar{\Lambda}$$

Cor. 2: power-law $G \propto \xi^{\gamma}$, quadratic surplus

$$\boxed{\kappa(G) = \underbrace{\bar{\Lambda}}_{\text{frequency}} + \underbrace{2\gamma\,\bar{\Lambda}}_{\text{selection}} = (1+2\gamma)\,\bar{\Lambda}}$$

Selection share $= 2\gamma/(1+2\gamma)$ — set by cost-concentration alone; same $2\gamma = \nu$ as the previous slide.

SELECTION SHARE ACROSS THE COST-CONCENTRATION SPECTRUM

With $\kappa = (1 + 2\gamma)\bar{\Lambda}$, the *selection share* $2\gamma/(1 + 2\gamma)$ depends *only* on the tail parameter γ (not on $\bar{\Lambda}$, B, or w) — a single number indexes the entire Calvo-to-Golosov-Lucas continuum.

Model	γ	**Selection share**
Calvo	0	0%
Kurtosis match	0.4	44%
ALO benchmark	1	$\frac{2}{3} \approx 67\%$
Near Golosov-Lucas	4	89%
Golosov-Lucas	∞	100%

Takeaway: the empirically relevant ALO case puts **two-thirds of the Phillips slope on selection** — frequency alone (the Calvo benchmark) captures only one-third of the response to a monetary shock.

RATIONAL INATTENTION AS A MICROFOUNDATION OF STICKY PRICES

Established. Reis (2006), Mackowiak-Wiederholt (2009), and the broader sticky-price RI tradition derive sticky prices from informational frictions with Shannon-style information costs in place of menu costs.

What the duality adds — sharper. For any cost distribution G:

Menu-cost economy and inattention economy with the *same* G produce the same firm-level decisions, f_{ss}, $\bar{\Lambda}$, $\kappa = (1 + 2\gamma)\bar{\Lambda}$, and selection wedge $2\gamma\bar{\Lambda}$.

Relabel ξ: physical resource cost $\rightleftharpoons$ cognitive attention cost. The threshold rule "adjust iff $w\xi \leq S$" reads as "update iff cognitive value $\geq$ attention cost". Theorem 1 makes the relabelling exact at the ex-ante extensive margin.

The Calvo–Golosov-Lucas spectrum reads as a family of attention rules:

- Calvo (low γ) $\sim$ Reis (2006)'s time-dependent updating
- Golosov-Lucas (high γ) $\sim$ threshold attention in the Sims (2003) tradition
- Mackowiak-Wiederholt and broader RI sticky-price models occupy interior points

Identification. Micro pricing data identifies G, not what G measures. The menu-cost vs RI debate has been about an *unidentified* object: same primitive, two interpretations.

GLOBAL SOLUTION

GLOBAL SOLUTION: CONFIRMING THE ANALYTICAL DECOMPOSITION

The analytical decomposition $\kappa = (1 + 2\gamma)\bar{\Lambda}$ rests on two approximations:

- *Static surplus*: $S(x) = Bx^2$ — repricing gain is quadratic in the price gap
- *Interior support*: stationary gap distribution f_{ss} sits strictly inside the inaction band, with boundary mass negligible

Purpose of the global solution: two objectives.

- **Verify** the closed-form $\kappa = (1 + 2\gamma)\bar{\Lambda}$ holds in the full dynamic GE model where neither approximation is exact (dynamic surplus only approximately quadratic, local curvature $\sim 4\times$ static; survives because Cor. 1's invariance $\kappa/\bar{\Lambda} = 1 + 2\gamma$ does not depend on B).
- **Generate state-dependent GIRFs** — the operational payoff: paired Monte-Carlo impulse responses conditioned on cross-sectional price-gap dispersion, delivering state-dependent monetary transmission across the spectrum that linearised methods cannot produce.

Relation to Costain-Nakov. Costain-Nakov (2011, JME) solve smoothly-state-dependent pricing computationally via Reiter linearisation. The present solution is via RTM (Lee 2026) without aggregate linearisation.

RTM: SOLUTION METHOD AND OUTPUTS

Using the *Repeated Transition Method* (Lee, 2026), I globally solve the dynamic pricing model.

What the method produces:

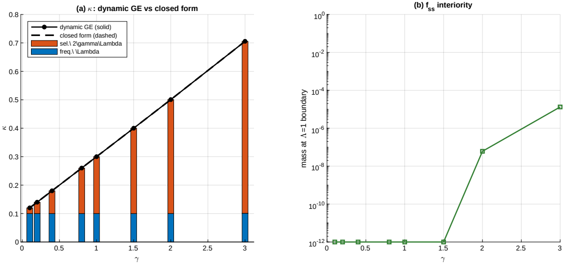
1. Solves at the benchmark and traces the spectrum across γ
2. Tracks the cross-sectional distribution $\boldsymbol{f_t}$ along its simulated path (no forecasting rule)
3. Produces aggregate dynamics without log-linearisation around steady state
4. Preserves the value-function kink at the inaction-band boundary

Two ingredients make this possible:

- Convex-cost representation $\Rightarrow$ smooth FOC without threshold machinery
- RTM matching on lagged price-dispersion Δ_{t-1} as the matching state

Note on scope. State-dependence in IRFs has prior precedent (Costain-Nakov-Petit); the dynamic exercise here serves as a consistency check on the static decomposition and as a vehicle for state-dependent IRFs at the benchmark.

SPECTRUM ACROSS γ IN THE DYNAMIC MODEL



Notes: (a) dynamic GE solution (solid) tracks the closed-form $(1+2\gamma)\bar{\Lambda}$ (dashed) across the spectrum. (b) Boundary mass $\leq 10^{-5}$ even at $\gamma = 3$, so the correction to κ is numerically zero.

- **Static closed-form is quantitatively accurate in the dynamic GE model:** hazard retains power-law form $\Lambda(x) = (Bx^2/(w\bar{\xi}))^\gamma$; local curvature $\approx 4\times$ static (discounted accumulation), and $\kappa = (1+2\gamma)\bar{\Lambda}$ holds as a tight numerical approximation.
- **Selection share preserved:** curvature-invariance $\kappa/\bar{\Lambda} = 1+2\gamma$ is independent of B; range ∼17% (low γ, Calvo-like) to ∼86% (high γ, GL-like), with the $\frac{2}{3} \approx 67\%$ benchmark at $\gamma = 1$ intact.

STATE-DEPENDENT GIRF: COMPARATIVE STATIC ACROSS γ

RTM generates **generalised impulse responses** (paired Monte-Carlo, conditioning on lagged price-dispersion Δ_{t-1} quartiles within TFP).

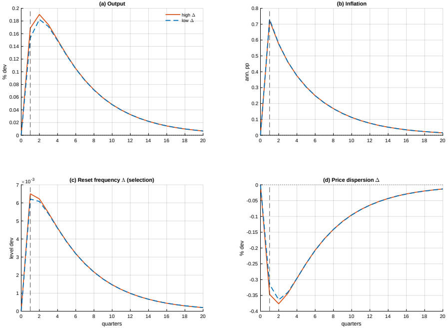
Impact response to +1-SD preference (demand) shock, by regime:

	$\hat{\gamma} = 0.4$ (sel. 44%)		$\hat{\gamma} = 0.5$ (sel. 50%)		$\hat{\gamma} = 1$ (sel. $\frac{2}{3}$)	
	hi-Δ	lo-Δ	hi-Δ	lo-Δ	hi-Δ	lo-Δ
Y (%)	+0.169	+0.154	+0.161	+0.147	+0.127	+0.120
π (ann. pp)	+0.723	+0.731	+0.692	+0.699	+0.584	+0.586
$\bar{\Lambda}$ (lvl)	+0.0065	+0.0062	+0.0069	+0.0066	+0.0074	+0.0073
Δ (%)	−0.347	−0.315	−0.321	−0.294	−0.236	−0.225
Y hi/lo gap	+10.0%		+9.0%		+5.6%	

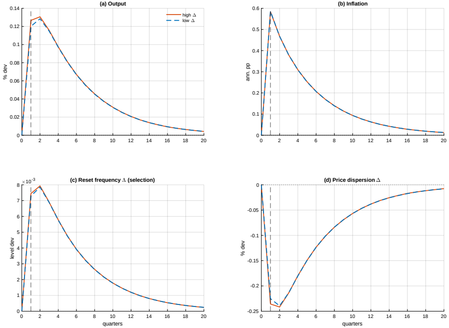
Takeaway: state-dependence is modest at all three calibrations and weakens monotonically as γ rises.

- Demand shock raises both Y and π; selection responds on impact via $\bar{\Lambda} \uparrow$ and $\Delta \downarrow$
- High-Δ output amplification: 10% → 9% → 6% as γ rises from 0.4 to 0.5 to 1
- Mechanism: as selection becomes dominant, adjustment concentrates on the most-mispriced firms regardless of cross-section composition ⇒ less state-dependence

GIRF FIGURES: $\gamma = 0.4$ VS $\gamma = 1$



(a) $\hat{\gamma} = 0.4$ (kurtosis-matching alt.)



(b) $\hat{\gamma} = 1$ (benchmark)

- High-Δ vs low-Δ paths fan out at $\gamma = 0.4$ but nearly collapse at $\gamma = 1$.
- On impact, $\bar{\Lambda}$ jumps and pulls Δ down — the selection channel is visible, not just inferred.
- Benchmark $\gamma = 1$ is the headline: ALO-consistent, residual state-dependence is small.

FURTHER APPLICATIONS

The duality applies to **any extensive-margin decision with heterogeneous costs**:

- **Firm entry/exit**: free-entry equates marginal entrant's cost to surplus; heterogeneous entry costs G micro-found the smooth extensive margin
- **Vacancy creation**: convex costs from heterogeneous vacancy creation costs; $C''(\varphi)$ reflects the density g
- **Labour force participation**: smooth supply elasticity $\approx g(G^{-1}(\varphi))/w$, without Rogerson (1988)-type lotteries
- **Technology adoption**: hazard $\Lambda(x)$ derived from the cost distribution, not assumed
- **Credit default**: same structure when costs are drawn from G

Identification: γ from $\log \Lambda \propto 2\gamma \log |x|$; G from size-distribution moments of adjustment events.

CONCLUSION

TAKEAWAYS

Three central insights:

1. **Fixed and convex costs are dual representations.** The distinction is not between two kinds of economic friction but between two parametrisations of the same friction, linked by Fenchel conjugacy. The structural primitive is the distribution G of heterogeneous fixed costs.
2. **The Rosetta Stone.** Five literatures — menu costs, lumpy investment, discrete choice, rational inattention, control costs — each assume a shape for G. Making this assumption explicit converts a free modelling choice into a testable distributional restriction.
3. **The Calvo–Golosov-Lucas spectrum.** A single parameter γ (the power-law exponent) governs the degree of cost concentration and traces a continuous spectrum from dispersed costs (Calvo-like, weak selection) to concentrated costs (Golosov-Lucas-like, strong selection). The structural mapping $\nu = 2\gamma$ ties γ to ALO's hazard exponent. **ALO's $\nu \approx 2$ implies the benchmark $\hat{\gamma} = 1$, with selection contributing $\frac{2}{3} \approx 67\%$** of the Phillips curve slope — selection as the dominant flexibility margin.

CONCLUSION

- fixed $\equiv$ convex — Fenchel duality
- $\nu = 2\gamma$ — benefit × cost shape
- ALO's $\nu \approx 2 \Rightarrow \hat{\gamma} = 1$; selection $\frac{2}{3}$ of PC slope
- Smooth FOC — global solvers for lumpy models

APPENDIX

RELATED LITERATURE ▸ back

- Geometric analysis of dynamic equilibrium
  - Solow (1956); Swan (1956); Ramsey (1928); Cass (1965); Koopmans (1963)
- Global stochastic equilibrium solution framework
  - Marcet (1988); Den Haan and Marcet (1990); Krusell and Smith (1998); Den Haan, (1996, 1997); Reiter (2001); Algan et al. (2008, 2010); Den Haan and Rendahl (2010); Reiter (2010); Ahn et al. (2018); Boppart et al. (2018); Elenev et al. (2021); Auclert et al. (2021); Cao et al. (2023); Azinovic et al. (2022); Fernández-Villaverde et al. (2023); Han et al. (2025); Payne et al. (2025); Lee (2026)
- Nonlinear equilibrium dynamics — state dependence
  - Kaplan and Violante (2014); Vavra (2014); Berger and Vavra (2015); Basu and Bundick (2017); Bloom et al. (2018); Kaplan et al. (2018); Petrosky-Nadeau et al. (2018); Baley and Blanco (2019); Pizzinelli et al. (2020); Berger et al. (2021); Melcangi (2024); Winberry (2021); Lee (2026)
- Random dynamical system
  - Arnold (1998); Schenk-Hoppé (1998); Schenk-Hoppé (2001); Yannacopoulos (2011)

BENCHMARK NUMBERS AT $\hat{\gamma} = 1$

Plugging the benchmark $\hat{\gamma} = 1$, $\bar{\Lambda} = 0.24$ into the closed-form decomposition $\kappa = (1 + 2\gamma)\bar{\Lambda}$:

$$\kappa = 3 \times 0.24 = 0.72$$

$$\text{Frequency component} = \bar{\Lambda} = 0.24 \quad (33\% \text{ of } \kappa)$$

$$\text{Selection wedge} = \kappa - \bar{\Lambda} = 0.48 \quad \left(\tfrac{2}{3} \approx 67\% \text{ of } \kappa\right)$$

Non-neutrality interpretation:

- Nominal non-neutrality: $1 - \kappa = 0.28$
- Relative to Calvo at same frequency: $(1 - \kappa)/(1 - \bar{\Lambda}) = 0.28/0.76 \approx 0.37$
- Selection reduces non-neutrality by $\approx 63\%$ relative to Calvo benchmark

Relation to Costain–Nakov. They decompose numerically across the spectrum; the closed-form selection share $2\gamma/(1 + 2\gamma)$ depending only on γ (not $\bar{\Lambda}$, B, or w) is the new layer.

THREE CANONICAL CORRESPONDENCES: DERIVATIONS

1. **Exponential** $G(\xi) = 1 - e^{-\lambda\xi}$
   - $C(\varphi) = \frac{w}{\lambda}[(1-\varphi)\ln(1-\varphi)+\varphi]; \quad C'(\varphi) = -\frac{w}{\lambda}\ln(1-\varphi)$
   - Information-theoretic counterpart: *generalized KL divergence*
   - Adjustment: $\varphi^* = 1 - e^{-\lambda S/w}$
2. **Power law** $G(\xi) = (\xi/\bar{\xi})^\gamma$ on $[0, \bar{\xi}]$
   - $C(\varphi) = \frac{\gamma}{\gamma+1} w\bar{\xi}\,\varphi^{1+1/\gamma}$ (iso-elastic, exponent $q = 1 + 1/\gamma$)
   - Information-theoretic counterpart: *Tsallis q-divergence*
   - γ controls cost concentration: $\gamma = 1 \to$ quadratic; $\gamma \to 0$ dispersed; $\gamma \to \infty$ concentrated
3. **Uniform** $G = U[0, \bar{\xi}]$ (equivalently $\gamma = 1$)
   - $C(\varphi) = \frac{w\bar{\xi}}{2}\varphi^2$ (quadratic, L^2 penalty)
   - Foundational in Khan-Thomas (2008) and Gabaix (2014)

POWER–LAW FAMILY: CURVATURE AND DENSITY

For $G(\xi) = (\xi/\bar{\xi})^{\gamma}$ with $\gamma > 0$:

$$\text{Density:} \quad g(\xi) = \frac{\gamma}{\bar{\xi}} \left(\frac{\xi}{\bar{\xi}} \right)^{\gamma - 1}$$

$$\text{Induced cost:} \quad C(\Lambda) = \frac{\gamma}{\gamma + 1} w \bar{\xi} \Lambda^{1 + 1/\gamma}$$

Shape interpretation:

- $\gamma < 1$: density *decreasing* — cheap adjustment abundant; C more curved than quadratic
- $\gamma = 1$ (uniform): $G = U[0, \bar{\xi}]$ gives $C(\Lambda) = (w\bar{\xi}/2)\Lambda^2$ — the Khan–Thomas case
- $\gamma > 1$: density *increasing* — cheap adjustment rare; C less curved than quadratic

γ alone governs the spectrum: frictionless ($\gamma \to 0$) to maximum selection ($\gamma \to \infty$).

IDENTIFYING γ: HAZARD-EXPONENT BENCHMARK

Benchmark identification (structural hazard exponent):

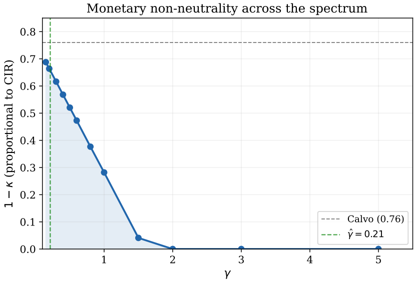
- Structural mapping (Remark after Thm. 1): $\nu = 2\gamma$ where $\Lambda(x) \propto |x|^{\nu}$
- ALO (2022) recover heterogeneity-corrected $\nu \approx 2$ from Cavallo scraped CPI data
- Implies $\hat{\gamma} = \nu/2 = 1$ (at the Khan-Thomas uniform-G point of the spectrum)

Implied benchmark calibration:

$$\hat{\gamma} = 1, \qquad \kappa = 3\bar{\Lambda}, \qquad \text{selection share} = \frac{2}{3}.$$

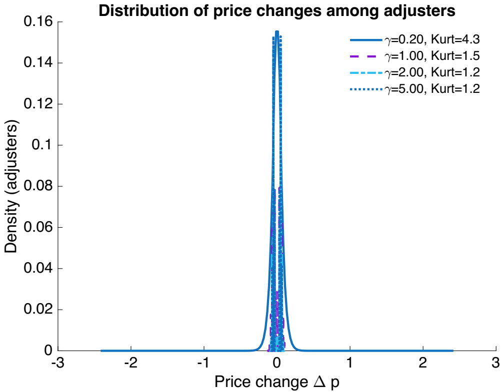
Alternative reading. Purely moment-matching against raw CPI kurtosis (heterogeneous-σ_z, CV=1) yields $\hat{\gamma}^{\text{kurt}} \approx 0.4$ as an alternative (selection share 44%); the structural hazard-exponent identification carries the benchmark.

CIR ACROSS THE SPECTRUM (ALI KURTOSIS IDENTITY)



- **ALI (2016) kurtosis-CIR identity:** for the power-law family, $\mathrm{CIR}(\gamma) = \mathrm{Kurtosis}(\gamma)/(6\,\bar{\Lambda})$ — the canonical reduced-form bridge from price-change moments to monetary non-neutrality.
- CIR collapses monotonically across the spectrum: Calvo ceiling at $\gamma \to 0$, Golosov–Lucas floor at $\gamma \to \infty$; at the benchmark $\hat{\gamma} = 1$ ($\nu = 2$), CIR already sits well below the Calvo ceiling.
- Connects to the empirical kurtosis-identification debate (Cavallo data, ALO's heterogeneity correction): the same kurtosis that pins down γ pins down monetary non-neutrality.

Backup: calibrated distribution of price changes



Heterogeneous-σ_z alternative ($\hat{\gamma} = 0.4$) reproduces the uncorrected leptokurtic CPI distribution; the benchmark $\hat{\gamma} = 1$ matches ALO's heterogeneity-corrected kurtosis ≈ 2.