Online Appendix

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A Testing the dividend channel

In this section, I test whether the state dependence of the consumption responsiveness to a TFP is through the dividend channel or not in the data. For this, I first factor out the variation of consumption orthogonal to the contemporaneous dividend variation:[1]

$$C_t = C_t^D + \widetilde{C}_t, \quad \text{s.t. } \widetilde{C}_t \perp D_t.$$

Then, I check whether the responsiveness of C_t^D is dependent on the cash stocks. Table A.1 shows the state dependence of C_t^D. The consumption that co-varies with the dividend still displays significant state-dependent shock responsiveness. Even in the data before 1980, this dividend channel displays a significant role in consumption state dependence. However, once the whole consumption is considered, this channel is muted down (Table 4), possibly due to the relatively low importance cash stocks before 1980.

Table A.1: State-dependent consumption responses to negative and positive shocks through the dividend channel in data

	Dependent variables:			
	$\lvert log(c_t)\rvert$ $(p.p.)$ before 1980		$\lvert log(c_t)\rvert$ $(p.p.)$ after 1980	
	Neg. (1)	Pos. (2)	Neg. (3)	Pos. (4)
$Cash_{t-1}(s.d.)$	-0.045 (0.021)	0.036 (0.022)	-0.072 (0.036)	0.035 (0.027)
Constant	Yes	Yes	Yes	Yes
Observations	63	49	77	79
R^2	0.068	0.055	0.05	0.021

Notes: The table reports the results of the regression of consumption responses to a negative and positive aggregate TFP shock on the lagged aggregate cash stocks using the consumption varying along with the dividend. The first column is for the negative aggregate TFP shock before 1980; the second is for the positive aggregate TFP shock before 1980; the third is for the negative aggregate TFP shock after 1980; the last is for the positive aggregate TFP shock after 1980. The numbers in the bracket are standard errors.

[1] I projected C_t onto the polynomials of D_t up to the sixth order.

B Fixed parameters

The fixed parameters are set at the following levels:

$$
\begin{aligned}
\text{(Span of control)} \quad & \gamma = 0.8500 \\
\text{(Corporate saving technology)} \quad & r^{ca} = 0.0081 \\
\text{(Idiosyncratic shock persistence)} \quad & \rho_z = 0.9000 \\
\text{(Idiosyncratic shock volatility)} \quad & \sigma_z = 0.0500 \\
\text{(Household's discount factor)} \quad & \beta = 0.9900.
\end{aligned}
$$

The internal discount rate r^{ca} is set at the 80% level of the stationary equilibrium's interest rate $1/0.99-1$. The stochastic aggregate productivity process is from Krusell and Smith (1998):

$$
\Gamma_A = \begin{bmatrix} 0.8750 & 0.1250 \\ 0.1250 & 0.8750 \end{bmatrix}
$$

$$
A \in \{0.99, 1.01\}.
$$

C Definition: Aggregate cash stocks from the Flow of Funds

The aggregate cash stocks are defined as sum of following items in the Flow of Funds:

- (FL103091003) Foreign deposits
- (FL103020000) Checkable deposits and currency
- (FL103030003) Time and savings deposits
- (FL103034000) Money market fund shares
- (LM103064203) Mutual fund shares
- (FL102051003) Security repurchase agreements
- (FL103069100) Commercial paper
- (LM103061103) Treasury securities

D Propositions and proofs

Proposition 1 (A sufficient condition for the sufficient statistic)**.**
For a sufficiently large T*, if there exists a time series of an aggregate allocation* $\{e_t\}_{t=0}^{T}$ *such that for each time partition* $\mathcal{T}_S = \{t | S_t = S\}$*,* $\forall S \in \{B, G\}$ *and for* $\forall (a, z)$*,*

$$
\begin{aligned}
&(i) \quad e_{\tau_0} < e_{\tau_1} \iff V_{\tau_0}^{(n)}(a,z) < V_{\tau_1}^{(n)}(a,z) \text{ for any } \tau_0, \tau_1 \in \mathcal{T}_S \\
&or \\
&(ii) \quad e_{\tau_0} < e_{\tau_1} \iff V_{\tau_0}^{(n)}(a,z) > V_{\tau_1}^{(n)}(a,z) \text{ for any } \tau_0, \tau_1 \in \mathcal{T}_S,
\end{aligned}
$$

then e_t *is the sufficient statistic of the endogenous aggregate state* Φ_t *for* $\forall t$*. In other words, for* $\forall t \in \mathcal{T}_S$*,*

$$
\arg \inf_{\tau \in \mathcal{T}_S} ||\Phi_\tau^{(n)} - \Phi_t^{(n)}||_\infty = \arg \inf_{\tau \in \mathcal{T}_S} ||e_\tau - e_t||_\infty.
$$

Proof.

Lemma 1 states that if and only if the endogenous aggregate states of two periods are the closest and their exogenous states are identical, the corresponding value functions are the closest. Lemma 2 states that if and only if sufficient statistic is the closest and the exogenous states are identical, the corresponding value functions are the closest. That is, the two periods with the closest value functions to each other share the closest endogenous aggregate states and the closest sufficient statistic if the exogenous aggregate states are identical. Therefore, the closest sufficient statistic imply the closest endogenous aggregate states, and the converse is also true.

■

Lemma 1 (Value function equivalence from the sufficient statistic)**.**
Define $\tau_1^* := \arg\inf_{\tau \in \mathcal{T}_S} ||V_\tau^{(n)} - V_t^{(n)}||_\infty$ *and* $\tau_2^* := \arg\inf_{\tau \in \mathcal{T}_S} ||\Phi_\tau^{(n)} - \Phi_t^{(n)}||_\infty$*, for* $t \in \mathcal{T}_S$*. Then,*

$$\tau_1^* = \tau_2^*.$$

Proof.
Due to the restriction $\tau \in \mathcal{T}_S$, the state realizations satisfy the following:

$$S_{\tau_1^*} = S_{\tau_2^*} = S_t.$$

If the path is long enough, τ_2^* almost surely satisfies

$$\Phi_{\tau_2^*}^{(n)} = \Phi_t^{(n)}.$$

Then,

$$X_{\tau_2^*} = \{S_{\tau_2^*}, \Phi_t^{(n)}\} = \{S_t, \Phi_t^{(n)}\} = X_t,$$

which implies the economy at period τ_2^* and the economy at period t are identical. Thus, the following identity holds:

$$V_{\tau_2^*}^{(n)} = V_t^{(n)}.$$

Therefore,

$$\tau_1^* = \tau_2^*.$$

■

Lemma 2 (Value function equivalence from the endogenous state variable)**.**
Suppose (i) or (ii) in Proposition 1 holds. Define $\tau_1^* := \arg\inf_{\tau \in \mathcal{T}_B} ||V_\tau^{(n)} - V_t^{(n)}||_\infty$ *and* $\tau_2^* := \arg\inf_{\tau \in \mathcal{T}_B} ||e_\tau - e_t||_\infty$ *for* $t \in \mathcal{T}_S$*. Then,*

$$\tau_1^* = \tau_2^*.$$

Proof.
Due to the restriction $\tau \in \mathcal{T}_B$, the state realizations satisfy the following:

$$S_{\tau_1^*} = S_{\tau_2^*} = S_t.$$

If the path is long enough, τ_2^* almost surely satisfies

$$e_{\tau_2^*} = e_t.$$

Due to (i) and (ii),

$$V_{\tau_2^*}^{(n)} = V_t^{(n)}.$$

The result above is proven by contradiction. Suppose $V_{\tau_2^*}^{(n)} \neq V_t^{(n)}$. Then there exists (a, z) such that $V_{\tau_2^*}^{(n)}(a, z) \neq V_t^{(n)}(a, z)$. Therefore, $e_{\tau_2^*} \neq e_t$ due to (i) or (ii), which is contradiction.

Therefore, from $V_{\tau_2^*}^{(n)} = V_t^{(n)}$,

$$\tau_1^* = \tau_2^*.$$

■

Proposition 2 (The existence of the target cash-holding level)**.**

Suppose policy functions are non-trivial: $ca'(ca_1, z) > 0$ *for some* $ca_1 > 0$ *and* $d(ca_2, z) > 0$ *for some* $ca_2 > 0$*, given* z*. Then, there exists* $\overline{ca}(z) > 0$ *such that* $ca'(ca, z) \leq \overline{ca}(z)$ *for* $\forall ca \geq 0$*.*

Proof.

To prove the proposition by contradiction, suppose there is no such $\overline{ca}(z)$. That is, $ca'(ca, z) < ca'(ca + \epsilon, z)$ for $\forall(ca, z)$ and $\forall \epsilon > 0$.

I define the liquidity on hands $m(ca, z) = \pi(z) + ca$. Then,

$$d(ca, z) + \frac{1}{1 + r^{ca}} ca'(ca, z) = m(ca, z).$$

$m(ca, z)$ strictly increases in ca. Due to the monotone preference on greater d and ca' and strict monotonicity of m on ca, d and ca' weakly increases in ca. I consider $\widetilde{ca}$ such that $ca'(\widetilde{ca}, z) > 0$ and $d(\widetilde{ca}, z) > 0$. Such $\widetilde{ca}$ exists as ca' and d weakly increases in ca. For example $\widetilde{ca} = \max\{ca_1, ca_2\}$.

Then, for a marginal incremental ϵ in cash, the marginal cost of hoarding cash is 1 (forgone dividend), while the marginal benefit out of hoarding cash is $\frac{1+r^{ca}}{1+r}$:[2]

$$\underbrace{1}_{\text{Marginal cost}} > \underbrace{\frac{1 + r^{ca}}{1 + r}}_{\text{Marginal benefit}}.$$

where, $1 + r = 1/\beta$ in the stationary equilibrium. This implies that for the extra cash, the firm does not have an incentive to hoard it in the cash reserve. Therefore,

[2]In this argument, the non-negativity constraint does not matter, as $ca'(\widetilde{ca}, z) > 0$.

$d(ca+\epsilon, z) = d(ca, z) + \epsilon$, if $d(ca, z) > 0$. Then, from a firm's budget constraint,

$$\begin{aligned}\frac{1}{1+r^{ca}} ca'(\widetilde{ca}+\epsilon, z) &= \widetilde{ca} + \epsilon + \pi(z) - d(\widetilde{ca}+\epsilon, z) \\ &= \widetilde{ca} + \pi(z) - (d(\widetilde{ca}+\epsilon, z) - \epsilon) \\ &= \widetilde{ca} + \pi(z) - d(\widetilde{ca}, z) \\ &= \frac{1}{1+r^{ca}} ca'(\widetilde{ca}, z).\end{aligned}$$

Therefore, any extra increase in the current cash stock $\widetilde{ca}$ does not change the future cash stock:

$$ca'(\widetilde{ca}, z) = ca'(\widetilde{ca}+\epsilon, z),$$

which is a contradiction. Therefore, there exists the target cash stock $\overline{ca}(z)$. ∎