Striking While the Iron Is Cold: Fragility after a Surge of Lumpy Investments[†]

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Abstract

In this paper, I argue that synchronized large-scale investments of large firms can significantly amplify productivity-driven aggregate fluctuations and lead to investment cycles even in the absence of aggregate shocks. Using U.S. Compustat data, I show that the years preceding recessions display investment surges among large firms. Furthermore, after the investment surges, large firms become inelastic to interest rates and display persistent inaction duration. I then develop a heterogeneous-firm real business cycle model in which a firm needs to process multiple investment stages for large investments and can accelerate it at a cost. In the model, following a TFP shock, the synchronized timings of lumpy investments are persistently synchronized. And TFP-induced recessions are especially severe after the surge of large firms' lumpy investments. In support of this prediction, I present evidence for the investment cycle in post-shock periods in macro-level data on nonresidential fixed investment.

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1 Introduction

1980, 1998, and 2007 were the three years with the largest surges in the fraction of large firms making large-scale investments since 1980. And these three years were followed by recessions within two years.[1] Is it merely a coincidence that investment surges of large firms precede recessions?

This paper studies a mechanism that makes an economy more fragile to a negative TFP shock after a surge in lumpy investments of large firms. I develop and analyze a business cycle model with heterogeneous firms that reflects empirical findings from micro-level data. Then using the model, I qualitatively and quantitatively analyze the amplification of productivity-driven aggregate fluctuations. Through lumpy investment decisions at firm level, the interaction between endogenous and exogenous sources of aggregate fluctuations builds the core of this analysis.

I document two empirical characteristics of firm-level lumpy investments that matter for aggregate fluctuations: the interest-inelasticity of large-scale investment timings and the highly persistent inaction durations. First, using the U.S. Compustat data, I show that large firms' timings of lumpy investments are inelastic to interest rate changes. Therefore, if a negative aggregate TFP shock hits an economy after a previous surge of large firms' lumpy investments, new aggregate investment drops substantially. These large firms are not willing to make another large capital adjustment on top of their recent investments despite a lowered interest rate.

Second, I document that inaction periods of a firm's capital adjustment are highly persistent across periods. The observed persistence for all firms is substantially higher than the level implied by the stochastic investment (S, s) cycle in models with fixed costs in the literature. This high persistence has an important aggregate implication: the mean-reversion of the synchronized investment timings across firms is sluggish. Thus, an aggregate TFP shock effect lasts longer when the high persistence in the length of inaction periods at firm level is higher.

Therefore, it is necessary to capture these two empirical findings in the model to study how firm-level lumpy investments affect business cycle. Based on existing evidence from the literature (Yang et al., 2020), I argue that large firms' structured decision-making process such as capital budgeting accounts for the observed inelasticity and the persistence. In particular, capital budgeting is a universal tool for CFO's to plan and evaluate an investment project. Almost 99.5% of CFO's from Fortune 1,000 firms rely on capital budgeting. I provide

[1] Following Cooper and Haltiwanger (2006), I define an investment in a year beyond 20% of existing capital stock as a large-scale capital adjustment. Firms that hold capital stocks greater than the 90th percentile of the capital distribution in each industry based on two-digit NAICS code are defined as large firms.

a suggestive evidence that firms become inelastic to interest rate change and insensitive to investment opportunities during the capital budgeting process. From this, I claim implementation lags from structured decision-making process is a critical component to be modeled to capture the empirical findings.

Then I develop and analyze a model with lumpy investment in which a firm needs to process a required number of investment stages for a large-scale investment. Here the investment stages capture the bureaucratic steps in capital budgeting such as meetings of the board of directors for the large-scale investment decision or auditing procedures for large-scale investments (Malenko, 2019). Each firm decides the optimal number of stages to process each period. The processing cost convexly increases both in the number of stages to be processed and in the size of a firm's capital stock. I name this cost as "acceleration cost." The convexity of acceleration cost in the number of stages disturbs firms' nimble capital adjustment. The convexity in the size of capital stock makes large firms face larger cost of agile capital adjustment. These features make large firms' lumpy investments inelastic to interest rate changes.

When an aggregate TFP shock hits the economy in the model, the timing of lumpy investments is synchronized across firms. Then, aggregate investments nonlinearly respond to the aggregate TFP shock because synchronized lumpy investments are not mitigated by changes in the interest rate. Furthermore, high persistence in inaction duration persistently synchronizes future investment timings. This leads to long-run echo effects in the economy. In this model, the impulse response of the economy depends on the aggregate state of the economy. Specifically, the mass of large firms that are ready to adjust their own existing capital is the key conditioning state variable.

Using the calibrated model, I decompose the total response of aggregate investments to an aggregate TFP shock into an exogenous effect and an endogenous effect. The endogenous effect accounts for substantial portion of the aggregate investment response: it explains up to 15% of the total response. The endogenous effect is largest when a negative aggregate TFP shock hits the economy after a surge of lumpy investments: the same negative aggregate TFP shock has up to 29% greater impact on aggregate investment after a surge of lumpy investments than in other aggregtae states.

In the model, if a group of firms is extremely inelastic to interest rate changes and has an extremely high persistence of inaction duration, the echo effect can permanently persist in the post-shock period. The synchronized investment timings are then permanently synchronized, leading to a stationary cycle.[2] To characterize this stationary cycle formally, I first define a

[2]Under the calibrated baseline model, the echo effects die out after around 25 years from the point of an aggregate TFP shock. A permanent echo is generated under a parameterization with higher acceleration cost

cyclical competitive equilibrium that conceptually extends stationary recursive competitive equilibrium. In this equilibrium, aggregate allocations fluctuate without relying on exogenous shocks. Different endogenous fluctuations can arise depending on the synchronized pattern in the initial distribution. I explore this theoretical possibility in Section 6.

I found the model prediction of echo effects is empirically supported by macro-level data. First, I analyze echo effects from historical events that were followed by large aggregate TFP shocks. According to Ohanian (2001), aggregate TFP dropped by around 18% in the Great Depression. Using a Fisher g-test, I show that there was significant deterministic periodicity in the manufacturing industries' investment growth rate in non-residential structures after the Great Depression. Similarly, after the oil crisis in 1979, the oil industry's investment growth rate in structures displays significant deterministic periodicity. Second, I provide an evidence of echo effects from the impulse response of non-residential structure investment from BEA data. These empirical results validate nonlinear dynamics implied by the acceleration cost model.

Related literatures This paper contributes to the literature that studies how firm-level lumpy investments affect business cycle. Within this literature, Abel and Eberly (2002) empirically showed that there are statistically and economically significant nonlinearities in firm-level investments. They point out that it is necessary to track the cross-sectional distribution of firm-level investments to account for aggregate investment. Cooper et al. (1999) and Gourio and Kashyap (2007) found aggregate investment is largely driven by establishment-level capital adjustment in extensive margin. Especially, Cooper et al. (1999) found synchronized lumpy investments can generate echo effect of aggregate shocks in partial equilibrium. Gourio and Kashyap (2007) pointed out that if a fixed cost is drawn from a highly concentrated non-uniform distribution, aggregated lumpy investments show different impulse response than frictionless models in partial equilibrium. In contrast, Khan and Thomas (2008) found that lumpiness in investment at the establishment level is washed out after aggregation, due to strong general equilibrium effect.

In this paper, I empirically show that there are firm-level lumpy investments that are inelastic to interest rate dynamics, thus not smoothed out by changes in the interest rate after aggregation. Therefore, irrelevance result does not hold even in general equilibrium if a model captures those interest-inelastic firms. I conclude that large firms' lumpy investments contributes to nonlinear aggregate fluctuations under the interest-inelasticity. This is a consistent result with Koby and Wolf (2020) which shows the observed dampening effect of factor price is not as strong as the implied level in models with fixed cost, using the semi-elasticity

where large firms become extremely inelastic to the operating environment.

estimates to the bonus depreciation from Zwick and Mahon (2017).

House (2014) pointed out that a conventional model with fixed cost cannot capture inelastic lumpy investments due to strong general equilibrium effect; model-implied lumpy investments are highly price-elastic. To overcome this limitation in the fixed cost model, Bachmann et al. (2013) introduce maintenance and replacement investments under the high fixed cost parameter. In their model, micro-level lumpiness does not wash away after aggregation, leading to state-dependent sensitivity of aggregate investment in general equilibrium. Winberry (2021) includes habit formation in the household's utility function so that aggregate TFP sensitivity of real interest rate becomes counter-cyclical. Combined with convex adjustment cost, counter-cyclically responsive real-interest rate does not dampen aggregated lumpy investments over the business cycle.

Differently from these approaches, I introduce a convex acceleration cost in the model which captures large firms' interest-inelastic behavior. Also, this model captures high persistence of inaction durations across periods. This feature is relatively less highlighted in the literature despite its important role in the aggregation. Specifically, high persistence of inaction durations across periods contributes to the persistent synchronization of firm-level lumpy investments in the post shock periods. This generates nonlinearity in the impulse response of the aggregate investment to an aggregate TFP shock that mimics echoes. In support of this theoretical prediction, I present evidence for the nonlinear investment dynamics from the macro-level data. This is closely related to Baley and Blanco (2021), which shows that two sufficient statistics can characterize aggregate investment dynamics: 1) the capital to productivity ratio's dispersion and 2) its covariance with the duration of inaction. Compared to this, I highlight the role of the marginal distribution of large firms' inaction duration over the business cycle, which is readily observable in the data in a timely manner due to their mandated financial disclosure.

Second, this paper contributes to nonlinear business cycle literature. A large body of researches has focused on the nonlinearity in aggregate fluctuations that arise when heterogeneous agents are subject to micro frictions. Bachmann et al. (2013) found firm-level lumpiness in investments leads to pro-cyclical sensitivity of aggregate investments to an aggregate shock. Similarly, Berger and Vavra (2015) concludes lumpiness in households' durable adjustment result in pro-cyclical responsiveness of aggregate durable expenditures to an aggregate shock. Fernandez-Villaverde et al. (2020) found that financial frictions can generate endogenous aggregate risk under the heterogeneous household model. In this setup, the aggregate allocations display state-dependent responsiveness to an aggregate TFP shock. Volatility shock in real interest rate studied in Fernandez-Villaverde et al. (2011) and uncertainty shock in Bloom et al. (2018) also lead to nonlinear aggregate fluctuations. To this literature, this paper contributes by modeling interest-inelastic firm's lumpy investments as an additional source of nonlinearity in the business cycle.

Third, this paper contributes to endogenous business cycle literature by generating aggregate fluctuations in general equilibrium without increasing-returns-to-scale technologies (Benhabib and Farmer, 1994; Farmer, 2016). After an aggregate TFP shock hits the economy, the equilibrium allocations fluctuate, forming echo patterns. This is due to persistently synchronized investment timings among interest-inelastic firms. Depending on the level of insensitivity to idiosyncratic shock process, this echo can be a decaying echo or a permanent echo. Both types of echoes are possible sources of endogenous fluctuations in an economy. I show that these echoes are empirically supported by statistically significant deterministic periodicity in the post-crisis period from macro-level data.

Roadmap Section 2 empirically analyzes characteristics of lumpy investments for large and small firms. Based on the empirical analysis, Section 3 develops a business cycle model with heterogeneous firms subject to acceleration cost. In Section 4, I explain calibration used for this model. Using the model under the calibrated parameters, Section 5 quantitatively analyze nonlinear effect of lumpy investments in business cycle. In Section 6, endogenous aggregate fluctuations arising from a permanent echo effects are studied as a theoretical possibility an acceleration cost model can lead to. Section 7 suggests empirical evidence for nonlinearity in macro-level data. Section 8 concludes. Proofs and other detailed figures and tables are included in appendices.

2 Firm-level empirical analysis

For the firm-level empirical analysis, I use U.S. Compustat data. While Compustat data covers only public firms, its coverage is relatively less an issue in this analysis because the focus is on firms with large capital stocks. Throughout the whole empirical analysis, large firms are defined as firms that hold capital stocks greater than the 90th percentile of the capital distribution in each industry of two-digit NAICS code. Sample period covers from 1980 to 2016. Firms with negative asset and zero employment are excluded from the sample. All the firm-level variables except capital stock and investment are deflated by GDP deflator. Investment is deflated by nonresidential fixed investment deflator available from National Income and Product Accounts data (NIPA Table 1.1.9, line 9). Firm-level real capital stock is obtained from applying perpetual inventory method to net real investment. Industry is categorized by the first two-digit NAICS code.[3]

	Large	Small
Total		
Aggregate Sales ($1 bil.)	9007.2	4641.8
Aggregate Employment (1 mil.)	30.3	22.3
Firm-level		
Avg. Sales ($1 mil.)	8143.9	332.2
Avg. Employment (1K)	28	1.7
Avg. Age after IPO	20.1	7.6
Num. Obs.	1111	13985
Financial constraint		
Total Liability / Total Asset (%)	61.7	98.4

Table 1: Large and small firms' summary statistics

Table 1 reports summary statistics for large and small firms during the sample periods. Under the given definition of large firms, around 60% of aggregate sales and employments belong to large firms. On average, large firms are 25 times greater than small firms in sales and employment. Large firms are on average old firms, having been listed around 13 years longer than small firms. Large firms' ratio of total liability out of total asset is around 61.7%, and is smaller than the small firms' fraction 98.4%. Thus, large firms are less financially constrained on average.

2.1 Motivating facts

I define an investment spike as a firm-specific event where a firm makes a large-scale investment greater than 20% of the firm's existing capital stock.[4] I refer to this investment spike as a lumpy investment or capital adjustment in extensive margin, interchangeably. Throughout the empirical analysis, the fraction of firms making lumpy investments is the key variable. I define the key variable, spike ratio as follows:

$$\text{Spike ratio}_{j,t} := \frac{\sum_{i \in j} \text{Investment spike}_{i,t}}{\#\text{ of } j\text{-type firms at } t}, \quad j \in \{small, large\}$$

[3]If only SIC code is available for a firm, I imputed NAICS code following online appendix D.2 of Autor et al. (2020). If both of NAICS and SIC are missing, I filled in the next available industry code for the firm.

[4]20% cutoff is from non-convex adjustment cost literature including Cooper and Haltiwanger (2006), Gourio and Kashyap (2007), and Khan and Thomas (2008). If a firm's acquired capital stock is greater than 20% of existing capital stock in a certain year, I do not count the year as the firm's lumpy investment period.

The numerator is counting all the investment spikes of firm type $j \in \{small, large\}$ at time t, and it is normalized by the total number of j type firms.

Figure 1 plots the time series of spike ratio of large firms. On average, 15.3% of large firms adjust their existing capital stocks in extensive margin in a year. As can be seen from Figure 1, since 1980 there have been only three periods (1980, 1998, and 2007) where the fraction of large firms making spiky investments surged beyond 20%. All three events were followed by recessions within two years.

Conversely, there were four recessions in the U.S. over the same periods, and three out of four recessions were preceded by the surge of large firms' lumpy investments. The exception was the recession in 1990, and it was the mildest recession among the four recessions.

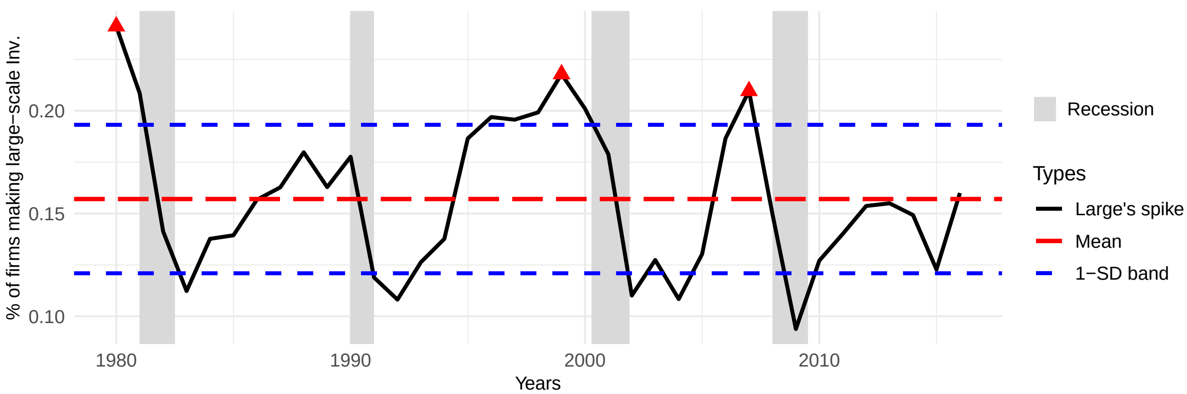


Figure 1: Three surges of large firms' lumpy investments preceded recessions

Variables (% dev. from average)	Before Recession I (1980)	Before Recession II (1989)	Before Recession III (1998)	Before Recession IV (2007)
$\Delta Spike_{Large}(\%)$	53.53	3.70	26.84	33.49
$\Delta Spike_{Small}(\%)$	11.78	-0.20	16.97	7.34

Table 2: Deviation of spike ratios from mean before recessions

Table 2 summarizes the deviation of large and small firms' spike ratios from the mean level in each year before the recessions.[5] Before the recessions in 1981, 2000 and 2008, the spike ratio of large firms were greater than the average level by more than 25%. In contrast, the spike ratio of small firms did not increase dramatically before each recession as shown in the second line of the table.

[5] $\Delta Spike_j(\%)$ is obtained from demeaning and normalizing spike ratio by the mean separately for large and small firms.

Relatedly, in the following analysis, I show aggregate investment rate is conditionally heteroskedastic on the average lagged spike ratio of large firms. That is, residualized volatility of aggregate investment rate is high if a great portion of large firms have made lumpy investments in the recent years.

For this analysis, I use aggregate data on non-residential investment (NIPA Table 1.1.5, line 9) and aggregate capital (Fixed Asset Accounts Table 1.1, line 4) from BEA. The thick line in Figure 2 plots logged estimates of standard deviation of residuals from autoregression of aggregate investment rates as a function of the recent average of large firms' spike ratio.[6] The recent average is based on the average spike ratio of past two years. As can be seen from this figure, aggregate investment rates are heteroskedastic conditional on the lagged average spike ratio. Table A.1 reports the regression coefficients for the fitted line. According to the regression result, one standard-deviation increase (3.18%) in the large firms' spike ratio is associated with 35% increase in the standard deviation of the residualized aggregate investments. From this result, I conclude that aggregate investments respond more strongly to an aggregate shock after a surge of lumpy investments of large firms. Consistent with the patterns in Figure 1, the three recession years of interest are located at the top-right corner in Figure 2.

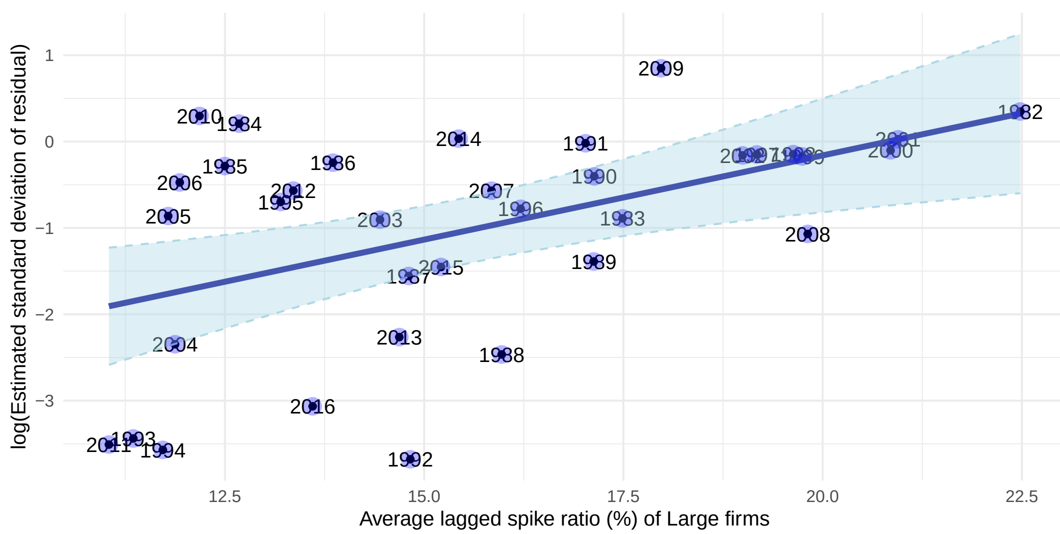


Figure 2: Conditional heteroskedasticity of aggregate investments

I claim this is not a mere coincidence that the surges of large firms' lumpy investments precede the recessions. I suggest a novel mechanism where an economy responds more strongly to a negative aggregate productivity shock after a surge of large firms' lumpy investments

[6] This empirical analysis is motivated from the conditional heteroskedasticity analysis in Bachmann et al. (2013) (Figure 1).

based on the empirical findings at the firm level. The key mechanism is in the interest-inelastic investment timings of large firms. I empirically investigate the characteristics of large firms' lumpy investments in the following sections.

2.2 Interest-inelasticity of large firms' lumpy investments

In this section, I run a Vector Autoregression (VAR) to analyze different investment behavior of large and small firms when the interest rate changes. Then, using high-frequency monetary policy shocks, I estimate the heterogeneous interest-elasticity of investments in the extensive margin for large and small firms.

In the VAR, aggregate TFP, federal funds rate, and the fraction of large/small firms that make large-scale investments ($SpikeRatio_{j,t}$) are included in the stated order; and one-period-ahead CPI is included as an exogenous control variable:[7]

$$X_{j,t+1} = \Phi_0 + \Phi_1 X_{j,t} + \Phi_2 C_t + \epsilon_t \quad j \in \{Small,\ Large\}$$

$$X_{j,t} = [TFP_t,\ FedFund_t,\ SpikeRatio_{j,t}]', \quad C_t = \mathbb{E}_t \tilde{CPI}_{t+1} \approxeq CPI_{t+1}$$

Figure 3 plots impulse responses of the fractions of large (solid line) and small (dot-dashed line) firms adjusting capital stocks in extensive margin ($SpikeRatio_{j,t}$) to an interest rate shock. Dashed line is 95% confidence interval of the estimated response.

Upon impact, there are no significant contemporaneous responses from large and small firms' lumpy investments. However, in the following years, small firms display significant drop in the fraction of adjusting firms. Two years from the shock period, the response drops by around 1.3%. In contrast, the large firms' response does not show any significant deviation from zero for the whole post-shock period. From this evidence, I claim large firms do not significantly change their lumpy investment timings in response to interest rate changes.

This is consistent with the finding of Cloyne et al. (2020) that the investment of large firms paying dividends are inelastic to interest rate changes.[8] Similarly, Crouzet and Mehrotra (2020) found large firms are less cyclically sensitive than small firms.[9] Also, there exists survey evidence that supports interest-inelasticity of firm-level investments. According to

[7]All the macro variables are at annual frequency and are HP-filtered with smoothing parameter 6.25 following Ravn and Uhlig (2002). In calculation of the fraction of investment spikes, a firm's consecutive two investment spikes are considered as one spike. The impulse response is obtained from the orthogonalized VAR. The impulse variable is the federal funds rate. The lag order $p = 1$ is chosen by AIC criterion.

[8]The result of Cloyne et al. (2020) combines both intensive and extensive margin responses, while the result in this paper singles out the response in extensive margin.

[9]According to Crouzet and Mehrotra (2020), this discrepancy between small and large firms is not driven by financial distress.

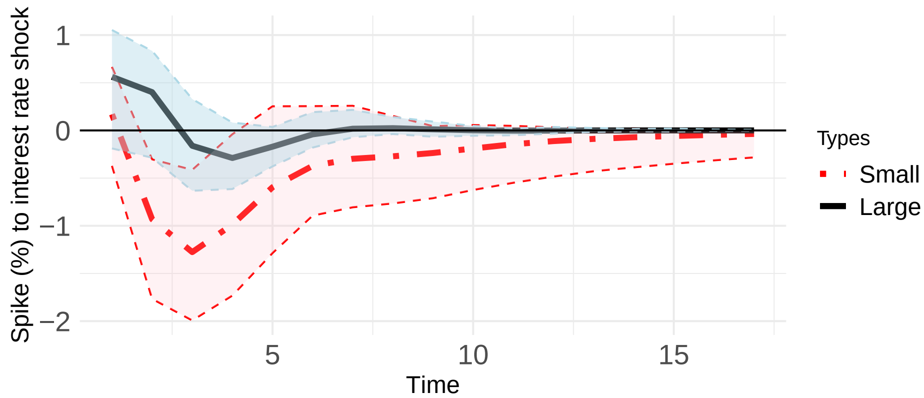


Figure 3: Impulse response of spike ratios to the interest rate shocks

the survey results in Sharpe and Suarez (2013), 68% of the respondent firms do not change their investment plan despite the interest rate drops.[10] And almost 80% of the respondent firms do not change their investment plans unless the interest rate jumps up more than 3%. Considering the survey respondents are large firms that hire CFO for their financial management, the reported inelastic investments to interest rate change are consistent with the result of VAR analysis in this paper.

However, the VAR analysis does not rule out the possibility that other exogenous variations than TFP can simultaneously affect the spike ratio and the interest rate. Therefore, the result obtained from the VAR is about a correlation rather than a response to the pure interest rate change. For the sharp identification of heterogeneous interest-elasticity in the extensive margin, I construct an exogenous monetary policy shock following Jeenas (2018) and Ottonello and Winberry (2020). The monetary policy shock is obtained by time aggregating high-frequency monetary policy shock identified from the unexpected jump (drop) in the federal funds rate during a 30-minutes window around the FOMC announcement.[11] To capture the unexpected component in the federal funds rate, I use the change in the rate implied by the current-month federal funds futures contract. All the data on the timings of the FOMC announcement and the high-frequency surprise are from Gurkaynak et al. (2005) and Gorodnichenko and Weber (2016). The sample period covers from March 1990 until December 2009. I follow the convention that the positive monetary policy shock is an unexpected increase in the federal funds futures rate, so it implies the contractionary monetary policy.

To match the data frequency between the firm-level data and the monetary policy shock, I time aggregate the monetary policy shocks. Specifically, I compute the one-year backward

[10]The survey was conducted by Duke University and CFO magazine, and around 1,000 companies responded to the survey. Table A.2 summarizes the key results of the survey.

[11]The result is robust over the choice of a wider window (one-hour window).

weighted average monetary policy shock at each firm's financial yearend. The weight of each surprise is determined by the number of days between the corresponding FOMC announcement and the next FOMC announcement.[12] If the next FOMC announcement was made after the financial yearend, the days are counted until the financial yearend. By this data joining process, a firm's balance sheet information and the monetary policy shock is matched at the same financial year. The weighted moving average monetary policy shock is plotted in Figure B.1.

To study the heterogeneous firm-level investment responses in the extensive margin to the monetary policy shock, I estimate the following probit regression separately for large firms and small firms.

$$\mathbb{P}(spike_{i,t}) = \beta MP_t + \alpha_i + \alpha_{s,t} + \Omega' Control_{i,t} + \eta_{i,t}, \quad \eta_{i,t} \sim_{iid} N(0,\sigma)$$

where MP_t is the monetary policy shock, α_i is the firm i fixed effect, and $\alpha_{s,t}$ is the sector-year fixed effect. The control variables include the current account and current liability normalized by total asset, log of total asset (size), and log of sales. The standard errors are two-way clustered across sectors and years.

	Dependent variable: $\mathbb{P}(spike_{i,t})$	
	Large	Small
MP_t	-0.0022	-0.0124
	(0.0306)	(0.0063)
Observations	7,635	84,300
Firm Fixed Effect	Yes	Yes
Sector-year Fixed Effect	Yes	Yes
Firm-level Control	Yes	Yes
Two-way Cluster	Yes	Yes
0	0	0

Table 3: Persistence in inaction durations

Table 3 reports the coefficient of the monetary policy shock in the probit regression separately for large and small firms with the standard errors in the bracket. In the estimated result, a contractionary (expansionary) monetary policy shock significantly reduces (increases) the probability of making large-scale investment for small firms while large firms stay unaffected. From the marginal effect analysis on the estimated probit regression, I find one basis point increase in the monetary policy shock (from zero) is associated with around 2% drop in the

[12] A higher weight is assigned for a monetary policy shock when there was greater amount of time for a firm to respond to the shock (Ottonello and Winberry, 2020).

probability of making lumpy investments for small firms. The same variation in the monetary policy shock is associated with only negligible variation in the large firms' investments in the extensive margin.

The interest-inelasticity of large firms' investments in the extensive margin has an important macroeconomic implication in the aggregation of micro-level investments. Under the presence of interest-inelasticity, micro-level lumpiness does wash out after aggregation as the timings of lumpy investments are not smoothed by the interest rate changes over the business cycle. Therefore, the micro-level lumpiness leads to macro-level lumpiness after aggregation. This macro-level lumpiness generates nonlinear aggregate fluctuations in the economy. In the quantitative analysis section, using the heterogeneous-firm business cycle model, I analyze the role of interest-inelastic lumpy investments of large firms on the business cycle.

2.3 Insensitivity of large firms' lumpy investments to idiosyncratic TFP shocks

In this section, I show investments of large and small firms have different sensitivity in extensive margin to their idiosyncratic TFP shocks. For this empirical analysis, I measure the firm-level TFP following Ackerberg et al. (2015). The detailed steps for the firm-level TFP estimation are described in Appendix C.

Specifically, I implement an event study separately for large and small firms to study the response of capital adjustment in extensive margin to the shock in the firm-level TFP. The probit regression for the event study is specified as follows:

$$\mathbb{P}(spike_{i,t}) = \sum_{\tau=-4}^{3} \beta_\tau \mathbb{I}\{\tau = t\} + \alpha_{industry} + \alpha_{year} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim_{iid} N(0, \sigma)$$

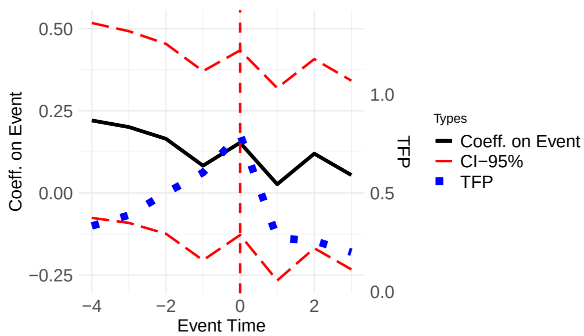
where $\tau = 0$ is the event time, and $spike_{i,t}$ is a binary variable indicating whether a firm makes a large-scale investment in extensive margin. The event is defined as a firm-specific year when an innovation in the firm-level TFP deviates more than one standard deviation from zero. The innovation in TFP ($TFPinnovation$) is obtained from the residuals after fitting the TFP process into $AR(1)$ process:

$$TFP_{i,t} = \rho TFP_{i,t-1} + TFPinnovation_{i,t}$$

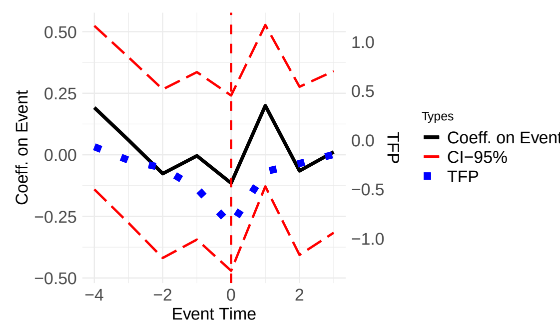
The periods of interest are from four years prior to the event until three years after the event. Full observations of eight years around the event (including the event year) are required to be included in the sample.

Each panel of Figure 4 plots the estimated coefficients β_τ of large and small firms (solid line) and its 95% confidence interval (dashed line) around the event time $\tau = 0$, for both positive event (panel (a) and (c)) and negative event (panel (b) and (d)). The dotted line is the time series of average idiosyncratic TFP across firms around the event.

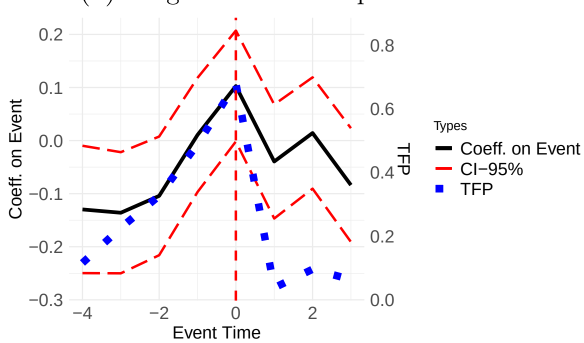
As can be seen from panel (a) and (b), large firms' extensive-margin adjustment does not significantly respond to idiosyncratic productivity shocks. In contrast, small firms display strong responsiveness to both positive and negative idiosyncratic productivity shocks as shown in panel (c) and (d). For a positive innovation in the firm-level TFP, the small firms' probability of making large-scale investment jumps up by 10%. For the negative innovation in the firm-level TFP, the small firms' probability of making large-scale investment drops by 14%.



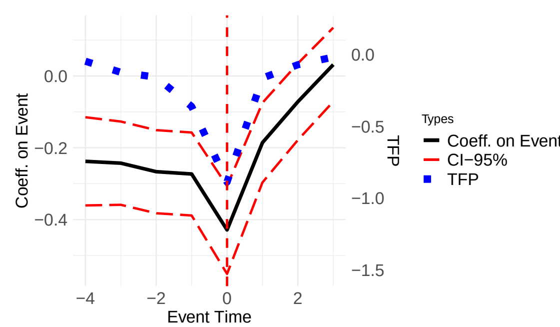
(a) Large firms around positive event



(b) Large firms around negative event



(c) Small firms around positive event



(d) Small firms around negative event

Figure 4: Event study: sensitivity to idiosyncratic TFP innovation

For the robustness check, I estimate the firm-level TFP in two other ways: one is from Solow residuals and the other is from Olley and Pakes (1996). The results stay unchanged for these alternative TFP measures. The results based on the other two TFP measured are reported in Appendix D.

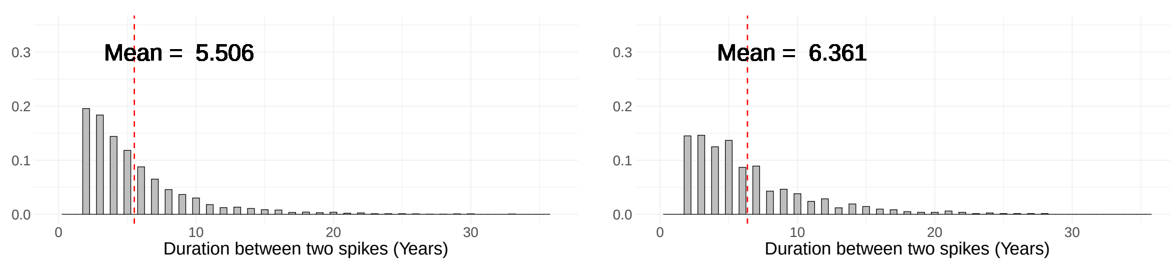
To sum up the results, the extensive-margin investments of small firms strongly respond to idiosyncratic productivity shocks. In contrast, large firms' extensive-margin adjustments do not strongly respond to idiosyncratic productivity shocks. This insensitivity possibly comes from difficulty of catching a sudden investment opportunity for large firms relying on capital budgeting for their internal resource allocations. The detailed discussion for why this insensitivity arises will be made in section 2.5.

Heterogeneous sensitivity to idiosyncratic TFP shock is important in aggregate investment dynamics because it determines an allocation's speed of reversion to a steady-state level. If an aggregate TFP shock hits an economy, the distribution of micro-level allocations departs from the stationary distribution. Then, large firms with low sensitivity to an idiosyncratic shock converge slower to the stationary allocation than small firms do.

2.4 Firm-level persistence in inaction duration

In this section, I document large firms' inaction duration of capital adjustment is highly persistent across periods.

Figure 5 plots the distributions of inaction periods between neighboring spiky investments for large and small firms.[13] Large firms' inaction durations are longer than small firms' inaction durations on average by around a year and have a fatter tail in the distribution.



(a) Small firms (b) Large firms

Figure 5: Distribution of inaction durations

To study underlying regularity in the lumpy investment timings at the firm level, I compare each firm's inaction duration with the lagged inaction duration. Specifically, I check how well aligned the inaction duration and the lagged duration are along the 45-degree line in a scatter plot. To this end, I fit the inaction duration into autoregressive process. Table 4 reports the AR(1) regression results for logged inaction periods ($t2Inv$) of large and small firms. Inaction duration ($t2Inv$) at t is defined as a time interval (in years) between the spike at period t and the most recent investment spike. The numbers in the bracket are the standard errors. Both types of firms have fairly high persistence in the inaction durations. The level of persistence is even higher than the measured persistence in the firm-level productivity shocks that is

[13] Consecutive investment spikes are assumed to have no inaction periods.

around 0.58 (Bachmann and Bayer, 2013). [14]

	Dependent variable: $log(t2Inv_{i,j})$		
	All	Large	Small
$log(t2Inv_{i,j-1})$	0.885	0.889	0.884
	(0.007)	(0.017)	(0.008)
Large - Small			0.005
(p-value)			(0.782)
Observations	5,338	842	4,496

Table 4: Persistence in inaction durations

To summarize the results, the inaction durations are highly persistent across periods for both large and small firms. The high persistence in the inaction duration has an important macroeconomic implication for aggregate investment dynamics: once firms' investment timings are synchronized, the timings are persistently synchronized. Also, it takes long time for firms to revert back to the stationary distribution once they deviate from it.

2.5 Why some firms are insensitive? Capital budgeting

> *It* [i.e., capital budgeting] *sucks the energy, time, fun, and big dreams out of an organization. It hides opportunity and stunts growth. It brings out the most unproductive behaviors in an organization, from sandbagging to settling for mediocrity.*
>
> — Jack Welch, *General Electric*

In this section, I suggest a possible economic explanation for insensitivity of large firms' lumpy investments to interest rate changes and investment opportunities. A chief financial officer (CFO) of a firm faces complicated inflow and outflow of capital during the firm's operation. Thus, having the capital flow under complete control is one of the most important things to do for the position. For this, most of CFO's rely on capital budgeting for their decision on capital allocation and investment plan for longer horizon than a year. According

[14]The implied level of average persistence in inaction duration in Khan and Thomas (2008) is around 0.7. Calibration used in Gourio and Kashyap (2007) gives slightly higher persistence around 0.75, but it does not achieve the observed high persistence level in the data.

to a survey conducted by Ekholm and Wallin (2000) towards 650 Finnish companies with a turnover greater than 16.7 million euros, 86% of respondents answered they use annual capital budgeting. In the survey conducted by Ryan and Ryan (2002) towards Fortune 1000 companies, they found 99.5% of the respondents answered they use capital budgeting. Likewise, capital budgeting is a universal tool for CFO's to plan and evaluate an investment project.

However, as pointed out in the quote from Jack Welch, a former CEO of General Electric, the budgeting process often involves inflexibility that possibly leads to decision lags. Yang et al. (2020) showed how structured decision making such as capital budgeting can affect the decision lags. According to Yang et al. (2020), CEOs at large firms make a decision based on significantly more structured style than CEOs at small firms. In their estimates, a one standard deviation increase in the score of structured style is associated with a 1.92-fold increase in firm size. Then, they found structured decision-making process takes longer time than unstructured (intuition-driven) process. A one standard deviation increase in the score of structured style is associated with 28% longer time required to reach a decision. This indicates that large firms tend to display decision lags on average due to their structured style of decision making. This does not imply the structured decision making is inefficient. Rather, Yang et al. (2020) points out that the structured style helps making a greater number of decisions than the unstructured style. It could be understood as large firms adopt the structured style due to a great number of issues to deal with, and this leads to decision lags. This gives an explanation on why large-scale investment timings of large firms are insensitive to a change in the real interest rate.

In a survey from Duke University/CFO Magazine Business Global Outlook completed by around 800 CFO's of the U.S. companies reported by Sharpe and Suarez (2013), 89% of respondents answered they would not change investment plan despite more than 3 percentage point decrease in the interest rate.[15] The reason for interest-inelasticity is summarized in Table 5. Among those who answered that their reasons are non-financing related, nearly half of CFO's answered that it is because their investment plans are set on long-term basis.[16] This answer shows inflexible lumpy investment timings due to long-run planning horizon in capital budgeting process. The second largest group of CFO's chose lack of profitable investment opportunities as a reason for their interest-inelastic investment plans. This can

[15] 80% of respondents answered they would not change investment plan despite more than 3 percentage point increase in the interest rate.

[16] For the financing related reasons, 32% of all respondents answered they are interest-inelastic because their firms are financially unconstrained, and 27% of all respondents answered that it is because their hurdle rate from capital budgeting is already higher than interest rate. Around 35% of respondents chose non-financing related answers.

be also attributed to a limitation in capital budgeting practice that might have masked good opportunities according to Jagannathan et al. (2016).

Q. Reasons for not changing investment plan despite the interest rate change (among respondents who chose non-financing related answers)		
Reasons	Despite price drop	Despite price jump
Based on long-term plan, not current rates	49%	47%
Lack of profitable opportunities	29%	31%
High uncertainty	9%	3%
Firm is not capital intensive / Other	14%	19%

Table 5: CFO survey results (Sharpe and Suarez, 2013): inelasticity to interest rate changes

Consistently, Ekholm and Wallin (2000) found from their survey that "incapability of signaling changes in the competitive environment" is the most agreed problem among CFO's in annual capital budgeting convention. This incapability makes firms insensitive not only to price fluctuations, but to investment opportunities. Then, it is natural that the firm sticks to their convention of investment routines, displaying high persistence in inaction periods.[17]

The insensitivity to competitive environment including investment opportunity and interest rate change, is not only an issue to CFO's in large firms. It matters also for the whole economy, as it leads to nonlinear dynamics of aggregate investments once aggregated. In the next section, I model this firm-level insensitivity by introducing a technological restriction that disturbs a nimble reaction to changes in operating environments including idiosyncratic productivity and interest rate.

3 Model

I develop and analyze a heterogeneous-firm real business cycle model that captures the empirical findings of this paper.

In the model, time is discrete, and lasts forever. There is a continuum of measure one of firms that own capital, produce business outputs, and make investments. The business output can be reinvested as capital, after a firm pays adjustment costs.

[17] The surveys I included in this section did not explicitly distinguish large and small firms except for Yang et al. (2020). However, all the respondents are CFO's of firms. Assuming firms that hire CFO are on average large firms, the evidence supports the claim of this paper.

3.1 Technology

A firm owns capital. It produces a unit of goods that can be converted to a unit of capital after an adjustment cost. The production technology is a Cobb-Douglas function with decreasing returns to scale:

$$z_t A_t f(k_t, l_t) = z_t A_t k_t^\alpha l_t^\gamma$$

where k_t is capital input; l_t is labor input; z_t is idiosyncratic productivity; A_t is aggregate TFP, and $\alpha + \gamma < 1$. Idiosyncratic productivity z_t and aggregate TFP A_t follow the stochastic processes as specified below:

$$\begin{aligned} ln(z_{t+1}) &= \rho_z ln(z_t) + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim_{iid} N(0, \sigma_z) \\ ln(A_{t+1}) &= \rho_A ln(A_t) + \epsilon_{A,t+1}, \quad \epsilon_{A,t+1} \sim_{iid} N(0, \sigma_A) \end{aligned}$$

where ρ_i and σ_i are persistence and standard deviation of $i.i.d$ innovation in each process $i \in \{z, A\}$, respectively. Both of stochastic processes are discretized using the Tauchen method for computation.

3.1.1 Investment stage policy

I assume a large-scale investment could be made only after $\bar{s} > 0$ investment stages are completed, and accelerating completion of stages takes time and costs. $\bar{s}$ could be interpreted as a number of bureaucratic steps in capital budgeting, such as meetings of the board of directors for the large-scale investment decisions.[18] From now on, I describe the model without time subscript for the simpler notation. Instead, a future period's allocation is marked with a prime. Without a prime, the variable is for the current period. Due to the recursive nature of the problem, my model can be fully characterized without time index.[19]

In the beginning of a period, a firm is given with the number of completed stages s. I assume s takes discrete nonnegative integer value.[20] If $s = \bar{s}$, the firm reached at the completion period of the large investment. A manager chooses the number of stages $b > 0$ to process within the current period. $s' = s + b$ is the number of total stages completed by the end of the current period. $b = 0$ implies no change in the given stage s. After a large investment is made, I assume the stage starts again from stage 1. Thus, the future stage $\tilde{s}'$

[18]In a framework of the optimal internal capital allocation studied in Malenko (2019), this could be understood as an auditing process for large-scale investment project.

[19]With time index, the notation in the model can become highly complicated due to coexistence of calendar time and planning horizon.

[20]All the results are unaffected in the choice of discrete or continuous stage assumptions.

is equivalent to s' such that $\tilde{s}' \equiv s' \pmod{\bar{s}}$.[21]

Completion of one stage per period does not incur a cost. However, completion of multiple stages in a period entails convexly increasing cost, which I name as acceleration cost, specified in the following form:

$$\text{(Acceleration Cost)} \qquad acc(s', s, k) := \mathbb{I}\{s' > s + 1\} \left(\frac{\mu^a}{2}(s' - s - 1)^2 \right) k^2$$

where s' is the targeted future stage, and μ^a is the acceleration cost parameter. The timing of capital adjustment in extensive margin has been only implicitly determined in the models with fixed cost. In contrast, firms that are subject to acceleration cost explicitly determine the optimal timing of lumpy investment. If a firm faces higher acceleration cost, a firm's nimble capital adjustment is costly. Thus, it becomes less sensitive to surrounding economic environment such as interest rate changes. To capture large firms' inelastic capital adjustment observed from data, I assume that 1) acceleration cost convexly increases over the size of a firm, and 2) hazard rate decreases over firm size. Therefore, large firms face large acceleration cost. The hazard rate is explained later more in detail.

Firms that face extremely high acceleration cost will behave as if they are strictly bound by timing constraints. For these firms, the acceleration cost imposes a similar restriction as time-to-build or time-to-plan constraints (Kydland and Prescott, 1982).

3.1.2 Stage-contingent investment

When the stage is incomplete, $s' \leq \bar{s}$, I assume a firm can invest/disinvest only a small portion of the owning capital stock, following Khan and Thomas (2008) and Winberry (2021). This is also a similar setup as Malenko (2019), where the optimal allocation of capital within a firm follows a threshold rule. According to the paper, divisional managers are allocated with a discretionary account below a threshold in the optimal budgeting. Large-scale investments beyond the threshold needs to be audited by headquarters. The costly auditing process is equivalent to the acceleration cost in this paper.

A firm's capital stock evolves in the following law of motion if $s' \leq \bar{s}$:

$$k' = (1 - \delta)k + I, \quad I \in \Omega(k) := [-\nu k, \nu k]$$

where investment entails a convex adjustment cost $c(k, I) = \frac{\mu_I}{2} \left(\frac{I}{k}\right)^2 k$ as in Winberry (2021). The convex adjustment cost is considered to mitigate intensive-margin elasticity of firm-level investment to the interest rate change. Note that investment is restricted to $[-\nu k, \nu k]$, and

[21] According to this notation, $s = 0$ is equivalent to $s = \bar{s}$.

$0 < \nu < \delta$. In this setup, a firm's capital stock does not reach a steady-state, and a firm's investment follows (S, s) rule in the optimal policy.

When the stage is complete, $s' > \overline{s}$, a firm can make a large-scale investment or disinvestment. Thus, a firm's capital stock evolves in the following law of motion if $s' > \overline{s}$:

$$k' = (1-\delta)k + I, \quad I \in (-\infty, \infty)$$

where investment entails a convex adjustment cost $c(k, I) = \frac{\mu_I}{2}\left(\frac{I}{k}\right)^2 k$.

3.1.3 Hazard function

I introduce hazard function that determines exit rate for firms. According to Clementi and Palazzo (2016), the exit rate exponentially decreases as a firm grows older. In my model, age is not explicitly considered as a state variable. However, by introducing an exponentially decreasing hazard function over firm size proxied by capital stock k, old firms are large on average, consistent with the empiric observation. On top of this, I assume that exiting firms are replaced by the same new firms. This is to purely focus on lumpy investments' role on aggregate fluctuations without heterogeneous entry and exit over business cycle. The assumed functional form of hazard function h is as follows:

$$h(k) := \overline{h} * \left(1 + \frac{1}{exp(k)}\right)$$

where $\overline{h}$ is the parameter that determines the entire level of exit rate. I calibrate this parameter by matching the average exit rate 6.2% in Clementi and Palazzo (2016).

3.2 Timing decision for large-scale investment: intensive margin in the extensive margin

Given the acceleration cost, a firm's timing decision for large-scale investment becomes substantially different from the one in the model with fixed cost. In the latter, firm-level large-scale investment is a binary decision to make it today or not. In contrast, in the model of acceleration cost, there is an additional dimension: an intensive margin in the extensive margin. On top of the decision on whether to make a large-scale investment today or not (extensive margin), a firm needs to decide how further to go with respect to investment stages (intensive margin in the extensive margin).

If a firm processes multiple stages today, a firm can reach a better stage in the future for a large-scale capital adjustment. However, the trade-off is convexly increasing acceleration cost.

Regardless of whether a firm makes a large-scale investment today or not, this decision is necessary in every period. This captures the long-run horizon of investment plans consistent with the survey results in section 2.5.

The investment timing decision can be summarized as the following problem. To capture the core mechanism, I assume the small-scale investment is zero ($\nu = 0$) and the hazard rate is zero in this formulation. Given a firm's value function $J(k, z, s)$ where k is capital stock; z is firm-level idiosyncratic productivity; and s is the number of stages completed, the investment timing decision is as follows:

$$\max\{ \underbrace{\max_{s'>\bar{s}}}_{\text{Ext. margin - large}} -I^{*} - \overbrace{acc(s', s, k)w}^{\text{Int. margin in ext. margin}} + \frac{1}{1+r}\mathbb{E}J(\underbrace{k(1-\delta)+I^{*}}_{\text{Int. margin - large}}, z', \overbrace{s' \pmod{\bar{s}}}^{\text{Int. margin in ext. margin}}),$$
$$\underbrace{\max_{s'\leq\bar{s}}}_{\text{Ext. margin - small}} - \overbrace{acc(s', s, k)w}^{\text{Int. margin in ext. margin}} + \frac{1}{1+r}\mathbb{E}J(\underbrace{k(1-\delta)}_{\text{Int. margin - small}}, z', \overbrace{s' \pmod{\bar{s}}}^{\text{Int. margin in ext. margin}})\}$$

where I^{*} stands for the optimal large-scale investment; r and w are interest rate and wage. In this formulation, a firm first decides whether to make a large-scale investment today ($s' > \bar{s}$) or not ($s' \leq \bar{s}$). This decision problem is the choice between payoffs from the first and the second line. Then, the firm needs to decide how many stages to process given the tradeoff between acceleration cost and the value gain from future investment stage. In this decision, an acceleration of investment stage does not give a flow payoff to the firm in the next period. Instead, it guarantees a better capital adjustment stage in the next period. In this regard, the model with acceleration cost captures a firms' long-run preparation steps for investments.

Therefore, the nature of a firm's problem is starkly distinguished from the problem in the models with fixed cost. In the models with fixed cost, firms determine whether to make a large-scale investment in the current period or not. In this decision making, large-scale investment does not require a preparation step, so firms respond more sensitively in extensive margin to interest rate changes.

Specifically, in the acceleration cost model, the spike ratio for firms greater than a size threshold $\overline{k}$, is as follows:[22]

$$SpikeRatio(\overline{k}) = \int \mathbb{I}\{s'(k,z,s) > \overline{s}\} \underbrace{\frac{\mathbb{I}\{k > \overline{k}\}}{\Phi(k > \overline{k})} \mathbb{I}\left\{\frac{I(k,z,s)}{k} > 0.2\right\}}_{=:\, M(k,z,s)} d\Phi$$

where the first indicator function specifies the condition that firms complete the whole investment stages. The second indicator specifies the firm size requirement, and the third indicator is for investment size requirement. The given distribution of each firm's individual state (k, z, s) is denoted by Φ. For brevity, I define $M(k, z, s)$ as the product of the last two indicator functions.

Firms that satisfy the condition in the first indicator function $s' > \overline{s}$ can be categorized into two groups: 1) firms that are ready for a large-scale investment ($s = \overline{s}$) and 2) firms that accelerate the stages for a large sale investment ($s' < \overline{s}$ and $s' > \overline{s}$).

$$SpikeRatio(\overline{k}) = \int \left(\underbrace{\mathbb{I}\{s = \overline{s}\}}_{\text{Firms that are ready}} + \underbrace{\mathbb{I}\{s < \overline{s}\}\mathbb{I}\{s'(k,z,s) > \overline{s}\}}_{\text{Firms that accelerate}} \right) M(k,z,s) d\Phi$$

For the notational brevity, I denote the expected value when a firm makes large-scale investment as $\mathbb{E}J$ and the expected value when a firm does not make a large-scale investment as $\mathbb{E}J^c$. For firms that accelerate for their large-scale investment, the marginal benefit of acceleration is greater than the marginal cost from the acceleration:

$$\underbrace{\frac{1}{1+r}(\mathbb{E}J - \mathbb{E}J^c) - I^*}_{\text{Marginal benefit}} > \underbrace{\frac{\mu^{acc}}{2}(\overline{s} - s)k^2}_{\text{Marginal cost}}$$

Therefore, the spike ratio could be formulated as follows:

$$SpikeRatio(\overline{k}) = \\ \int \left(\underbrace{\mathbb{I}\{s = \overline{s}\}}_{\text{Invariant over } \Delta r} + \mathbb{I}\{s < \overline{s}\} \underbrace{\mathbb{I}\left\{\frac{1}{1+r}(\mathbb{E}J - \mathbb{E}J^c) - I^* > \frac{\mu^{acc}}{2}(\overline{s} - s)k^2\right\}}_{\text{For large } k \text{, few firms are responsive to } \Delta r} \right) M d\Phi$$

The first term in the bracket is invariant over the contemporaneous interest change. As $\overline{k}$ increases, a mass of firms that accelerate decreases. It is because large capital stock $k > \overline{k}$

[22] Consistent with the empirical section, I define spike ratio as the fraction of firms making investment greater than 20% of existing capital stock.

makes the marginal cost of acceleration greater than the marginal benefit for a great portion of the firms. Therefore, for large firms, the spike ratio is dominantly driven by firms that are already at the last stage for their lumpy investments, so it is highly interest-inelastic in the model with acceleration cost. On the other hand, the spike ratio in the fixed cost model becomes highly responsive to the interest rate change regardless of the size as formulated in Appendix E.

3.3 Nonlinear size effect on interest-inelasticity

In the model, the acceleration cost is assumed to convexly increase in the size of a firm's capital stock. In this section, I study whether this convexity assumption is empirically supported from the data.

Figure 6 illustrates the stationary distribution of the capital stocks in the model and interest-inelasticity in the thick curve. Due to convexly increasing acceleration cost in size, the interest-inelasticity of a firm's investment timing convexly increases. Hence, the model predicts that medium sized firms and small sized firms are not distinguishable in terms of their interest-inelasticity. This is an empirically testable model implication. So, I set two cutoffs $\overline{k}_0$ and $\overline{k}_1$ in the capital distribution to define small and medium firms. Specifically, I set $\overline{k}_0$ as the 50th percentile and $\overline{k}_1$ as the 80th percentile of capital distribution for each two-digit NAICS industry. Thus, small firms are the firms holding capital stock k such that $k < \overline{k}_0$, and medium sized firms are the firms holding capital stock k such that $\overline{k}_0 < k < \overline{k}_1$.

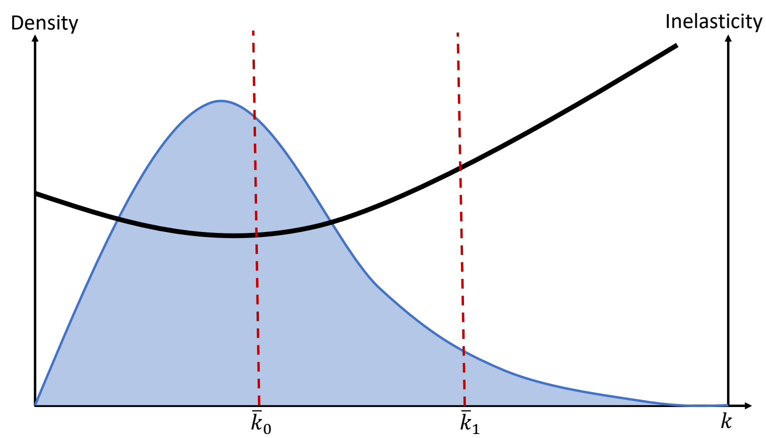


Figure 6: Capital distribution and interest-inelasticity in the model

Then I run the same VAR analysis as in the section 2.2 for small and medium size firms for an interest rate shock. Figure 7 plots the impulse responses of the spike ratio of small and medium firms. Both of the firms display significant drops in the spike ratios, and the difference between two responses are statistically insignificant. The responses are starkly different from the inelastic response of the large firms as shown in Figure 3. From this evidence, I claim the acceleration cost's convexity in capital size is empirically supported.

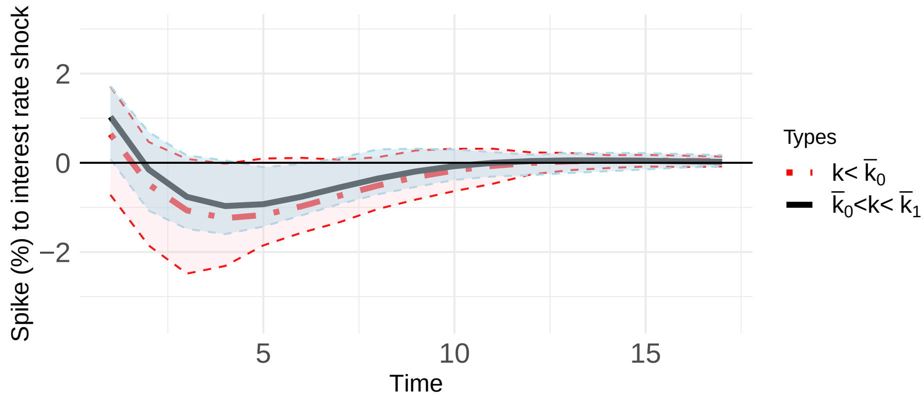


Figure 7: Impulse response of spike ratio for small and medium firms

3.4 Firm's problem: recursive formulation of baseline model

A firm is given with capital k, an idiosyncratic productivity z, and the number of completed stages s in the beginning of a period. Also, they are given with the knowledge on the contemporaneous distribution of firms Φ and the aggregate TFP level A. For each period, firm determines investment level I, labor demand l_d, and when to make a large investment by choosing next period's investment stage s'. A manager of a firm can decide either to get closer to the larger investment period ($s' > s$) or delay ($s' = s$) the process. A firm's problem is formulated in the following recursive form:

$$
\begin{aligned}
&J(k, z, s; \Phi, A) = \pi(z, k; \Phi, A) + \max\{ \\
&\max_{s' > \bar{s}, I}\{-I - c(k, I) - acc(s', s, k)w(\Phi, A) + \frac{1 - h(k)}{1 + r(\Phi, A)}\mathbb{E}J(k', z', s' \pmod{\bar{s}}; \Phi', A')\}, \\
&\max_{s \leq \tilde{s}' \leq \bar{s}, I^c \in \Omega(k)}\{-I^c - c(k, I^c) - acc(\tilde{s}', s, k)w(\Phi, A) + \frac{1 - h(k)}{1 + r(\Phi, A)}\mathbb{E}J(k'^c, z', \tilde{s}'; \Phi', A')\}\}
\end{aligned}
$$

$$
\begin{aligned}
&\text{(Operating Profit)} && \pi(z,k;\Phi,A) := \max_{l_d} zAk^{\alpha}l_d^{\gamma} - w(\Phi,A)l_d \ (l_d\text{: labor demand})\\
&\text{(Convex Adjustment Cost)} && c(k,I) := \frac{\mu^I}{2}\left(\frac{I}{k}\right)^2 k\\
&\text{(Acceleration Cost)} && acc(s',s,k) := \left[\mathbb{I}\{s' > s+1\}\left(\frac{\mu^a}{2}(s'-s-1)^2\right)\right]k^2\\
&\text{(Constrained Investment)} && I^c \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta)\\
&\text{(Aggregate Law of Motion)} && \Phi' := H(\Phi,A),\ A' = G_A(A)\ \text{(AR(1) process)}\\
&\text{(Hazard rate)} && h(k) := \overline{h} * \left(1 + \frac{1}{exp(k)}\right)\\
&\text{(Idiosyncratic Law of Motion)} && z' = G_z(z)\ \text{(AR(1) process)}
\end{aligned}
$$

where J denotes the value function of a firm; l_d is a labor demand; w is wage; r is real interest rate; $c(k,I)$ is a convex adjustment cost, and $acc(\tilde{s}', s)$ is an acceleration cost. z and A are idiosyncratic and aggregate productivities, respectively. The prime in superscript of each variable indicates that the variable is for the next period.

3.5 Household

A stand-in household is considered. The household consumes, supplies labor, and saves. In the beginning of a period, the household is given with wealth level a, information on the contemporaneous distribution of firms Φ, and the aggregate TFP level A. The household problem is as follows:

$$
\begin{aligned}
V(a;\Phi,A) = \max_{c,a',l_H} \quad & log(c) - \eta l_H + \beta \mathbb{E}^{A'} V(a';\Phi',A')\\
\text{s.t. } & c + \frac{a'}{1+r(\Phi,A)} = w(\Phi,A)l_H + a\\
& G(a,\Phi) = \Phi'\\
& G_A(A) = A'
\end{aligned}
$$

where V is the value function of the household; a is a current saving level; Φ is a distribution of firms; A is an aggregate productivity; c is consumption; a' is a future saving level; l_H is labor supply; w is wage, and r is real interest rate. Household is holding the equity of firms as their asset. Following Bachmann et al. (2013) and Khan and Thomas (2008), I assume labor supply is indivisible.

3.6 Cyclical competitive equilibrium

I define cyclical competitive equilibrium that conceptually extends conventional stationary recursive competitive equilibrium. This equilibrium includes aggregate allocations' stationary cycle as a possible equilibrium outcome. When the length of stationary cycle's period is one, the cyclical competitive equilibrium collapses to a stationary recursive competitive equilibrium. A stationary endogenous cycle is one theoretical possibility an acceleration cost model can lead to when a group of firms' lumpy investment timings are independent from idiosyncratic stochastic process. The stationary cycle in cyclical competitive equilibrium will be studied in Section 6. I provide a version without aggregate uncertainty. The extension to a stochastic version is not different from an extension of the stationary competitive equilibrium to the recursive competitive equilibrium. The cyclical competitive equilibrium is defined as follows.

Definition 1 (Cyclical competitive equilibrium)**.**

$(g_c, g_a, g_{lH}, g_k, g_l, g_b, V, J, G, r, w, \Phi, n^*)$ are cyclical general equilibrium if

1. $g_c, g_a, g_{lH}, V : \mathbb{R} \times \mathbb{D} \times \mathbb{R} \to \mathbb{R}$, solve the household's problem. Note that $\mathbb{D}$ is a set of all probability measures Φ defined on the cartesian product of the sigma algebras $\mathcal{K} \times \mathcal{Z} \times \mathcal{S}$ generated from $(\mathbb{K}, \mathbb{Z}, \mathbb{S})$.

2. $g_k, g_l, J : \mathbb{K} \times \mathbb{Z} \times \mathbb{S} \times \mathbb{D} \times \mathbb{R} \to \mathbb{R}$, $g_b : \mathbb{K} \times \mathbb{Z} \times \mathbb{S} \times \mathbb{D} \times \mathbb{R} \to \{0, 1, 2, ...\}$ solve a firm's problem.

3. Define $g_s : \mathbb{K} \times \mathbb{Z} \times \mathbb{S} \times \mathbb{D} \times \mathbb{R} \to \mathbb{S}$, s.t. $g_s(k, z, s; \Phi) = s + g_b(k, z, s; \Phi)$.

   $$(g_k, g_s)(\Phi)(k, z, s) := \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'} dz' \right) \mathbb{I}\{g_k(k, z, s) \in K\} \mathbb{I}\{g_s(k, z, s) \in S\} d\Phi(k, z, s)$$

   for any set (k, z, s) in the σ-algebra $(\mathcal{K}, \mathcal{Z}, \mathcal{S})$ generated from the domains $(\mathbb{K}, \mathbb{Z}, \mathbb{S})$ and $(g_k, g_s)^n(\Phi) = (g_k, g_s)((g_k, g_s)^{n-1}(\Phi))$, for any $n \in \{1, 2, 3, ...\}$, and $\Phi \in \mathbb{D}$.

   There exist $n^* \geq 1$, and $\Phi_0 \in \mathbb{D}$, s.t. $(g_k, g_s)^{n^*}(\Phi_0) = \Phi_0$.
   And define $\Phi_n := (g_k, g_s)^n(\Phi_0)$ for $n \in \{0, 1, 2, ..., n^* - 1\}$.

4. Market Clearing: for $\forall n \in \{0, 1, 2, ..., n^* - 1\}$

$$\text{(Labor Market)} \quad g_{lH}(a; \Phi_n) = \int g_l d\Phi_n$$

$$\text{(Equity Market)} \quad a = \int J(k, z, s; \Phi_n) d\Phi_n$$

5. Consistency Condition:

$$\Phi' = G(a, \Phi) = (g_k, g_s)(\Phi), \text{ for } \forall \Phi \in \mathbb{D}$$

It is worth to note that the length of the equilibrium cycle n^* is an endogenous equilibrium object in the definition. When $n^* = 1$, the equilibrium allocations are at a stationary point. If $n^* > 1$, the equilibrium allocations form a stationary cycle. The markets are required to clear for entire n^* periods within a cycle.

For convenient computation, I use a technique in Khan and Thomas (2008) that solves a firm's problem with normalized value function $\tilde{J}$ instead of J, where $\tilde{J}(\cdot; \Phi, A) := p(\Phi, A)J(\cdot; \Phi, A)$ and $p(\Phi, A) = u'(c(\Phi, A))$. Then, wage and real interest are simultaneously determined by dynamics of $p(\Phi, A)$. Therefore, $p(\Phi, A)$ is the only price to be computed in the outer loop.

Under the aggregate uncertainty, stochastic general equilibrium is hard to compute due to two problems: 1) infinite dimension of state variable Φ, and 2) nonlinear dynamics in aggregate allocations and prices. Due to the latter concern, the celebrated algorithm of Krusell and Smith (1998) is not helpful in the computation of stochastic general equilibrium.

To overcome this difficulty, I use a computation method called repeated transition method which I am concurrently developing in Lee (2021). This method can solve heterogeneous agent model under aggregate uncertainty without relying on parametric form of the law of motion. I elaborate the method in section 5.4.

4 Calibration

The core parameters to be calibrated are acceleration cost and adjustment cost parameters. All the parameters are set at the level that matches simulated moments with target moments except for the parameters of firm-level idiosyncratic productivity process. I fix non-core parameters at the reasonable level consistent with the literature. The labor supply parameter η is set at the level that gives labor participation rate around 60%. The fixed parameters are summarized in Table F.3.

Target Moments	Data	Model	Reference
Persistence of inaction periods	0.88	0.88	Compustat data
Average inaction periods (years)	5.98	5.54	Compustat data
Cross-sectional average of i_t/k_t ratio (%)	10.4	9.9	Zwick and Mahon (2017)
Cross-sectional dispersion of i_t/k_t ($s.d.$)	0.16	0.15	Zwick and Mahon (2017)
Cross-sectional average spike rate (%)	14.4	17.7	Zwick and Mahon (2017)
Cross-sectional average hazard rate (%)	6.20	7.23	Clementi and Palazzo (2016)
Autocorrelation of Y_t	0.94	0.90	NIPA data (Annual)
$sd(I_t)/sd(Y_t)$	1.98	1.79	NIPA data (Annual)

Table 6: Fitted Moments

First, I estimated parameters for firm-level idiosyncratic productivity process outside of the model from Compustat data. The detailed steps for firm-level TFP estimation is explained in Appendix C. Using the estimated firm-level TFP, I run a pooled autoregression, and the parameters of the autoregressive process are $\rho = 0.55$ and $\sigma = 0.18$. In the computation, the idiosyncratic process is discretized using the Tauchen method with 7 grid points.

Then, I calibrate parameters from stationary equilibrium allocations. Matching the average inaction period of 5.98 years, I calibrate the required number of stages $\overline{s} = 4$. From the average persistence of inaction duration 0.88, the acceleration cost parameter is set as $\mu^{acc} = 0.052$.[23]

Zwick and Mahon (2017) summarizes statistics on firm-level investment rates using IRS data. I used the empirical moments reported in Appendix B.1 in Zwick and Mahon (2017) as the target moments for investment rates. From the average investment rate 14.4%, I set $\mu^I = 0.580$. From the average spike rate (%), the small investment range parameter $\nu = 0.030$ is calibrated.[24] The hazard rate parameter $\overline{h} = 0.0565$ is identified from average exit rate, and the level is matched to 6.2% as studied in Clementi and Palazzo (2016). I found this cross-sectional parameter setup gives a close match in an untargeted moment: the cross-sectional dispersion of investment rate.

Aggregate moments are matched in the dynamic stochastic general equilibrium. From the autocorrelation of output obtained from BEA data, I calibrate the autocorrelation parameter of the aggregate TFP process $\rho_A = 0.8145$. From the volatility of private domestic investment relative to output volatility, I set the aggregate TFP volatility parameter $\sigma_A = 0.027$. Based

[23] Persistence of inaction duration is autoregression coefficients of inaction duration obtained from U.S. Compustat data.

[24] Spike rate (%) is the percentage of firms making lumpy investments.

on these paramters, the aggregate TFP process is discretized using the Tauchen method with 5 grid points.

The fitted moments are summarized in Table 6, and the fitted parameters are reported in Table 7.

Parameters	Description	Value
μ^{acc}	Baseline acceleration cost	0.052
$\overline{s}$	Investment completion stage	4
μ^{I}	Baseline adjustment cost	0.580
ν	Small investment range	0.030
h	Hazard rate	0.0565
σ_A	Standard deviation of aggregate TFP shock	0.027
ρ_A	Persistence of aggregate TFP	0.8145

Table 7: Calibrated Parameters

5 Quantitative analysis

5.1 Echo effects in post-shock period

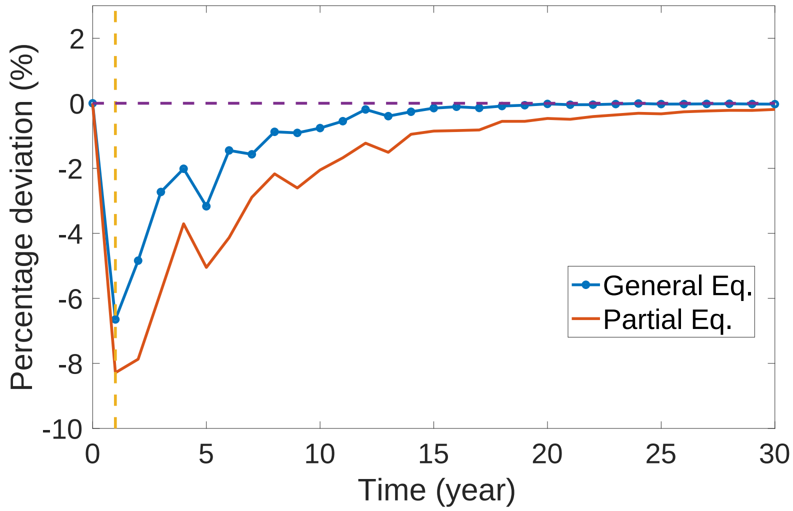
In this section, I quantitatively analyze state-dependent impulse response of aggregate investments to an aggregate TFP shock in the general equilibrium framework.

A novel feature of the acceleration cost model is that impulse response of aggregate investment displays echoes in the post shock periods in general equilibrium. Figure 8 plots impulse responses of aggregate investment in the baseline model for both partial and general equilibrium (panel (a)); its growth in general equilibrium (panel (b)); and impulse response of average investment stages in general equilibrium (panel (c)). The impulse response is obtained from nonlinear method that computes transition path from a shock period to the stationary period as described in Boppart et al. (2018).[25] All the responses are expressed in terms of percentage deviations from the steady-state level.

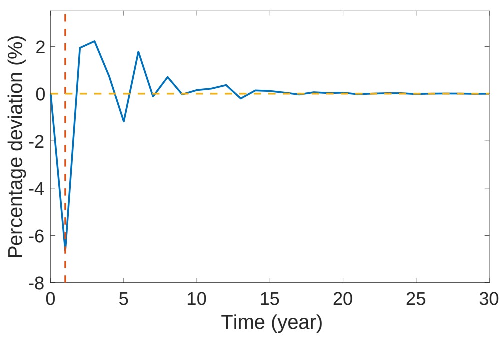
As can be seen from panel (a), general equilibrium effect only partially dampens the response of aggregate investment. Upon impact, aggregate investment drops by 8.3% in partial equilibrium and drops by 6.6% in general equilibrium. Thus, the factor price decreases contemporaneous response by only 20%($\approx 100 * (1 - 6.6/8.3)$). In contrast, in models with fixed cost, general equilibrium effect dampens the contemporaneous response by around 6 folds ($\approx$ 83%).[26] This difference is due to large firms' inelasticity to interest rate fluctuations

[25]Certainty equivalence is assumed in the impulse response by the nature of MIT shock.

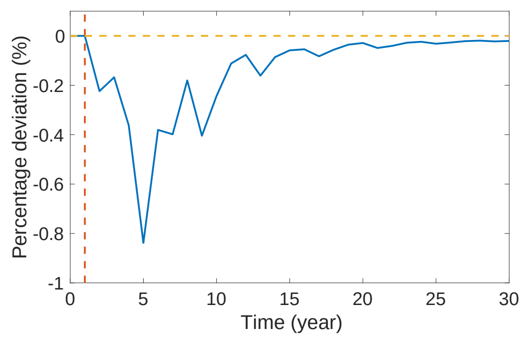
[26]Strong general equilibrium effects in models with fixed cost are compared in Figure 13



(a) Aggregate investment



(b) Aggregate investment growth



(c) Average investment stages

Figure 8: Echo effects in the impulse responses

in the acceleration cost model.

In the post-shock period, aggregate investment gradually recovers to the stationary level. Along the recovery path, there are both trend of recovery and oscillation around the trend. I refer to the oscillation as echo effect. The magnitude of the oscillation in aggregate investment decays overtime, and its magnitude ranges from -6.6% to 2.2% of the stationary level. As shown in panel (b) and (c), the impulse responses of aggregate investment growth rate and average investment stages also display echo effects. For both of the responses, the lowest is at the fifth period from the shock after a shock period. This lowest point is the timing where an economy becomes the most fragile to another negative aggregate TFP shock. This will be studied more in detail in the next section.

On impact of a negative aggregate shock, firms that are ready to adjust capital stock in extensive margin tend to delay their adjustment to escape from low aggregate TFP.[27] There-

[27] If a positive aggregate TFP shock hits the economy, firms that have not considered large-scale investment

fore, the aggregate TFP shock synchronizes firms' large-scale investment timings. Against this synchronized investment timing, there are two mitigating forces that spread out the timings back to the stationary equilibrium distribution. The first is factor price, and the second is stochastic mean reversion. The first force works by making firms' investments costlier when more firms are investing together. In the acceleration cost model, large firms that face high acceleration cost become insensitive to the first force because large acceleration cost already strongly constrained these firms' investment timing. Thus, there is only little room for factor price to affect further the constrained investment timing. Therefore, interest rate dynamics does not fully mitigate the synchronized investment timings among large firms.

The second force, stochastic mean reversion flattens the synchronized timings by spreading out the distribution of lengths of inaction periods. Depending on the mixing rate implied by the idiosyncratic stochastic process, the distribution of lumpy investment timings quickly or slowly move back to stationary distribution. In other words, the speed of convergence in the law of large numbers is the key condition to determine whether synchronized timings of lumpy investments can persist or not. In the calibrated acceleration cost model, the average persistence of inaction periods are as high as in the observed level in the data (≈ 0.9). Thus, the timings of lumpy investments revert back to stationary distribution slowly.

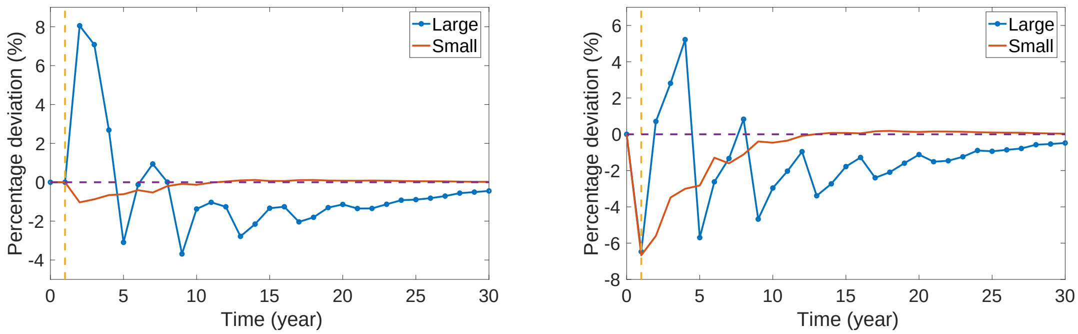
For the decomposition analysis, I define large firms as the top 20% largest firms. Under this approach, 26.7% of total capital belong to large firms. Compustat space covers around half of the total U.S. private fixed investment, and around 60% of capital stocks in Compustat data are from large firms defined in the empirical section. So, around 30% of total capital stock in the U.S. belongs to the large firms. Therefore, the definition of large firms as the top 20% largest firms is consistent with the definition in the empirical analysis.[28] Figure 9 visualizes heterogeneous echo effect for large and small firms in the impulse responses of average investment stages (panel (a)) and aggregate investment (panel (b)). Heterogeneous echo effects under the alternative definitions of large firms are reported in Figure G.4 (Top 30%) and Figure G.5 (Top 40%). Despite the difference in the magnitudes, the qualitative results stay unchanged over the different proxies.

The echo effect is mostly driven by large firms. Panel (a) shows that large firms' investment timings are persistently synchronized in the post-shock period. By around 25 years later from the shock, the synchronization is mitigated, showing the flattened path of average investment stages. Small firms barely shows persistent synchronization due to fast mean-reversion of inaction duration.

Panel (b) shows that large firms' aggregate investment bounce up quickly right after the

in the shock period newly launch or accelerate their projects to utilize high aggregate TFP level.

[28] The model does not capture the thick tail of the firm distribution observed in the data.



(a) Average investment stage

(b) Aggregate investment

Figure 9: Heterogeneous echo effects for large and small firms

aggregate shock. Then, the aggregate investment of the large firms display oscillation ranging from -6% to 5.7% deviation from the steady state. On the contrary, small firms' aggregate investment slowly recovers to the steady-state level without much oscillation.

5.2 Fragility after a surge of lumpy investments

In this section, I study how differently aggregate investments respond to a TFP shock depending on the aggregate state. When an aggregate TFP shock hits the model economy, there arise echoes of the shock in aggregate investment during the post-shock period. Then I hit the economy with another aggregate TFP shock separately at each period on the recovery path. For each experiment, I control the magnitude of aggregate TFP shock to equalize the level of aggregate TFP at the shock period across the experiments.[29]

Figure 10 compares different impulse responses of aggregate investment depending on where the economy is located at the time of shock. The response is strongest when the negative aggregate TFP shock hits the economy right after the surge of aggregate investment; the aggregate investment drops by 7.5%. The response is weakest if a shock arrives at the surge of lumpy investments; the aggregate investment responds by 5.8%. Therefore, the response of aggregate investment is stronger by 29% ($\approx 100 * (7.5 - 5.8)/5.8$) when a shock hits after the surge of lumpy investments than when it does at the surge.

I make the same experiment for large and small firms, separately. Figure 11 visualizes the state-dependent impulse response of aggregate investment for large firms (panel (a)) and small firms (panel (b)). Large firms' immediate response is stronger for a shock after the surge than for a shock at the surge by around 9.7%. In contrast, small firms' response is stronger only by 0.5% for the same comparison. Regardless of where the economy is located,

[29]Specifically, I set the magnitude to set the aggregate TFP at shock period equivalent to one-standard-deviation drop from stationary level.

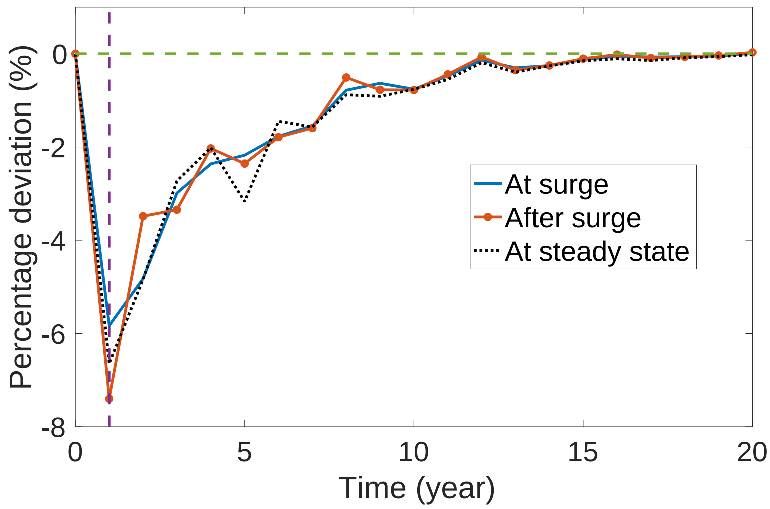
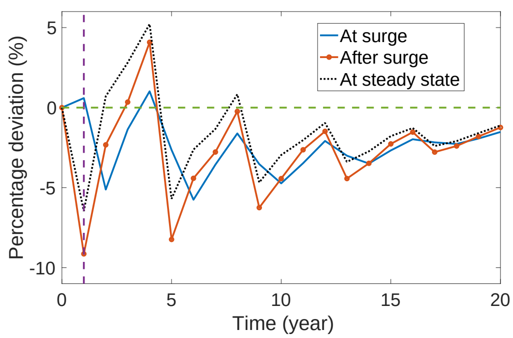
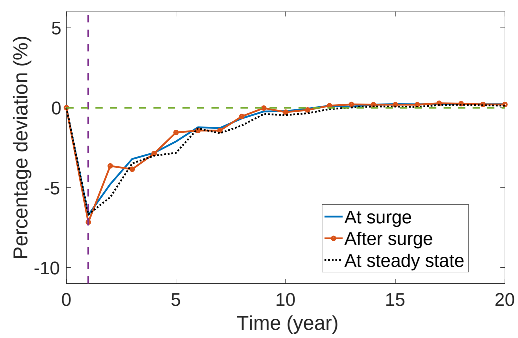


Figure 10: State-dependent impulse response of aggregate investment

small firms' timings of large-scale investment are strongly smoothed out by real interest rate. Thus, they display almost constant contemporaneous investment sensitivity to the aggregate productivity shock.



(a) Large



(b) Small

Figure 11: Heterogeneous endogenous effect

Table 8 summarizes the state-dependent contemporaneous impulse responses of investments. The i^{th} column reports the conditioning states of the i^{th} period from the initial aggregate TFP shock and the responses when a TFP shock arrives at the i^{th} period. The first row represents the lagged aggregate investment expressed in terms of percentage deviations from the steady-state level; The second row represents the lagged aggregate investment of large firms in terms of percentage deviations from the steady-state level; The third row represents the contemporaneous impulse response of aggregate investment; The fourth row represents the contemporaneous impulse response of aggregate investment of large firms; And the fifth row represents the contemporaneous impulse response of aggregate investment of small firms.

The contemporaneous impulse response of aggregate is the largest at the fifth period after the initial aggregate TFP shock. The fourth period display the smallest contemporaneous impulse response.

	t=+2	t=+3	t=+4	t=+5	t=+6	t=+7	t=+8	t=+9	t=+10
ΔI_{t-1} (%)	-6.6	-4.8	-2.7	-2.0	-3.2	-1.5	-1.6	-0.9	-0.9
$\Delta I_{large,t-1}$ (%)	-6.5	0.7	2.8	5.2	-5.7	-2.6	-1.3	0.8	-4.7
$\Delta I_{response,t}$ (%)	-7.3	-6.0	-5.8	-7.5	-6.2	-6.5	-6.2	-6.4	-6.4
$\Delta I_{largeresponse,t}$ (%)	-2.6	-1.5	0.6	-9.1	-7.0	-5.7	-3.9	-9.0	-7.6
$\Delta I_{smallresponse,t}$ (%)	-8.0	-6.6	-6.7	-7.2	-6.1	-6.6	-6.5	-6.0	-6.3

Table 8: Summary of state-dependent impulse responses

The core mechanism of the state-dependent responsiveness of aggregate investment is in the large firms' heterogeneous investment decision. There are two situations in which a large firm makes a lumpy investment in a period t:

(1) Firm enters period t with only one stage remaining ($s = \overline{s}$)

(2) Firm enters period t with more than one stage remaining ($s < \overline{s}$)

If a negative aggregate TFP shock hits the economy, large firms at situation (1) still invest, while large firms at situation (2) do not. Therefore, the state-dependent responsiveness is crucially determined by the fraction of large firms that have only one stage remaining for a large-scale investment, $s = \overline{s}$. Then, I define an investment fragility measure as follows:

$$\text{Investment fragility} := \sum_{Large} \mathbb{I}\{s < \overline{s}\} / \sum_{Large} \mathbb{I}\{s \leq \overline{s}\}$$

The investment fragility measure is the fraction of firms that are not in the last stage for a lumpy investment. In the model, a negative aggregate TFP shock has greater effect when the investment fragility is higher. And the investment fragility becomes high after a surge of lumpy investments.

Figure 12 plots large firms' marginal distributions of stages at different states of the model economy. It is worth to note that the stages are determined one period before the aggregate shock hits the economy by each firm's inter-temporal stage policy. Therefore, the contemporaneous stage distribution is an exogenous condition to an aggregate shock even if they share the same time index. As can be seen from the most right bars in the graph, at the surge of lumpy investments, many firms are at the last stage $\overline{s}$ in the beginning of the period. Therefore, despite a negative aggregate shock, large firms make large-scale investments, and this results in the weak response of aggregate investments to the negative shock. The investment fragility is 90.1% at the surge of lumpy investments. On the other hand, after the surge of lumpy investments, the least fraction of firms are at the last stage. The investment fragility is 93.3% at the surge of lumpy investments.

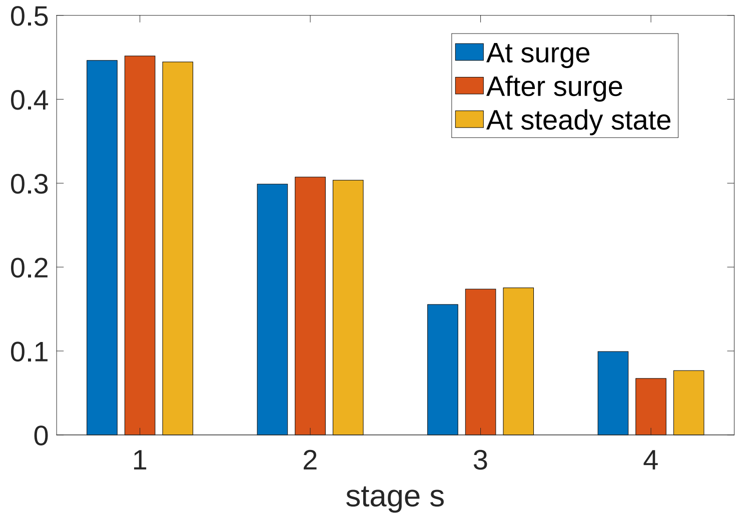


Figure 12: Large firms' stage distributions ($\bar{s} = 4$)

Then, I decompose the total contemporaneous change of the aggregate investment into exogenous component and the endogenous component. The endogenous component is from the direct effect from aggregate TFP shock. The endogenous component accounts for all the other remaining variation unaccounted by the direct effect.

Aggregate investment responds differently to an aggregate TFP shock depending on the aggregate states of the economy. Therefore, we can write aggregate investment $\mathcal{I}_t$ as a function of TFP level A_t and a vector of sufficient statistics of the aggregate state of the economy X_t:

$$\mathcal{I}_t = \mathcal{I}_t(A_t, X_t)$$

Then, the total variation in the aggregate investment $\mathcal{I}_t$ after an aggregate TFP shock can be further decomposed into exogenous component and the endogenous component.

$$\begin{aligned} \Delta log\mathcal{I}_t(A_t, X_t) \equiv & \underbrace{\left(\frac{\partial log\mathcal{I}_t}{\partial logA_t}\right)\Delta logA_t}_{\text{Direct effect}} && \textbf{(Exogenous component)} \\ & + \underbrace{\left(\frac{\partial log\mathcal{I}_t}{\partial logX_t}\right)\Delta logX_t}_{\text{Nonlinear effect}} && \textbf{(Endogenous component)} \end{aligned}$$

In the identity above, exogenous component is driven purely by exogenous direct variation in aggregate TFP; and the endogenous component indicates all other variations residualized after the exogenous variation.

From the experiments of the TFP shocks hitting the economy at different conditioning states, variations in the conditioning states ($\Delta log X_t$) and the contemporaneous total change of the aggregate investment ($\Delta log \mathcal{I}_t(A_t, X_t)$) are available.[30] The exogenous component is directly from the TFP shock and it is common across all observations as the shock magnitude is controlled.

Then, I non-parametrically approximate the conditioning states X_t using the following measures:[31]

1. Lagged aggregate investment $\mathcal{I}_{t-1}$
2. Investment fragility $\mathcal{S}_t$

Hence,

$$X_t = X_t(\mathcal{I}_{t-1}, \mathcal{S}_t)$$

The variation in the conditioning states X_t can be non-parametrically approximated by the variation in $\mathcal{I}_{t-1}$ and $\mathcal{S}_t$:

$$\Delta log X_t = \chi(\Delta log \mathcal{I}_{t-1}, \Delta log \mathcal{S}_t)$$

From the decomposition equation,

$$\begin{aligned}\Delta log \mathcal{I}_t(A_t, X_t) &\equiv \underbrace{\left(\frac{\partial log \mathcal{I}_t}{\partial log A_t}\right) \Delta log A_t}_{\text{Direct effect}} + \underbrace{\left(\frac{\partial log \mathcal{I}_t}{\partial log X_t}\right) \Delta log X_t}_{\text{Nonlinear effect}} \\ &= \underbrace{\left(\frac{\partial log \mathcal{I}_t}{\partial log A_t}\right) \Delta log A_t}_{\text{Intercept}} + \underbrace{\tilde{\chi}(\Delta log \mathcal{I}_{t-1}, \Delta log \mathcal{S}_t)}_{\text{Non-parametric approx.}}\end{aligned}$$

Based on the equation above, I run the non-parametric regression of aggregate investment variation $\Delta log \mathcal{I}_t(A_t, X_t)$ on $\mathcal{I}_{t-1}$ and $\mathcal{S}_t$ to identify the direct effect which is the intercept term in the regression. The intercept is estimated as -6.4, and the R^2 is around 96%. From this high R^2, I confirm $\mathcal{I}_{t-1}$ and $\mathcal{S}_t$ non-parametrically approximate X_t well.

The nonlinear effect amplifies or mitigates the direct effect based on the conditioning states. From the total change we have after surge of lumpy investments and at the surge of lumpy investments, the following decomposition is obtained:

[30] To obtain enough number of samples, I utilize the variations from three rounds of the TFP shock experiments. Specifically, the second round gives 10 observations, and the third round gives 10 observations for each 10 observations of the second round. Thus, I obtain total $10 + 10 \times 10 = 110$ observations.

[31] For robustness check, I use average investment stages $\overline{S}_{t-1}$ instead of $\mathcal{S}_t$. The result stays unchanged for this alternative choice.

After a surge of lumpy investment, the total change of aggregate investment could be decomposed as

$$100\%\ (7.5\%) = \underbrace{85\%\ (-6.4\%)}_{\textbf{Direct effect}} + \underbrace{15\%\ (-1.1\%)}_{\textbf{Nonlinear effect}}$$

At the surge of lumpy investment, the total change of aggregate investment could be decomposed as

$$100\%\ (-5.8\%) = \underbrace{110\%\ (-6.4\%)}_{\textbf{Direct effect}} - \underbrace{10\%\ (+0.6\%)}_{\textbf{Nonlinear effect}}$$

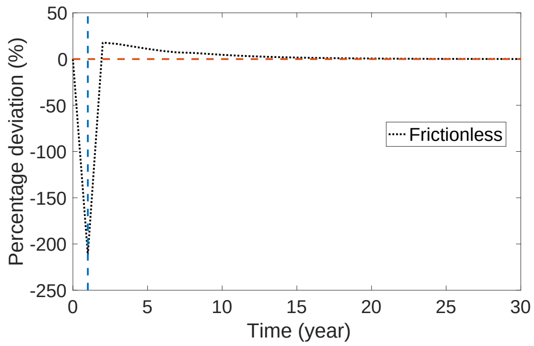
where the numbers in the bracket indicate the absolute effect in percentage.

From this decomposition analysis, I conclude that aggregate investment responds to an aggregate TFP shock substantially differently depending on the conditioning states of the economy. After the surge of lumpy investments from large firms (by 5.2%), the negative aggregate shock effect is amplified by 15% through the nonlinear endogenous channel. This is the upper bound of the amplification effect among the simulated responses. On the other hand, at the surge of lumpy investment, the negative aggregate shock effect is diminished by 10% through the nonlinear endogenous channel. I found this is the lower bound of the amplification effect among the simulated responses.

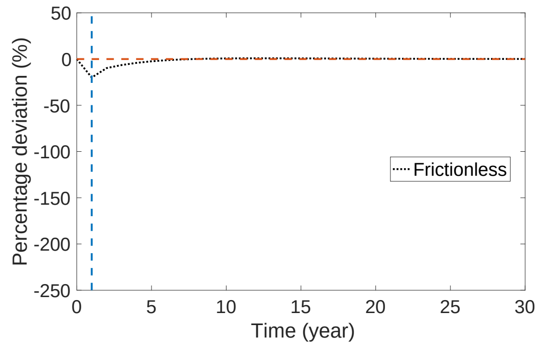
5.3 Comparison with other models

The timing synchronization upon an aggregate TFP shock is not a unique feature of the acceleration cost model; timing synchronization also happens in the models with fixed cost. However, in those models, firms' capital adjustment timing is highly elastic to factor prices. Therefore, in the post-shock period, firms have a strong tendency of not making large-scale investment together with other firms. By flexibly adjusting their investment timings, these firms spread out their lumpy investment schedules to have no lumpiness in the response of aggregate investments. Due to this flexibility allowed in the model, the persistence in the length of inaction periods are significantly (≈ 0.70) lower than the level observed from the data.

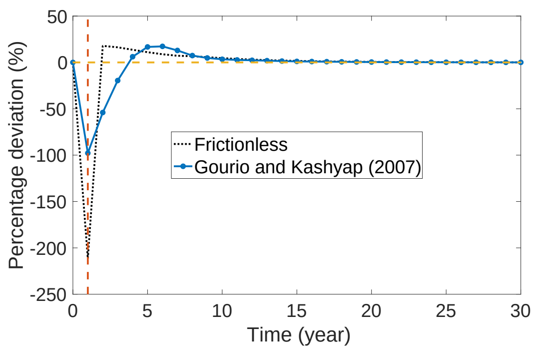
Figure 13 compares the impulse responses of aggregate investment in three different models including a non-lumpy frictionless investment model, Gourio and Kashyap (2007), and Khan and Thomas (2008). For the computation of these models, I use the parameters reported in Khan and Thomas (2008). For the non-uniform fixed cost distribution in Gourio and Kashyap (2007), I use a truncated normal distribution with the mean matched to the uniform



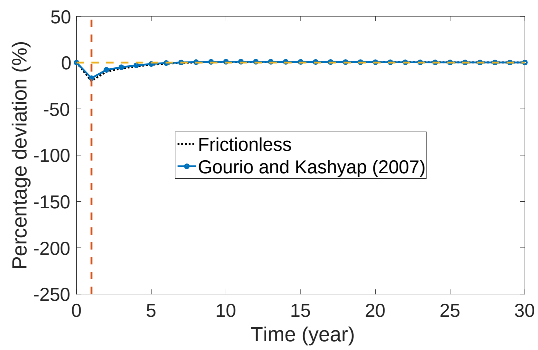
(a) Partial equilibrium



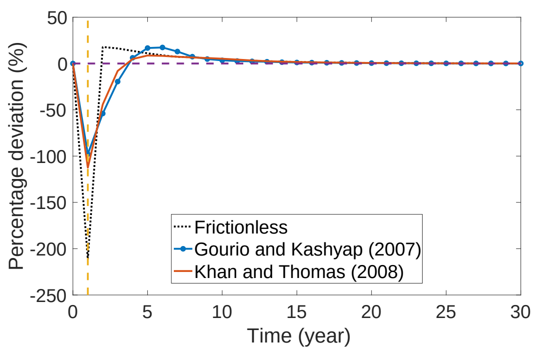
(b) General equilibrium



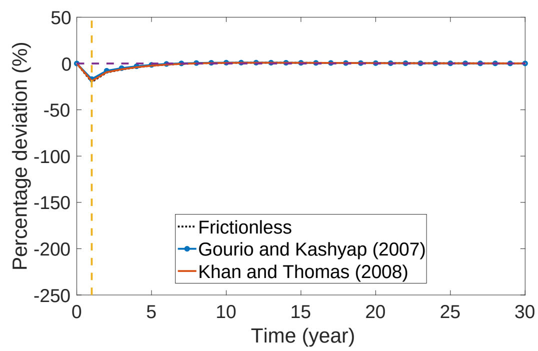
(c) Partial equilibrium



(d) General equilibrium



(e) Partial equilibrium



(f) General equilibrium

Figure 13: Strong general equilibrium effect on aggregate investment in models with fixed cost: irrelevance results

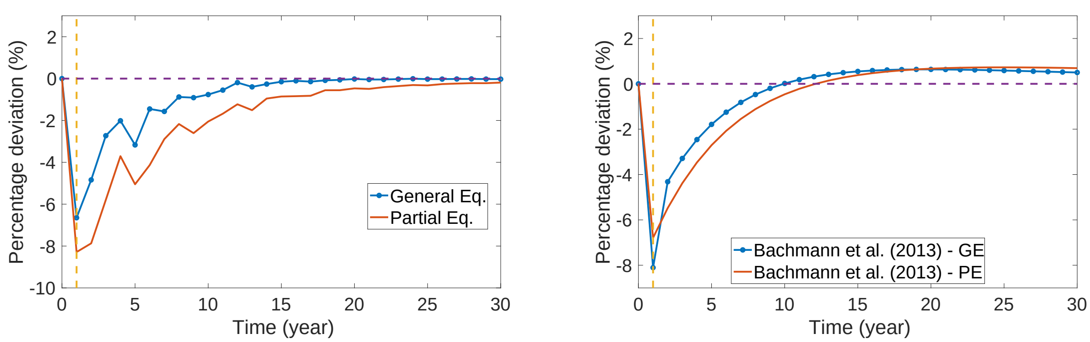
distribution in Khan and Thomas (2008). I used a small standard deviation (=0.001) for the highly concentrated mass around the mean.[32]

The first row of Figure 13 (panel (a),(b)) plots impulse responses of aggregate investment in the frictionless model. When the factor price is not considered (panel (a)), investment drops by more than 200% of steady-state level. However, in general equilibrium (panel (b)),

[32]The result for Gourio and Kashyap (2007) is robust over other parameter choices that give concentrated distributions of fixed cost.

simultaneous drop in the interest rate in the shock period incentivize firms to make more investments in the shock period; this reduces the response of the aggregate investment by more than 10 folds. In the second row (panel (c),(d)), the impulse response of aggregate investment in Gourio and Kashyap (2007) are added. As pointed out by the authors, fixed cost plays an important role to smooth the reactions of aggregate investments to the aggregate TFP shock in partial equilibrium (panel (c)). The aggregate investment drops by around 100% compared to the steady-state level, and smoothly recover. On the recovery path, aggregate investment forms a smooth hump before it converges to steady-state level due to synchronized lumpy investment timings in partial equilibrium. However, when the factor prices are considered (panel (d)), the impulse response of aggregate investment in Gourio and Kashyap (2007) becomes similar to frictionless model due to strong general equilibrium effect. In general equilibrium, the initial response of aggregate investment is dampened by around 6 folds due to factor price fluctuations. As shown in the third row of Figure 13 (panel (e),(f)), Khan and Thomas (2008) model results in similar impulse responses to that of Gourio and Kashyap (2007) in general equilibrium.[33]

Motivated from empirical findings on pro-cyclical sensitivity of aggregate investment to an aggregate shock, Bachmann et al. (2013) suggests a model with fixed cost with maintenance and replacement cost. Figure 14 compares the impulse responses in the acceleration cost model (panel (a)) and Bachmann et al. (2013) (panel (b)). The flattening effect from general equilibrium is substantially smaller in these two models as shown from the small difference between partial and general equilibrium response.



(a) Acceleration cost model

(b) Bachmann et al. (2013)

Figure 14: Factor price's partial smoothing

However, the implied persistence of inaction duration in Bachmann et al. (2013) does not achieve the level observed from the data because investment spikes beyond 20% of existing

[33]Due to the concentrated fixed cost distribution, Gourio and Kashyap (2007) has a stronger smoothing effect in partial equilibrium than Khan and Thomas (2008).

capital stock are modeled to happen more sparsely in general equilibrium to result in significantly lower persistence. Therefore, this model captures interest-inelastic investment spikes while it cannot capture firms' persistent inaction patterns. So the impulse response does not feature echo effect.

5.4 Business cycle analysis

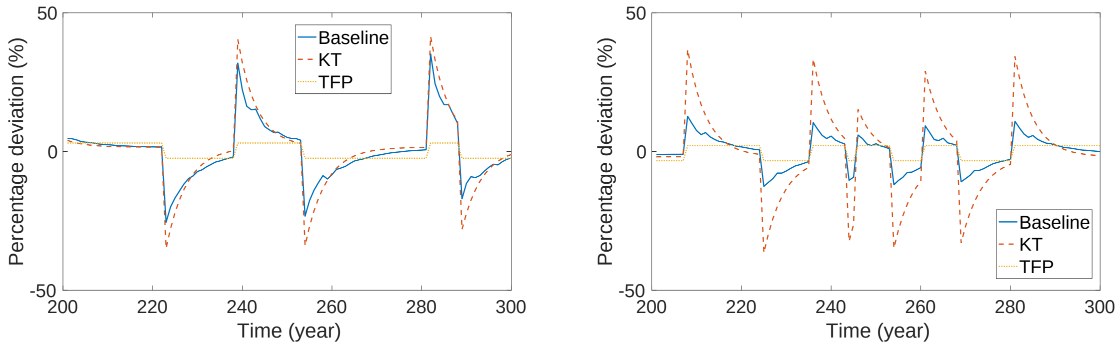
In this section, I analyze business cycle characteristics implied by the dynamic stochastic general equilibrium from the acceleration cost model, and compare these with the results in Khan and Thomas (2008) (hereafter, KT). There are two computational hurdles in this exercise: 1) curse of dimensionality in aggregate state variable and 2) nonlinearity in the aggregate dynamics.

In KT, due to strong general equilibrium effect, the true dynamics of aggregate capital stocks closely follows log-linear prediction rule. So, the dynamic stochastic general equilibrium is obtained by tracking only one moment as in the algorithm suggested by Krusell and Smith (1998).[34] However, as shown from the previous section, aggregate fluctuations implied by the acceleration cost model is highly nonlinear. Therefore, to use Krusell and Smith (1998) algorithm, more moments need to be considered potentially in nonlinear form in the predicted law of motions at large computational cost.

To overcome this difficulty, I use another algorithm, named as the repeated transition method to solve the acceleration cost model under aggregate uncertainty concurrently developed in Lee (2021). In the algorithm, I update an agent's prediction rule for aggregate states repeatedly from transition dynamics on a single simulated path until the prediction rule converges to the simulation. This method does not rely on parametric assumption on the predicted law of motions for the future aggregate states; market clearing prices, expected future aggregate states, and value functions on the transition path are explicitly computed. Then, I back out the prediction rule implied by the fitted outcomes on the sample path and check the validity from the out-of-sample simulation paths. I leave the detailed explanation on the algorithm to Lee (2021). The length of simulated path is 1,000 periods. I use histogram method for transition of the cross-sectional distribution of firms following Young (2010).

Figure 15 plots simulated aggregate investment rates (I_t/K_t) obtained from the baseline model (solid line), KT (dashed line), and simulated aggregate TFP path (dotted line) for different aggregate shocks. Panel (a) is based on the calibrated aggregate TFP shock in this paper, and the TFP shock in panel (b) is from KT. The aggregate investment rates

[34] Khan and Thomas (2003) found that there is no difference in the approximated dynamics of aggregate states between tracking one central moment and tracking two central moments of the partitioned state distribution.



(a) Calibrated TFP shock   (b) Khan and Thomas (2008) TFP shock

Figure 15: Comparison on dynamics of investment rates (I_t/K_t) in business cycle

are expressed in terms of percentage deviations from the steady-state level. The baseline results plotted in panel (a) and (b) are separately obtained from applying the repeated transition method to each of simulated paths from the two different aggregate shocks.[35] The law of motions in KT are obtained from the exact replication of the paper following their computation methodology explained in the paper.

In panel (b), the shock effect in aggregate investment rate of the baseline decays slower than KT. High persistence in lengths of inaction periods contributes to this slowly decaying shock effect. Also, the investment rate is less responsive to a shock in the baseline model than in KT. The acceleration cost makes large firms insensitive to exogenous shocks, weakening the responsiveness of aggregate investment rate. Due to nonlinearity in aggregate dynamics in the acceleration cost model, the baseline result includes echo effects after a jump (drop) in the TFP, as shown in the zig-zag patterns.

Under the calibrated TFP shock in panel (a), the implied dynamics of aggregate investment rate becomes closer between baseline and KT. However, the baseline result still features higher persistence and lower responsiveness than KT.

Table 9 summarizes the business cycle statistics for the simulated allocations in comparison with the statistics in the macro-level data at annual frequency. The data other than employment is from National Income and Product Accounts data (NIPA Table 1.1.5). Employment (L_t) (not an hour) is from Current Employment Statistics.[36] The sample period covers from 1955 to 2018. I use private domestic investments as investment (I_t). All variables are real at annual frequency, and I linearly detrend these variables after taking log.

The numbers in the first column of Table 9 are the statistics from the data; the second is from the calibrated baseline model; the third is from the baseline model computed with an

[35]For the other parameters, I use the same parameters as calibrated in this paper.

[36]Both of the models assumed an indivisible labor supply in the household's utility.

	Data	Baseline	Baseline + KT	KT (2008)
$corr(Y_t, Y_{t-1})$	0.941	0.900	0.867	0.825
$corr(I_t, I_{t-1})$	0.742	0.788	0.755	0.685
$corr(I_t, Y_t)$	0.795	0.924	0.939	0.873
$corr(L_t, Y_t)$	0.898	0.819	0.718	0.805
$corr(C_t, Y_t)$	0.978	0.989	0.993	0.899
$sd(I_t)/sd(Y_t)$	1.976	1.792	1.651	3.079

Table 9: Business cycle statistics

aggregate TFP shock used in KT; and the last column is from the result in KT. In both of the baseline and the baseline combined with KT shock, the autocorrelations of investment are higher than KT, and they are closer to the level observed from the data. High persistence in inaction periods implied by the acceleration cost model contributes to high persistence in aggregate investments. Correlations between employment and output, and between consumption and output are better captured in the acceleration cost model, while the correlation between investment and output are explained better in KT. Also, the relative volatility of aggregate investments in the acceleration cost model is closer to the level in the data.

6 A theoretical limit case: permanent echo and endogenous business cycle

In this section, I explore a theoretical possibility the acceleration cost model can lead to: a permanent echo after an aggregate TFP shock forms synchronization of spiky investment timings across firms.

In the empirical distribution of lengths of inaction periods, there are firms with extremely persistent inaction periods and that are inelastic to interest rate changes.[37] These firms could be understood as having strict investment timing policies for their investments that is almost invariant over time. In this section, I model these highly persistent inaction duration as a nature of investment technologies of large firms. Large firms' inaction periods are modeled to have a strict persistence at one.

Then, if timings of these firms are synchronized, the echo effects from these firms will not be muted by factor-prices. Also, idiosyncratic stochastic force does not flatten the comovements of these firms' lumpy investments.[38] Therefore, the echoes of aggregate TFP will

[37] Around 3% of firms have identical inaction periods across years.

[38] Event analysis shown in Figure 4 also points out large firms' lumpy investments are almost unaffected by idiosyncratic forces. Therefore, the speed of convergence in the law of large numbers is extremely low for large firms.

last forever. Depending on how investment timings among these firms are synchronized, the endogenous fluctuations in the permanent echo can take various patterns.

I compute the cyclical competitive equilibrium with a synchronized initial distribution that might have been initialized by some large aggregate TFP shocks in the prior history. I mute aggregate TFP fluctuations to solely focus on endogenous component of aggregate fluctuations. Without exit and entry, large and small firms are assumed to be permanently separated for simplicity.

I assume heterogeneous parameters for adjustment cost and acceleration cost. Specifically, for a firm with type $j \in \{small, large\}$, the investment technologies are modelled as follows:[39]

$$\text{(Convex Adjustment Cost)} \qquad c(k, I, j) := \frac{\mu_j^I}{2}\left(\frac{I}{k}\right)^2 k$$

$$\text{(Acceleration Cost)} \qquad acc(s', s, j) := \left[\mathbb{I}\{s' > s+1\}\left(\frac{\mu_j^a}{2}(s' - s - 1)^2\right)\right]$$

Note that differently from the baseline model, acceleration cost now does not depend on the size of capital stock. Instead, I assume ex-ante heterogeneous investment technology characterized by different parameters. When I bring the model to fit into the data, I obtain following strict orders between parameters:

$$\mu_{large}^a > \mu_{small}^a,\ \mu_{large}^I < \mu_{small}^I$$

The larger acceleration cost is needed for large firms to capture large firms' interest-inelasticity and more persistent inaction duration. The smaller convex adjustment cost is to match the fact that large firms are greater in size than small firms.[40]

If a large firms' acceleration cost is large enough, a firm might not choose to accelerate its investment stage at all and stick to one-stage-per-period rule despite the fluctuations in the factor prices and idiosyncratic productivities. In this case, the firm's capital adjustment policy will follow semi-deterministic (S, s) cycle: adjustment timing in extensive margin is deterministic while the intensive margin stochastically changes depending on the factor price and idiosyncratic productivity realizations. The following proposition formally states the existence of such large acceleration cost parameter that warrants semi-deterministic (S, s) cycle of large firms.

[39]The full formulation of heterogeneous large and small firms' problem is available in Appendix I.

[40]If $\mu_{large}^I \geq \mu_{small}^I$ holds, large firms' size become smaller than small firms. This is because large firms make less frequent capital adjustment in extensive margin ($\mu_{large}^a > \mu_{small}^a$), and the size of adjustment is smaller for large firms due to larger convex adjustment cost. This is counterfactual in that large firms are bigger firms than small firms on average.

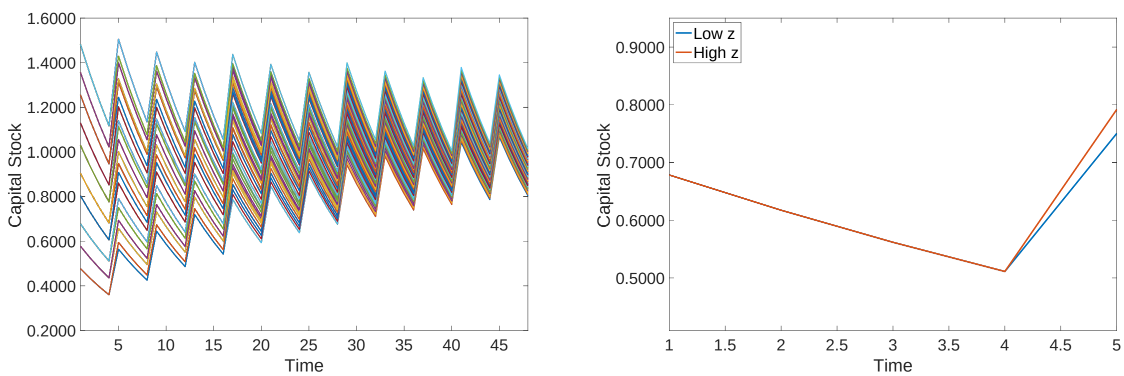
Proposition 1 (Isolated stage policy).
Given an idiosyncratic productivity process $G_z(z)$ with a bounded support $\mathbb{Z}$, there exists $\overline{\mu}_{G_z} > 0$ such that

$$\mu^a_{large} \geq \overline{\mu}_{G_z} \implies \hat{s}'(k, z, s, large; \Phi, A) = \hat{s}'(s, large) \text{ for } \forall (k, z, s) \in (\mathbb{K}, \mathbb{Z}, \mathbb{S})$$

where $(\mathbb{K}, \mathbb{S})$ denotes the domains of capital and investment stages, respectively.

Proof. See Appendix K.1. ■

It is worth to note that the threshold of large acceleration cost $\overline{\mu}_{G_z}$ is specific to idiosyncratic stochastic process G_z. If z can take an extreme value with positive probability, investment stage policy $\hat{s}'$ depends on shock realizations. However, if the idiosyncratic productivity process has a bounded support, there exists a sufficiently large level of acceleration cost parameter that makes investment stage policy independent from the shock process and interest rate fluctuations. Hereafter, given G_z with a bounded support $\mathbb{Z}$, I assume $\mu^a_{large} > \overline{\mu}_{G_z}$.



(a) Heterogeneous firms

(b) A single firm

Figure 16: Large firms' semi-deterministic (S, s) cycle and capital adjustment in intensive margin

Figure 16 shows large firms' semi-deterministic (S, s) capital adjusting rule. In this exercise, heterogeneous firms are given with different level of capital stocks at period 0, and the trajectory of each firm's optimal level of capital stocks is tracked over time. I use $\overline{s} = 4$ as in the calibration of baseline parameters. Panel (a) highlights the deterministic extensive-margin rule for capital adjustment. Due to large acceleration cost, large firms follow one-stage-per-period rule, and this makes capital stocks jumps up in every $\overline{s}$ periods, regardless of idiosyncratic productivity realizations and interest rate fluctuations. However, the magnitude of jumps changes depending on the idiosyncratic productivity realizations as shown in the single firms' capital adjusting rule in panel (b). Thus, the capital adjusting rule follows a semi-deterministic (S, s) rule.

Large firms' semi-deterministic (S, s) capital adjusting rule leads to a stationary capital cycle after aggregation in general equilibrium. This is because the initially synchronized firm's adjusting timings permanently stay synchronized without being mitigated by either factor prices or stochastic forces. This could be understood as a limit case of baseline model where stochastic mean reverting forces gradually flatten the echo effect.

Specifically, if the initial distribution of large firms' investment timings is non-uniform, there will be a stationary cycle of aggregate investments. This class of initial distributions is formally defined as follows:

Definition 2 (Class of synchronized distributions)**.**
Given $(\mathbb{K}, \mathbb{Z}, \mathbb{S})$*,* $\mathbb{D}$ *denotes a set of all probability measures* Φ *defined on the cartesian product of the sigma algebras* $\mathcal{K} \times \mathcal{Z} \times \mathcal{S}$ *generated from* $(\mathbb{K}, \mathbb{Z}, \mathbb{S})$*. Define a partition* $\{\mathbb{D}_0, \mathbb{D}_1\}$ *of* $\mathbb{D}$ *as follows:*

$$\mathbb{D}_1 := \{\Phi \in \mathbb{D} \mid \textit{for } \forall s \in \mathbb{S}, \int_{\mathbb{K}\times\mathbb{Z}\times\{s\}\times\{y,o\}} d\Phi\,(k, z, s, j; \Phi, A) = \frac{1}{\bar{s}}\}, \quad \mathbb{D}_0 := \mathbb{D} \setminus \mathbb{D}_1$$

The partition $\mathbb{D}_0$ is a class of firm distributions that support stationary cycle of aggregate investments once they become an initial distribution. In Proposition 2, I show that if large firms' investment stage policy is independent from price fluctuations and idiosyncratic productivity shocks, and initial distribution belongs $\mathbb{D}_0$, there does not exist a stationary recursive competitive equilibrium. In Corollary 1, I show that under the same condition, the cyclical competitive equilibrium with $n^* > 1$ exists.[41] Before the theoretical results, I define an implied sequence of distributions which is useful for throughout the theoretical statements.

Definition 3 (Implied sequence of distributions)**.**
Given firms' policy k'*,* s'*, and an initial distribution* Φ_0*, I define the implied sequence of distributions as* $\{\Phi_\tau\}_0^\infty$ *such that*

$$(\Phi_{\tau+1})(K, Z, S, j; \Phi, A) := \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'} dz'\right) \mathbb{I}\{\hat{k}'(k, z, s, j; \Phi, A) \in K\}$$
$$\mathbb{I}\{\hat{s}'(k, z, s, j; \Phi, A) \in S\} d\Phi_\tau(k, z, s, j; \Phi, A)$$

for any set (K, Z, S) *in the* σ*-algebra* $(\mathcal{K}, \mathcal{Z}, \mathcal{S})$ *generated from the domains* $(\mathbb{K}, \mathbb{Z}, \mathbb{S})$*.*

It is worth to note that hazard rate, type transition, and new entry are only implicitly considered because type distribution is assumed to stay the same after replacement for

[41] See Definition 3.

simplicity.

Proposition 2 (Breaking the law of large numbers)**.**
If $\mu^a_{large} \geq \overline{\mu}_{G_z}$*, and* $\Phi_0 \in \mathbb{D}_0$*, the implied sequence of distributions* $\{\Phi_\tau\}_0^\infty$ *does not have a limit point.*

Proof. See Appendix K.2. ∎

Therefore, Proposition 2 states that there does not exists a stationary recursive competitive equilibrium. Then, I show there exists cyclical competitive equilibrium in the following corollary.

Corollary 1 (Endogenous stationary cycle)**.**
Given $\mu^a_{large} \geq \overline{\mu}_{G_z}$ *and the initial distribution* $\Phi_0 \in \mathbb{D}_0$*, for* $\forall\ \epsilon > 0$*, there is a sufficiently large* $\overline{\tau} \in \{1, 2, 3, ...\}$ *such that the implied sequence of distributions* $\{\Phi_\tau\}_{\tau=0}^\infty$ *satisfies following property:*

$$||\Phi_{\tau+\overline{s}} - \Phi_\tau||_{\sup} < \epsilon, \ \text{for } \forall\tau > \overline{\tau}$$

Proof. See Appendix K.3. ∎

From Corollary 2, I show that the synchronized distribution $\mathbb{D}_0$ includes nearly all possible distributions of state variables. Thus, under the perfect isolation of large firms' stage policy from price fluctuations and idiosyncratic shock process, a stationary cycle arises in almost every initial distribution. Therefore, any slight synchronization of lumpy investment timings resulting from an aggregate TFP shock will lead to aggregate fluctuations due to a permanent echo.

Corollary 2 (Commonness of aggregate cycles)**.**
Consider a non-degenerate atomless distribution Ψ *defined on* σ*-algebra* $\mathcal{D}$ *generated from* $\mathbb{D}$*, where* $\mathbb{D}$ *is the support of* Ψ*. Then,* $\Psi(\mathbb{D}_1) = 0$*, and* $\Psi(\mathbb{D}_0) = \Psi(\mathbb{D}) = 1$*.*

Proof. See Appendix K.4. ∎

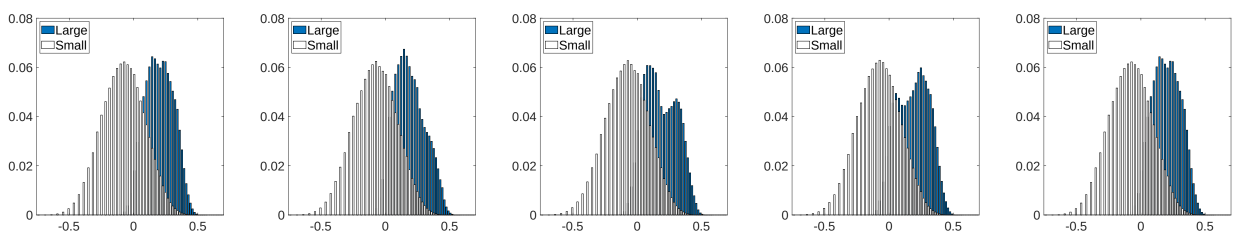
Given these theoretical results, I compute the cyclical competitive equilibrium using the parameters reported in Table I.4. I set the fraction of total large and small firms at the level where large firms hold the half of total capital stocks in the economy.[42] These parameters give similar cross-sectional moments to the target moments in the baseline.

[42] This is based on the summary statistics reported in Table 1.

For the stationary cycle, important parameters to be specified are the initial distribution on completed investment stages for large firms.[43] The distribution could be estimated from data by matching the fraction of firms making large investments. For the computation exercise, I use $\phi_S = (0.2211, 0.2412, 0.2613, 0.2764)$ as an initial synchronized distribution of investment stages for large firms.[44] As theory predicts, the initial distribution of completed stages is not mixed in the stationary cycle, and it moves in the circular pattern to make aggregate fluctuations. For the other parameters, I use the same parameters as in the calibrated baseline model.

The cyclical competitive equilibrium requires market clearing for the whole periods within a cycle. Computing market clearing prices for the entire cycle is a difficult task because firms' inter-temporal policies are sensitive to price rankings across periods. For this, I introduce a novel algorithm that solves the cyclical competitive equilibrium by preserving relative rankings of prices over the convergence path. I describe the details of the computation method in Appendix 8.

Figure 17 plots the endogenous fluctuations in the marginal distributions of large and small firms' logged capital stocks in the cyclical competitive equilibrium. Small firms' capital distribution shows little fluctuations, while large firms' capital distribution dramatically fluctuate endogenously without reliance on any exogenous aggregate forces.



(a) Period 1 (b) Period 2 (c) Period 3 (d) Period 4 (e) Period 1

Figure 17: Endogenous fluctuations in capital distributions for large and small firms

Along these aggregate fluctuations in the state distributions, aggregate allocations also move forming a stationary cycle in general equilibrium. Figure 18 plots the time path of aggregate allocations in the cyclical competitive equilibrium. As I set the required stages for large-scale investment $\bar{s} = 4$, the length of a period in the stationary cycle is also four periods. In the endogenous cycle, aggregate investment (i), employment (l), and real interest rate (r) are pro-cyclical, and aggregate capital stocks (k), consumption (c), and wage (w) are

[43] Small firms' initial distribution converges to an ergodic distribution following the law of large numbers. Therefore, initial distribution does not have to be specified for small firms.

[44] Small firms' initial distribution is not specified. It is because small firms' stage distribution converges to ergodic distribution regardless of the initial distribution.

counter-cyclical.

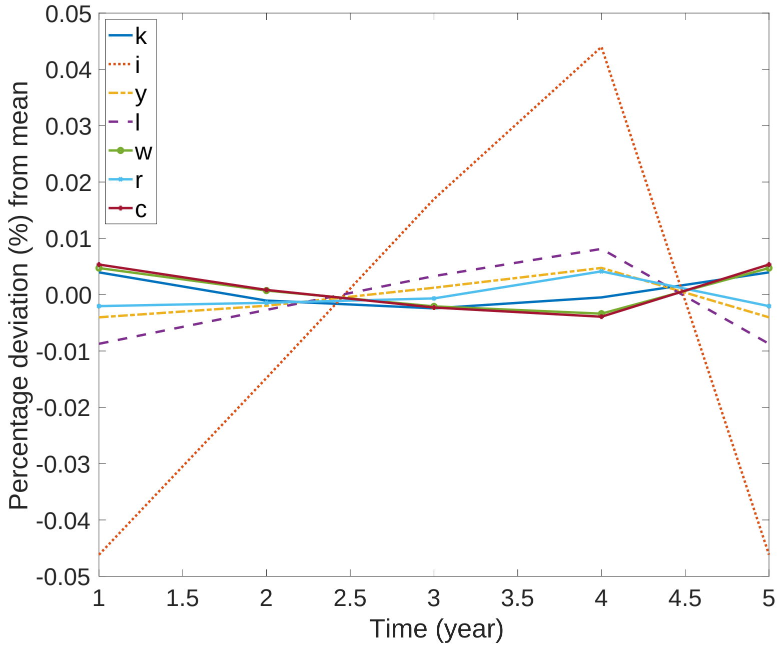


Figure 18: Endogenous stationary cycle in cyclical competitive equilibrium

These endogenous aggregate fluctuations in the stationary cycle are permanent echoes from large aggregate TFP shock that might have happened in the prior history that is not specified in the model. The initiation mechanism of these endogenous cycle is checked by impulse response of the economy to the large aggregate shocks such as a negative aggregate TFP shock during the Great Depression; According to Ohanian (2001), aggregate TFP dropped by around 18% during the Great Depression.

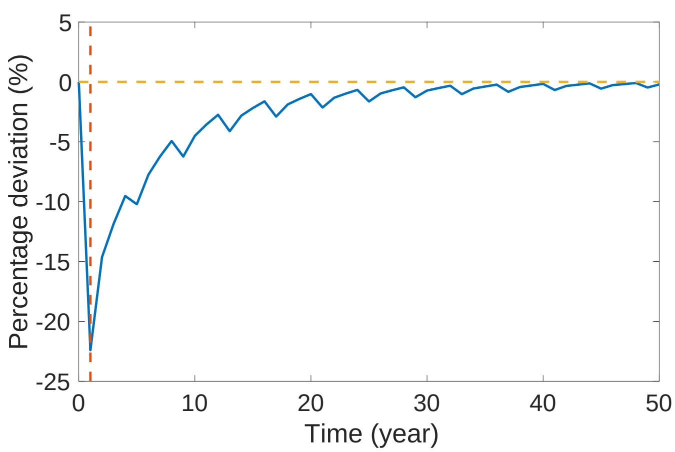


Figure 19: Permanent echo of a Great Depression

Figure 19 shows permanent echoes in the aggregate investments after a sudden 18% drop in the aggregate TFP. In this exercise, I assume the economy's investment stages were uniformly distributed (unsynchronized) before the shock.[45] Then, after the large aggregate shock, the firms' investment timings are synchronized, and it generates a permanent echo in the economy.

To sum up, large firms' extreme inelasticity to interest rate and extreme persistence in inaction durations lead to a permanent echo that generates endogenous aggregate fluctuations. This is a limit case of the baseline model which features decaying echo in the post shock periods. When a large shock hits the economy, the acceleration cost model predicts that an echo effect will not decay in the short run both in decaying echo and permanent echo setup. Specifically, a permanent echo model could be potentially used to analyze short run business cycle after a large aggregate TFP shock such as the Great Depression or COVID-19 pandemic.

In the short run business cycle analysis, a permanent echo model is particularly useful because the initial distribution of investment stages is a free parameter that can be estimated; the model could be fitted into large cross-sectional data. This characteristic is unique among general equilibrium models that studies aggregate fluctuations.

However, there are limitations in this theoretical argument: permanent echoes are difficult to detect empirically from data because the endogenous stationary cycle is not a response to any impulse. Another difficulty is that permanent echo patterns are subject to change depending on the arrival of different aggregate TFP shocks. Thus, I leave this endogenous cycle as a theoretical possibility an acceleration cost model can lead to, with a possibility to be used in short run business cycle analysis in future researches.

7 Empirical evidence from aggregate-level data

General equilibrium in acceleration cost model features echoes in aggregate investment after an aggregate TFP shock. This is due to interest-inelastic firm-level lumpy investments and high persistence in the length of inaction periods. These two characteristics are based on micro-level observations from the U.S. Compustat data. In this section, I show the echo effect is empirically supported in the macro-level data.

Figure 20 plots time series of the growth rate of investment in non-residential structures in manufacturing industry from 1935 to 2014 (thick solid line). The data is from BEA (NIPA Table 5.4.1, line 14). According to Ohanian (2001), the aggregate TFP has dropped around 18% during the Great Depression. Thus, if there were interest-inelastic firms with persistent

[45] Thus, the aggregate allocations are at the stationary competitive equilibrium.

inaction periods, there must have been large echoes in the post-crisis period according to acceleration cost model prediction.

Testing whether certain fluctuations are from echo effects or from stochastic shock process is a demanding task. However, there is a clear difference between fluctuations from two sources: echo effects results in deterministic periodicity across the humps, while stochastic shocks lead to random periodicity.[46] Therefore, the key to detect echo effects from a time-series hinges on the existence of deterministic periodicity.

After the Great depression in 1933, the growth rate in non-residential structures for manufacturing industry has fluctuated dramatically, as shown from the solid line in Figure 20. To statistically test the deterministic periodicity in these fluctuations, I use Fisher's g-test, following Wichert et al. (2004). Fisher's g-test tests deterministic periodicity in a time-series X_t by fitting the series into the following functional form:

$$X_t = \beta cos(\omega t + \phi) + \epsilon_t$$

where $\beta > 0$, $\omega \in (0, \pi)$, $\phi \sim U(-\pi, \pi]$, and ϵ_t is a serially uncorrelated noise which is assumed to be independent from ϕ. And the null hypothesis H_0 is as follows:

$$H_0 : \beta = 0$$

I apply this test to two sub-periods: 40 years right after the crisis (1933∼1972) and the

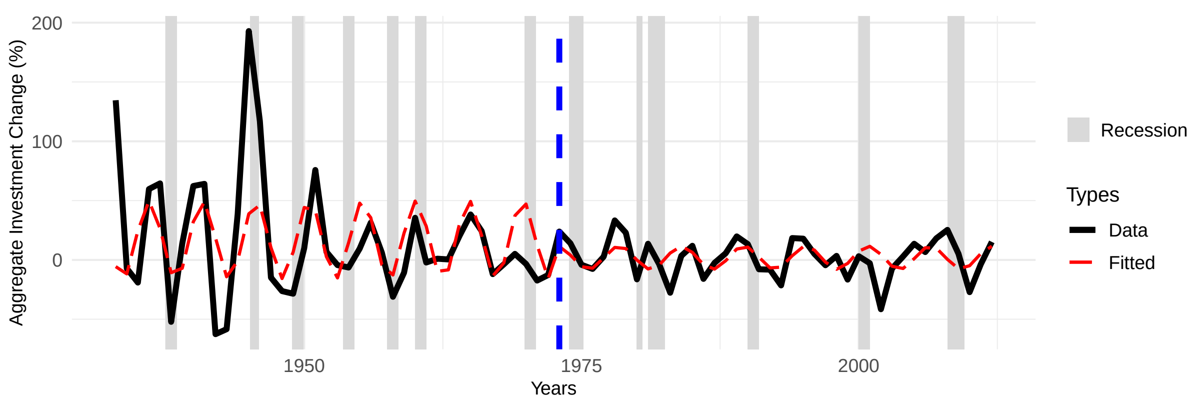


Figure 20: Echo effect in investments of manufacturing industry after the Great Depression

recent 40 years (1973∼2012). I refer to the former period as echo period, and the latter as non-echo period. As reported in Table 10, the large fluctuations after the Great Depression, featured a significant deterministic periodicity, and the length of a period is around 4.7

[46]Deterministic periodicity can happen in stochastic process with probability zero.

years.[47] However, the deterministic periodicity disappears in the recent years, which can be explained by decaying echo in the acceleration cost model.[48] The dashed line in Figure 20 displays fitted time-series in Fisher's g-test. During the echo period, fluctuations from the data and fitted series share the timings of ups and downs. However, this does not hold in the non-echo period. Consistently, Figure I.8 shows significant jump in the spectral density at the frequency of four to five years in the echo period. In the non-echo period, the spectral density does not display a peak. Table I.5 reports the serial correlation in the residuals. For echo periods, there was no significant serial correlation in the residuals. Thus, it validates the test result that is based on the assumption of serially uncorrelated errors.

	Echo period (1933 $\sim$ 1972)	Non-echo period (1973 $\sim$ 2012)
Estimated period	4.706	5.714
p-value	0.024	0.165

Table 10: Periodicity testing: echo after the Great depression

Additionally, I apply the same test procedure to oil industry. I use the nonresidential fixe investment growth data from BEA (NIPA Table 5.4.1, line 20). The large aggregate TFP shock of interest is the oil crisis at 1979.[49] Similar to the previous test, I divide the time-series into two sub-periods: 25 years right after the crisis (1933∼1972) as echo period and 25 years prior to the crisis (1973∼2012) as non-echo period.

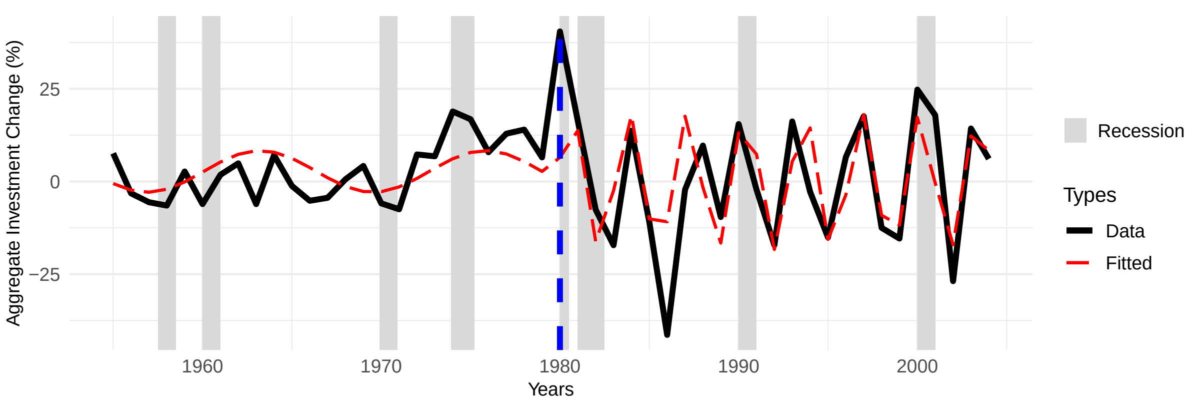


Figure 21: Echo effect in investments of oil industry after the oil crisis

[47]This result is robust over choices of sample periods and over test specifications such as extended g-test and likelihood-based tests.

[48]I apply the same g-test to the model-generated investment growth rate fluctuations in the post-shock period plotted in Figure 8. The estimated duration of deterministic period is 4.17 years, and it is statistically significant.

[49]Specifically, I test investment growth in non-residential structures for industries of mining exploration, shafts, and wells.

As can be seen from the solid line in Figure 21, after the oil crisis, investment growth in non-residential structures featured larger fluctuations compared to pre-crisis period. According to the test results reported in Table 11, the investment growth during the post-crisis periods features significant deterministic periodicity. However, in the period prior to crisis, there was no deterministic periodicity.

	Echo period (1980∼2004)	Non-echo period (1955∼1979)
Estimated period	3.333	25.000
p-value	0.030	0.820

Table 11: Periodicity testing: echo after the oil crisis in 1979

Next, I document evidence on echo effects from VAR analysis on aggregate investment. Specifically, using VAR, I study whether there are nonlinear responses to an output shock that has a similar pattern as echo effects from the macro-level data. For the VAR on macro-level investment data, I include HP-filtered real GDP and HP-filtered investments in non-residential structures for manufacturing industry from National Income and Product Accounts data (NIPA), in the stated order.[50] AIC criterion is used for the choice of optimal lags ($p = 4$) in the regression.

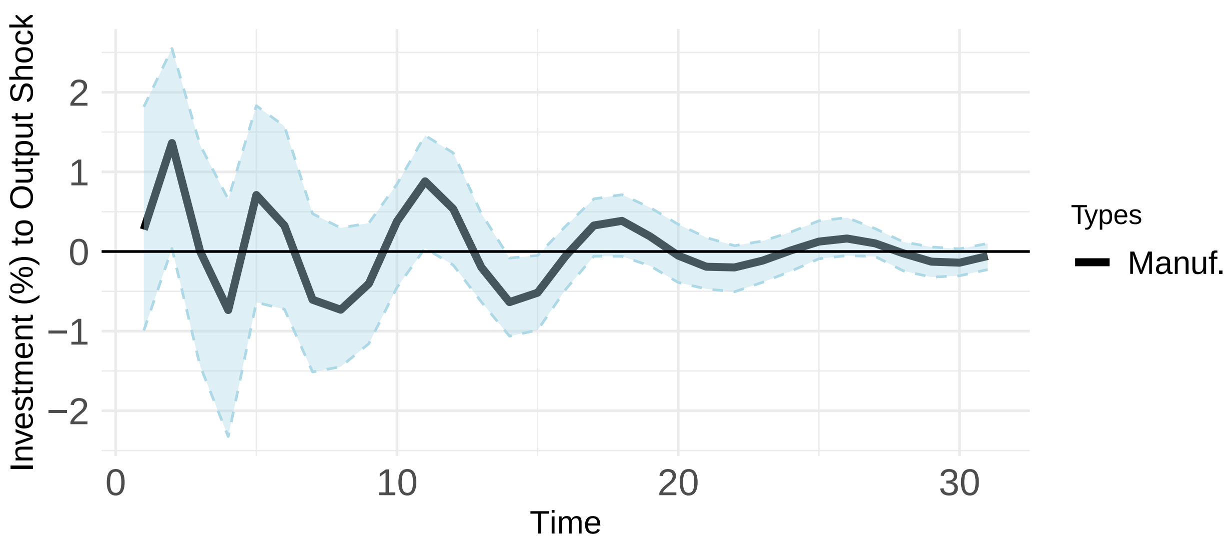


Figure 22: Impulse response of non-residential structure investment from NIPA data

Figure 22 plots impulse responses of investments in non-residential structures for manufacturing industries from BEA (Fixed Asset Accounts Table 4.8, line 11 and line 15). As

[50]All the variables are at an annual frequency. In the Hodrick-Prescott filter, I use 6.25 as a smoothing parameter following Ravn and Uhlig (2002).

can be seen from the figure, nonlinear echo effects are present in the impulse responses of the investments of manufacturing industries. This evidence supports the echo component in the response of the aggregate investment to an aggregate shock.

8 Conclusion

This paper studies how interest-inelastic lumpy investments at the firm level affect the business cycle. From empirical analysis, I show large firms' lumpy investments are inelastic to interest rate changes, and their inaction durations are highly persistent across periods. Then I develop a real business cycle model with heterogeneous firms that captures these two empirical facts. In the model, the aggregate investments display nonlinear impulse response to aggregate TFP shocks in general equilibrium. Specifically, there arise echoes of aggregate TFP shock in aggregate investment in the post-shock period. This is because synchronized timings of lumpy investments across large firms persistently synchronized over time due to weak flattening forces from factor prices and stochastic mean reversion. The endogenous fluctuations in large firms' lumpy investments generate a cycle of relaxation and contraction in the large firms' investment rate. After the high concentration, the economy becomes fragile to a negative aggregate TFP shock. The aggregate investment responds 29% stronger after the surge of large firms' lumpy investments than a shock at the surge of lumpy investments. Then I decompose the total response into exogenous effect and endogenous effect. The endogenous effect accounts for up to 15% of aggregate investment response.

The acceleration cost model gives a theoretical framework that explains the endogenous business cycle when large firms' lumpy investments are perfectly inelastic to factor prices and idiosyncratic shock in the extensive margin. The resulting stationary cycle in the cyclical competitive equilibrium is a limit case of the baseline model's decaying echoes in the post-shock periods.

For the echo effects and the state-dependent responsiveness of the aggregate investments, the key state variable is the fraction of large firms that are ready to make lumpy investments. A rise in this key state variable makes an economy fragile to a negative TFP shock in the following period. Therefore, this paper's findings point out the necessity of a state-contingent stabilization policy based on micro-level observations.

Also, the acceleration cost model provides a meaningful monetary policy implication. According to the model, the fraction of large firms ready to make large-scale investment fluctuates. This implies the efficacy of monetary policy through the interest rate channel would also fluctuate. If there are a great number of large firms that are at the last stage for their large-scale investment, the economy will respond strongly to the monetary policy through the interest rate channel. On the other hand, if only a few firms are ready for large-scale investment, the monetary policy will not effectively work. I leave the optimal monetary policy design under the presence of interest-inelastic firms to future research.

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A Appendix: tables and figures

A.1 Conditional heteroskedasticity: Regression result

	Dependent variable:	
	$\hat{\sigma}_t$	
	Large	Small
	(1)	(2)
$\overline{spike}_{t-1}$ (%)	0.068***	0.032
	(0.025)	(0.020)
Constant	−0.454	−0.174
	(0.396)	(0.510)
Observations	35	35
R^2	0.185	0.068
Adjusted R^2	0.161	0.040
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table A.1: Increasing sensitivity of aggregate investments along with the large firms' investment spike

Specifically, $\overline{spike}_{t-1}$ is defined as follows:

$$\overline{spike}_{t-1} := \frac{1}{J}\sum_{j=0}^{J-1} SpikeRatio_{t-1-j}$$

$$SpikeRatio_t := \frac{\#\text{Extensive-margin adjustment}_t}{\#\text{Firms}_t}$$

where J is the number of past years to be includes. In the reported result, I use $J = 3$. The result is robust over $J = 1, 2, 4$.

A.2 Survey result: Inelasticity of investments to interest rate change

Q1. By how much would your borrowing costs have to decrease to cause you to initiate, accelerate, or increase investment projects next year?		
Q2. By how much would your borrowing costs have to increase to cause you to delay or stop investment projects next year?		
Change in interest rate	Plan changing firms (Q1)	Plan changing firms (Q2)
0.5%	3%	6%
1%	5%	10%
2%	8%	16%
3%	5%	11%
More than 3%	11%	20%
No change	68%	37%

Table A.2: CFO survey results (Sharpe and Suarez, 2014): inelasticity to interest rate changes

B Appendix: Monetary policy shock

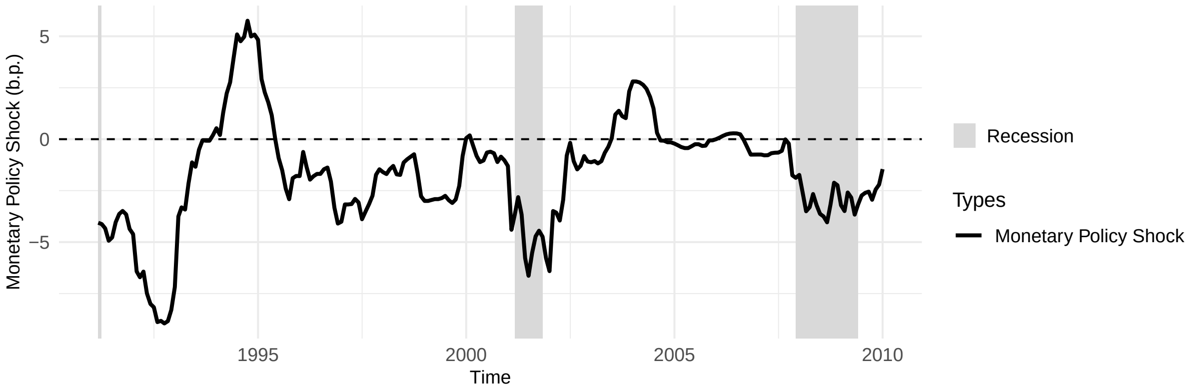


Figure B.1: One-year moving average monetary policy shock: March 1990 $\sim$ December 2009

C Appendix: Firm-level TFP estimation

I estimate firm-level TFP following Ackerberg et al. (2015). The estimation is based on the following model specification:

$$log(ValueAdd_{i,t}) = \overline{\alpha} + \alpha log(Capital_{i,t-1}) + \gamma log(Employment_{i,t}) + TFP_{i,t} + \epsilon_{i,t}$$

$$MaterialExpense_{i,t} = f(Capital_{i,t-1}, Employment_{i,t}, TFP_{i,t})$$

Then, I assume the following assumptions:

- Production, material expenditure and idiosyncratic TFP shocks are all realized simultaneously.

- Before the realization of the idiosyncratic TFP, a firm receives an idiosyncratic TFP signal ($sTFP_{i,t}$): a firm determines labor demand based on the signal. The idiosyncratic TFP follows a Markov process conditional on the signal of idiosyncratic TFP ($P(TFP_{i,t}|sTFP_{i,t})$).

- The idiosyncratic TFP signal follows a Markov process conditional on the past realization of the idiosyncratic TFP ($P(sTFP_{i,t}|TFP_{i,t-1})$).

- The function f is invertible with respect to $TFP_{i,t}$.

Then, the original model becomes

$$\begin{aligned} log(ValueAdd_{i,t}) &= \overline{\alpha} + \alpha log(Capital_{i,t-1}) + \gamma log(Employment_{i,t}) \\ &+ f^{-1}(Capital_{i,t-1}, Employment_{i,t}, MaterialExpense_{i,t}) + \epsilon_{i,t} \\ &= g(Capital_{i,t-1}, Employment_{i,t}, MaterialExpense_{i,t}) + \epsilon_{i,t} \end{aligned}$$

Then, I run a non-parametric regression of logged value-add on the capital, employment and material expenses to obtain $\widehat{g}(Capital_{i,t-1}, Employment_{i,t}, MaterialExpense_{i,t})$. Using the predicted value $\widehat{g}$, I estimate α and γ from the following conditional moment condition:

$$\mathbb{E}(\xi(\alpha,\gamma)|Capital_{i,t-1}, Employment_{i,t-1}) = 0$$

where $\xi(\alpha,\gamma) = TFP_{i,t} - \mathbb{E}(TFP_{i,t}|TFP_{i,t-1})$.

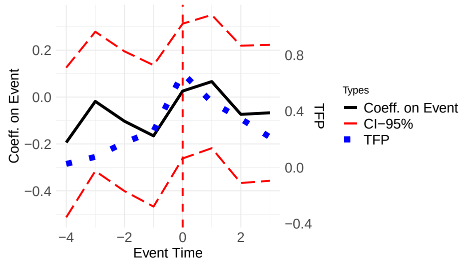
Specifically, $\widehat{TFP}_{i,t}(\widehat{\alpha},\widehat{\gamma}) = \widehat{g} - \widehat{\alpha} log(Capital_{i,t-1}) - \widehat{\gamma} log(Employment_{i,t})$. I obtain $\widehat{\xi}_{i,t}$ from the residuals of AR(1) regression of $\widehat{TFP}_{i,t}(\widehat{\alpha},\widehat{\gamma})$. The empiric analogue of the conditional moment is

$$\frac{1}{T}\frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N}\begin{pmatrix}\widehat{\xi}_{i,t} * Capital_{i,t-1} \\ \widehat{\xi}_{i,t} * Employment_{i,t-1}\end{pmatrix} = 0$$

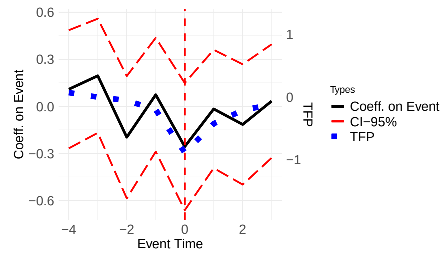
Each of the variables are obtained from firm-level balance sheet information in U.S. Compustat data combined with wage data by industry from the Current Employment Statistics (CES) survey. I join two datasets by matching the first two-digit NAICS codes. Specifically, each variable is defined as follows:

- $ValueAdd$ = Sale - Material Expense
- Material Expense = Total Expense - Wage × Firm-level Employment
- Total expense = Sale - Operating Income Profit Before Depreciation (OIBDP)
- *Capital* is obtained from applying perpetual inventory methods to the first available capital stock entry (PPEGT). Firm i's net real investment at period t is computed from $I_{i,t} - \delta k_{i,t-1} := (PPENT_{i,t} - PPENT_{i,t-1})/p_t$, where p_t is nonresidential fixed investment deflator available from National Income and Product Accounts data (NIPA Table 1.1.9, line 9). I assume $\delta = 0.1$ (annual) to get gross real investment at firm level. All the results stay robust over other reasonable choices of depreciation rates.

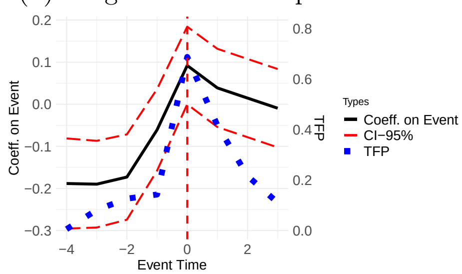
D Appendix: Event analysis using different idiosyncratic TFP measures



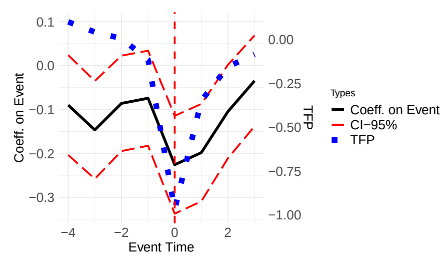
(a) Large firms around positive event



(b) Large firms around negative event

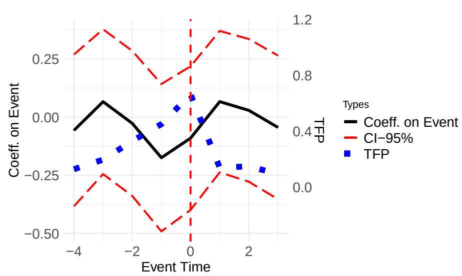


(c) Small firms around positive event

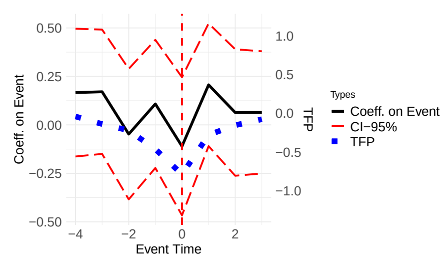


(d) Small firms around negative event

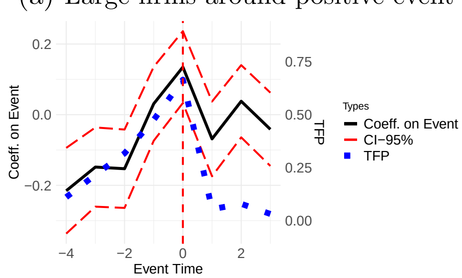
Figure D.2: Event study: sensitivity to idiosyncratic TFP innovation based on the Solow residuals



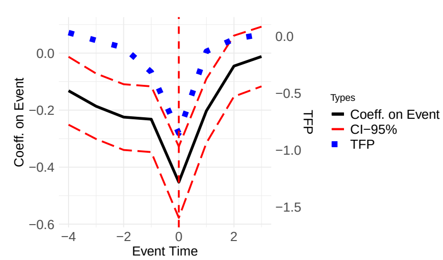
(a) Large firms around positive event



(b) Large firms around negative event



(c) Small firms around positive event



(d) Small firms around negative event

Figure D.3: Event study: sensitivity to idiosyncratic TFP innovation based on the method of Olley and Pakes (1996)

E Spike ratio in the model with fixed cost

In the model with fixed cost (Khan and Thomas, 2008), a firm's lumpy investment decision is characterized by a threshold rule $\xi^* = \xi^*(k, z)$:

$$I^*(k,z) = \begin{cases} I(k,z) & \text{if } \xi \leq \xi^*(k,z) \quad \text{(Unconstrained)} \\ I^c(k,z) & \text{if } \xi > \xi^*(k,z) \quad \text{(Constrained)} \end{cases}$$

where the fixed cost $\xi \sim_{i.i.d} G(\xi)$, and

$$\xi^* = \underbrace{\frac{1}{1+r(A,\Phi)}\mathbb{E}J(I+(1-\delta)k, z'; A', \Phi')}_{\text{Discounted Future value with lumpy investment}} - \underbrace{\frac{1}{1+r(A,\Phi)}\mathbb{E}J(I^c+(1-\delta)k, z'; A', \Phi')}_{\text{Discounted future value without lumpy investment}} - (I - I^c)$$

where I denotes the unconstrained investment, and I^c denotes constrained (small-scale) investment. I denote the value function in the first term as J and the second as J^c.

For firms greater than a size threshold $\overline{k}$, the spike ratio is

$$\begin{aligned} SpikeRatio(\overline{k}) &= \int\int \mathbb{I}\{\xi < \xi^*(k,z)\} \frac{\mathbb{I}\{k > \overline{k}\}}{\Phi(k > \overline{k})} \mathbb{I}\left\{\frac{I(k,z)}{k} > 0.2\right\} d\xi d\Phi \\ &= \int G\left(\frac{1}{1+r(A,\Phi)}(\mathbb{E}J - \mathbb{E}J^c)\right) \frac{\mathbb{I}\{k > \overline{k}\}}{\Phi(k > \overline{k})} \mathbb{I}\left\{\frac{I(k,z)}{k} > 0.2\right\} d\Phi \end{aligned}$$

Under the assumption that ξ follows a uniform distribution ($\xi \sim Unif[0, \overline{\xi}] = G$),

$$\begin{aligned} SpikeRatio(\overline{k}) &= \int \left(\frac{\xi^*(k,z)}{\overline{\xi}}\right) \frac{\mathbb{I}\{k > \overline{k}\}}{\Phi(k > \overline{k})} \mathbb{I}\left\{\frac{I(k,z)}{k} > 0.2\right\} d\Phi \\ &= \int \left(\frac{1}{1+r(A,\Phi)}\left(\frac{\mathbb{E}J - \mathbb{E}J^c}{\overline{\xi}}\right) - \frac{I - I^c}{\overline{\xi}}\right) \frac{\mathbb{I}\{k > \overline{k}\}}{\Phi(k > \overline{k})} \mathbb{I}\left\{\frac{I(k,z)}{k} > 0.2\right\} d\Phi \end{aligned}$$

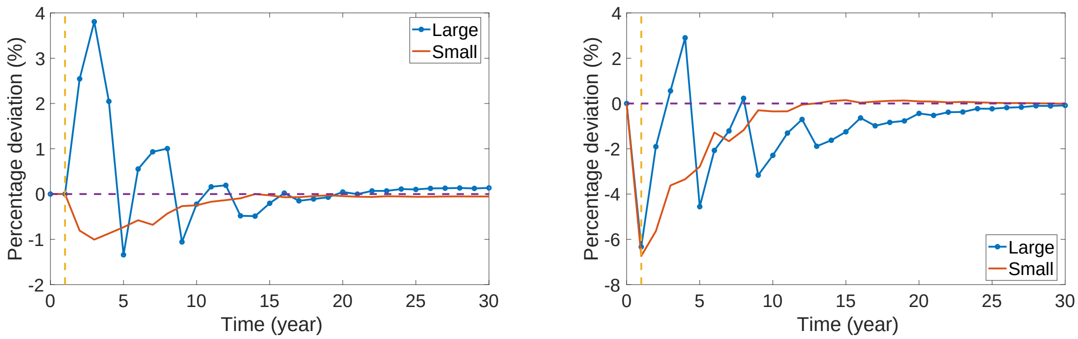
Thus, the spike ratio is strongly affected by the interest rate changes.

F Fixed parameters

Parameters	Description	Value
Firm Side Fundamentals		
α	Capital share	0.3
γ	Labor share	0.6
δ	Depreciation rate	0.09
Household Side		
β	Discount factor	0.96
η	Labor supply parameter	1.02

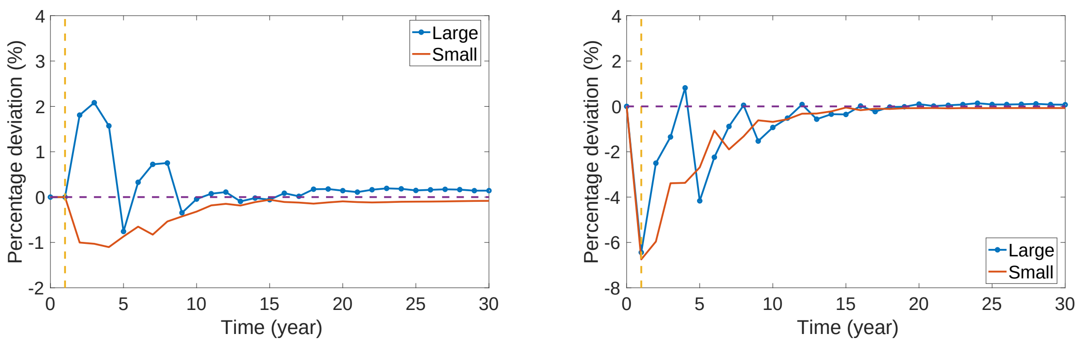
Table F.3: Fixed Parameters

G Appendix: heterogeneous echo effects with alternative definitions of large firms



(a) Average investment stages (b) Investment

Figure G.4: Heterogeneous echo effects: large firms are top 30%

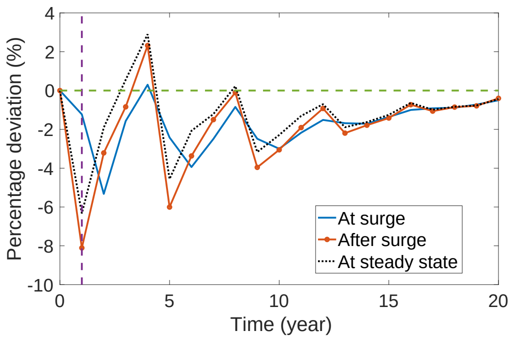


(a) Average investment stages (b) Investment

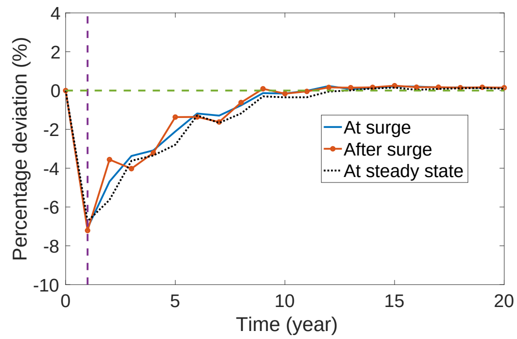
Figure G.5: Heterogeneous echo effects: large firms are top 40%

In the original definition where large firm are defined as top 20% largest firms, large firms take 26.7% of total capital. If large firm are defined as top 30% largest firms, then large firms take 38.5% of total capital. If large firm are defined as top 40% largest firms, then large firms take 49.6% of total capital.

H Appendix: heterogeneous nonlinear effects with alternative definitions of large firms

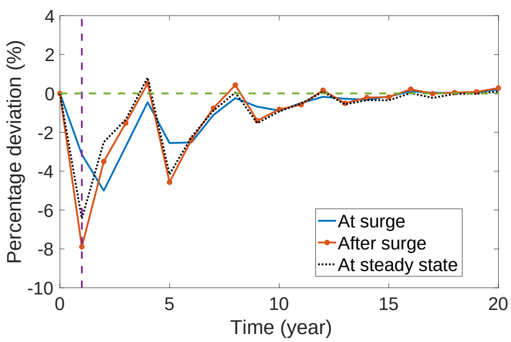


(a) Large

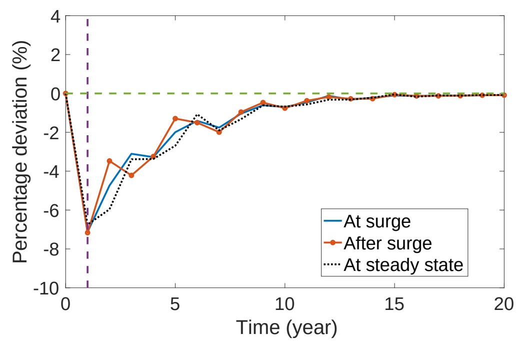


(b) Small

Figure H.6: Heterogeneous nonlinear effects: large firms are top 30%



(a) Large



(b) Small

Figure H.7: Heterogeneous nonlinear effects: large firms are top 40%

In the original definition where large firm are defined as top 20% largest firms, large firms take 26.7% of total capital. If large firm are defined as top 30% largest firms, then large firms take 38.5% of total capital. If large firm are defined as top 40% largest firms, then large firms take 49.6% of total capital.

I Appendix: Heterogeneous large and small firms' problem

A firm of type $j \in \{large, small\}$ solves the following problem:

$$
\begin{aligned}
&J(k, z, s, j; \Phi, A) = \pi(z, k; \Phi, A) + \max\{ \\
&\max_{s' > \bar{s}, I}\{-I - c(k, I) - acc_j(s', s)w(\Phi, A) + \frac{1-h}{1+r(\Phi, A)}\mathbb{E}J(k', z', s' \pmod{\bar{s}}, j'; \Phi', A')\}, \\
&\max_{s \leq \tilde{s}' \leq \bar{s}, I^c \in \Omega(k)}\{-I^c - c(k, I^c) - acc_j(\tilde{s}', s)w(\Phi, A) + \frac{1-h}{1+r(\Phi, A)}\mathbb{E}J(k'^c, z', \tilde{s}', j'; \Phi', A')\}\}
\end{aligned}
$$

$$
\begin{aligned}
&\text{(Operating Profit)} && \pi(z, k; \Phi, A) := \max_{l_d} zAk^\alpha l_d^\gamma - w(\Phi, A)l_d \ (l_d\text{: labor demand}) \\
&\text{(Convex Adj. Cost)} && c(k, I) := \frac{\mu_j^I}{2}\left(\frac{I}{k}\right)^2 k \\
&\text{(Acceleration Cost)} && acc_j(s', s) := \left[\mathbb{I}\{s' \geq s+1\}\left(\frac{\mu_j^a}{2}(s'-s-1)^2\right)\right] \\
& && (\mu^a_{large} > \mu^a_{small},\ \mu^I_{large} < \mu^I_{small}) \\
&\text{(Constrained Investment)} && I^c \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta) \\
&\text{(Agg. Law of Motion)} && \Phi' := H(\Phi, A),\ G_A(A) = A' \text{ (AR(1) process)}
\end{aligned}
$$

I.1 Parameters for heterogeneous large and small firms' problem

Parameters	Description	Value
Cost Parameters: hetergenous firms		
μ^{acc}_{large}	Large firms' acceleration cost	0.45
μ^{I}_{large}	Large firms' adjustment cost	2.50
μ^{acc}_{small}	Small firms' acceleration cost	0.18
μ^{I}_{small}	Small firms' adjustment cost	3.50
ν^{I}_{large}	Small investment range	0.01
ν^{I}_{small}	Small investment range	0.01

Table I.4: Parameters in heterogeneous large and small firms' problem

I.2 Testing serial correlation of residuals from fitting harmonic functions into the data

	Dependent variable:			
	ϵ_t			
	Echo (Manuf.)	Non-echo (Manuf.)	Echo (Oil)	Non-echo (Oil)
	(1)	(2)	(3)	(4)
ϵ_{t-1}	0.205	0.136	0.063	0.601***
	(0.152)	(0.161)	(0.181)	(0.173)
Constant	−2.433	−0.312	−1.385	0.055
	(7.186)	(2.342)	(2.365)	(1.273)
Observations	39	39	24	24
R^2	0.047	0.019	0.006	0.355
Adjusted R^2	0.021	−0.008	−0.040	0.326
Note:				*p<0.1; **p<0.05; ***p<0.01

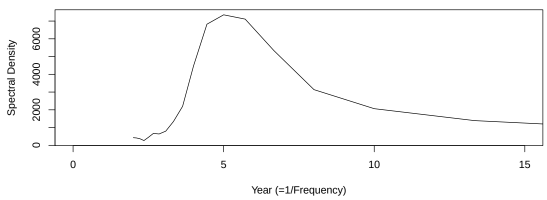
Table I.5: Serial correlations in residuals

Table I.5 reports the autoregression results of residuals from fitting a harmonic function into the investment growth data used in Table 10 and 11. The harmonic function is specified as follows:

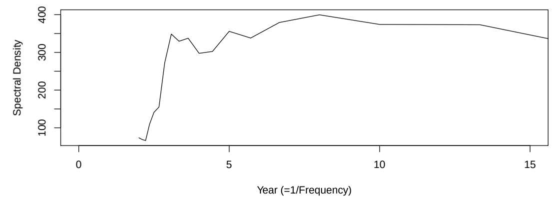
$$X_t = \beta cos(\omega t + \phi) + \epsilon_t$$

where $\beta > 0$, $\omega \in (0, \pi)$, $\phi \sim U(-\pi, \pi]$, and ϵ_t is a serially uncorrelated noise which is assumed to be independent from ϕ. The function is nonlienarly fitted into the data following Li (2010). For this estimation, I used *R* package "ptest"(Lai and McLeod, 2016).

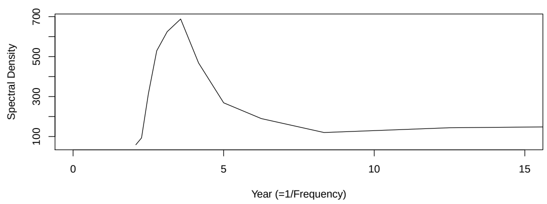
I.3 Spectral densities of growth rates of non-residential fixed investment



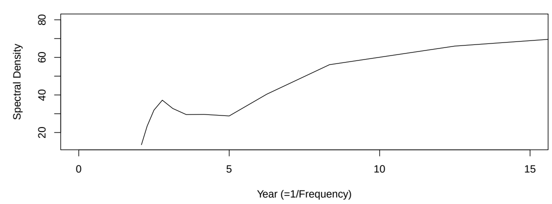
(a) Echo period for manufacturing



(b) Non-echo period for manufacturing



(c) Echo period for oil industry



(d) Non-echo period for oil industry

Figure I.8: Spectral densities of growth rates of non-residential fixed investment for manufacturing and oil industries

Spectral densities are estimated with modified Daniell smoothing parameter of 5.

J Appendix: Computation Methodology

J.1 Computation for cyclical competitive equilibrium I

To compute the cyclical competitive equilibrium, I take the following steps:

1. Set a capital grid $\mathbb{K}_0$, stage grid $\mathbb{S}_0 = \{1, 2, 3, ..., \bar{s}\}$, and a macro stage grid $\mathbb{T} = \{1, 2, 3, ..., n^*\}$. For the capital grid $\mathbb{K}_0$, note that the maximum and minimum values need to be distant enough to cover the entire closed capital domain $\mathbb{K}$ which will be obtained endogenously. Discretize the autoregressive productivity process z, to get the unconditional productivity support $\mathbb{Z}$.

2. Guess the number of periods within a cycle n^* and the corresponding number of price bundles $\{r_\tau, w_\tau\}_{\tau=1}^{n^*}$.

3. Solve a firm's problem using a value function iteration.

4. Come up with an initial distribution Φ_0 that has $\mathbb{K}_0 \times \mathbb{Z} \times \mathbb{S}_0$ as a support of the distribution.

5. Make Φ_0 evolve based on the policy functions from step 3 and transition rule of the autoregressive productivity process z to get $\{\Phi_t\}_{t=0}^{M}$ until there exists $M > N \geq 0$ such that $||\Phi_M - \Phi_N|| < tol$. $(M - N)$ is the implied length of the cycle in the solution given the initial guess.

6. Calculate $error1$ such that $error1 = |(M - N) - n^*|$. If $error1 > 0$, then go back to step 2 to start over with another initial guess for n^*. Otherwise, go the next step.

7. Compute the implied price bundles $(r_{implied,t}, w_{implied,t})_N^{M-1}$ from the inter-temporal and intra-temporal optimality conditions of the household using the endogenous aggregate allocation $\{c_t\}_N^{M-1}$ and optimal labor supply policy $\{l_t\}_N^{M-1}$:

$$\text{(Inter-temporal)} \quad \mathbb{E}\left(\beta \frac{u_c(c_t, l_t)}{u_c(c_{t+1}, l_{t+1})}\right) = 1 + r_{implied,t}$$
$$\text{(Intra-temporal)} \quad -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = w_{implied,t}$$

8. Calculate $error2 = max\{||r_{implied,t} - r||_{sup}, ||w_{implied,t} - w||_{sup}\}$, where $r_{implied,t}$ and r are vectors of the implied real interest rates and guessed real interest rates, respectively, and $w_{implied,t}$ and w are vectors of the implied real wage and guessed real wage, respectively. If $error2 > tol$ then go back to step 2 to start over with another initial guess for

$\{r_\tau, w_\tau\}_{\tau=1}^{n^*}$. Otherwise, the solutions obtained from step 3, $n^* = M - N$, and prices $\{r_\tau, w_\tau\}_{\tau=1}^{n^*}$ are the cyclical competitive equilibrium.

Note that in the baseline computation where firms are not heterogeneous in terms of the length of the firm-level (S, s) cycle, the equilibrium length n^* of the aggregate level cycle and the firm-level cycle $\overline{s}$ are identical. Therefore, with an initial guess $n^* = \overline{s}$, step 6 becomes unnecessary under the calibrated parameters because $error1 = 0$ always holds.

However, in the potential application of the model for firms with the heterogeneous cycle lengths, n^* might become different from $\overline{s}$. In this case, if the heterogeneity persists without shuffling across the firms, e.g. permanently different groups of firms with heterogeneous cycle lengths, $n^* = l.c.m.(\overline{s}_1, \overline{s}_2, ..., \overline{s}_G)$ is the correct guess for the aggregate cycle length, where G indicates the number of different groups.

J.2 Computation for cyclical competitive equilibrium II: a practical approach

The computation algorithm explained in Appendix 8 is implementable, but mathematical packages often fail to obtain a convergent solution to the fixed point due to the high sensitivity of the solution to the relative price levels across the periods. This issue becomes easier to understand, when compared to the stationary equilibrium case.

For example, suppose there is a stationary equilibrium, and the equilibrium real interest rate is $r^* = 0.04$. Let the initial guess for the real interest rate be $r_{guess} = 0.05$, which leads to an implied level of real interest rate $r_{implied} = 0.03$. During the approximation, it always holds that if $r_{implied} > r_{guess}$, then $r^* > r_{guess}$. This let the solver pick the next guess $r'_{guess} < r_{guess}$, i.e. $r'_{guess} = 0.038$, and by iterating these steps, the fixed point solution is obtained by the convergence.

However, consider a cyclical competitive equilibrium with a cycle length $n^* = 2$. In this case, the initial guess needs to be the real interest rates for two periods in a cycle. Suppose the equilibrium real interest rates are $(r_1^*, r_2^*) = (0.04, 0.045)$, and initial guess is $(r_{guess,1}, r_{guess,2}) = (0.045, 0.047)$ which leads to implied real interest levels of $(r_{implied,1}, r_{implied,2}) = (0.0361, 0.0362)$. Then, the prediction error is greater for the second period, even if the ranking of the guessed prices are correct across the periods. Then, a solver might recognize the current guess for the second period price is too high compared to the guess for the first period price. If it happens, for the next guess for the prices, a solver may choose to use a price bundle that has a flipped ranking of prices across the periods such as $(r'_{guess,1}, r'_{guess,2}) = (0.0442, 0.0432)$. In this case, the implied price jumps dramatically due to the flipped ranking because many firms change their investment decision in the extensive margin to utilize the price gain in the second period. These occasional jumps in the prediction errors make the solver fail to achieve a converged solution.

In the stationary equilibrium case, flipped price ranking across the periods are not an issue because there is only one price, which always preserves the monotone relationship among a guessed price, an implied price, and the fixed point price. However, in the cyclical competitive equilibrium, the flipped ranking is a challenging issue for the computation as it leads to a completely different implied equilibrium cycle due to the investment changes in the extensive margin.

To overcome this problem, I introduce another simple method for the computation which makes the guessed prices slowly and steadily converge to the equilibrium prices without flipping the ranking. The new method is implemented simply by changing step 2 and 8 in the previous method. I elaborate the new steps for the method as follows:

- Step 2*: Guess the number of periods within a cycle n^*. As an initial guess for the price bundles, consider a constant sequence of prices, that is $\{r_\tau, w_\tau\}_{\tau=1}^{n^*}$, s.t. $r_\tau = \overline{r}$, and $w_\tau = \overline{w}$, where $\overline{r}$ and $\overline{w}$ are taken to be large enough to be greater than any of possible equilibrium price levels.

- Step 8*: Calculate $error2 = max\{||r_{implied} - r||_{sup}, ||w_{implied} - w||_{sup}\}$, where $r_{implied}$ and r are vectors of the implied real interest rates and guessed real interest rates, respectively, and $w_{implied}$ and w are vectors of the implied real wage and guessed real wage, respectively. If $error2 > tol$ then go back to step 2 to start over with the specific initial guess $\{r'_\tau, w'_\tau\}_{\tau=1}^{n^*}$ such that $r'_\tau = \omega r_\tau + (1-\omega) r_{implied,\tau}$ and $w'_\tau = \omega w_\tau + (1-\omega) w_{implied,\tau}$, where ω is a price convergence parameter. If the price convergence prameter is close to 1, the prices converge slower while the convergence of the solution is more certainly guarenteed. I use the $\omega = 0.95$ for the assured convergence. If $error2 \leq tol$, the solutions obtained from step 3, $n^* = M - N$, and prices $\{r_\tau, w_\tau\}_{\tau=1}^{n^*}$ are the cyclical competitive equilibrium.

For example, suppose the initial guess for the real interest rate is $(\overline{r}, \overline{r}) = (0.06, 0.06)$ for a cyclical competitive equilibrium with a cycle length $n^* = 2$. Suppose it leads to an implied real interest rate level $(r_{implied,1}, r_{implied,2}) = (0.03, 0.032)$. Then, if the price convergence parameter $\omega = 0.95$, the next guess for the prices is $(r'_1, r'_2) = 0.95 * (\overline{r}, \overline{r}) + 0.05 * (0.03, 0.032) = (0.0585, 0.0586)$. Here the ranking of the prices is determined by the first iteration of the algorithm, and the ranking is likely to persist through the convergence. The ranking persistence is stronger for a higher price convergence parameter ω which gives slower but more certain convergence, and vice versa.

K Appendix: Proofs for the theoretical results

K.1 Proof for Proposition 1

Proposition 1 (Isolated stage policy)**.**
Given an idiosyncratic productivity process $G_z(z)$ with a bounded support $\mathbb{Z}$, there exists $\overline{\mu}_{G_z} > 0$ such that

$$\mu^a_{large} \geq \overline{\mu}_{G_z} \implies \hat{s}'(k, z, s, large; \Phi, A) = \hat{s}'(s, large) \text{ for } \forall (k, z, s) \in (\mathbb{K}, \mathbb{Z}, \mathbb{S})$$

where $(\mathbb{K}, \mathbb{S})$ denotes the domains of capital and investment stages, respectively.

Proof.
Define $\zeta(\mu^a_{large}) := \sup_{s' \in \{1,2,\ldots,\overline{s}\}, k' \in \mathbb{K}} \{\frac{1-h}{1+r(\phi,A)} \mathbb{E} J(k', z', s', large; \Phi', A'; \mu^a_{large})\}$.
Note that the equilibrium value function J is a weakly decreasing function of cost parameter μ^a_{large}. Thus, $\zeta(\mu^a_{large})$ is also weakly decreasing in μ^a_{large}.

$$Acc(s', s; \mu^a_{large}) = \left[\mathbb{I}\{s' > s+1\} \left(\frac{\mu^a_{large}}{2} (s' - s - 1)^2 \right) \right]$$

If $s' > s+1$, $Acc(s', s; \mu^a_{large}) \geq \dfrac{\mu^a_{large}}{2}$. Therefore, if $\exists \overline{\mu}_{G_z} > 0$ such that $\frac{\overline{\mu}_{G_z}}{2} > \zeta(\overline{\mu}_{G_z})$, optimal stage policy is always one-stage-per-period rule only if $\mu > \overline{\mu}_{G_z}$. This is because $\hat{s}'(k, z, s, large; \Phi, A) = \hat{s}'(k, s, large) = s + 1 \pmod{\overline{s}}$.
So, it is sufficient to show $\exists \overline{\mu}_{G_z} > 0$ such that $\frac{\overline{\mu}_{G_z}}{2} > \zeta(\overline{\mu}_{G_z})$.

Suppose $\nexists \overline{\mu}_{G_z} > 0$ such that $\frac{\overline{\mu}_{G_z}}{2} > \zeta(\overline{\mu}_{G_z})$. For $\forall \overline{\mu}_{G_z} > 0$, $\frac{\overline{\mu}_{G_z}}{2} \leq \zeta(\overline{\mu}_{G_z})$.
As $\zeta(\overline{\mu}_{G_z}) < \infty$, $\exists N < \infty$ such that $N > \zeta(\overline{\mu}_{G_z})$. Then, define $M := max\{\frac{\overline{\mu}_{G_z}}{2}, N\} + \epsilon$.
Hence,

$$\zeta(\overline{\mu}_{G_z}) < M \leq \zeta(2M)$$

This implies

$$\zeta(\overline{\mu}_{G_z}) < \zeta(2M) \text{ and } \overline{\mu}_{G_z} < 2M$$

This contradicts $\zeta(x)$ is weakly decreasing in x.
Therefore, $\exists \overline{\mu}_{G_z} > 0$ such that $\frac{\overline{\mu}_{G_z}}{2} > \zeta(\overline{\mu}_{G_z})$. ■

K.2 Proof for Proposition 2

Proposition 2 (Breaking the law of large numbers)**.**
If $\mu^a_{large} \geq \overline{\mu}_{G_z}$*, and* $\Phi_0 \in \mathbb{D}_0$*, the implied sequence of distributions* $\{\Phi_\tau\}_0^\infty$ *does not have a limit point.*

Proof.
Suppose there exists a limit point Φ^*, such that

$$(\Phi^*)(K, Z, S, j; \Phi^*, A) := \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'} dz'\right) \mathbb{I}\{\hat{k}'(k, z, s, j; \Phi^*, A) \in K\} \\ \mathbb{I}\{\hat{s}'(k, z, s, j; \Phi^*, A) \in S\} d\Phi^*(k, z, s, j; \Phi^*, A)$$

for any set (K, Z, S) in the σ-algebra $(\mathcal{K}, \mathcal{Z}, \mathcal{S})$ generated from the domains $(\mathbb{K}, \mathbb{Z}, \mathbb{S})$.
By the isolation proposition, for $\forall \tilde{s} \in S$

$$\begin{aligned}(\Phi^*)(K, Z, \tilde{s}, large; \Phi^*, A) &= \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'} dz'\right) \mathbb{I}\{\hat{k}'(k, z, s, large; \Phi^*, A) \in K\} \\ &\qquad \mathbb{I}\{\hat{s}'(s) = \tilde{s}\} d\Phi^*(k, z, s, large; \Phi^*, A) \\ &= \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'} dz'\right) \mathbb{I}\{\hat{k}'(k, z, s, large; \Phi^*, A) \in K\} \\ &\qquad \mathbb{I}\{s = \tilde{s} - 1 \pmod{\overline{s}}\} d\Phi^*(k, z, s, large; \Phi^*, A) \\ &= (\Phi^*)(K, Z, \tilde{s} - 1 \pmod{\overline{s}}, large; \Phi^*, A) \\ &= (\Phi^*)(K, Z, \tilde{s} - 2 \pmod{\overline{s}}, large; \Phi^*, A) \\ &= (\Phi^*)(K, Z, \tilde{s} - 3 \pmod{\overline{s}}, large; \Phi^*, A) \\ &= \dots \end{aligned} \tag{1}$$

Thus, $\Phi^* \in \mathbb{D}_1$. Therefore, it is sufficient to show that for $\forall \Phi_t \in \{\Phi_\tau\}_{\tau=1}^\infty$,

$$\Phi_t \in \mathbb{D}_0 \implies \Phi_{t+1} \in \mathbb{D}_0$$

From the same step as (1), we get

$$(\Phi_{t+1})(K, Z, \tilde{s}, large; \Phi_{t+1}, A) = (\Phi_t)(K, Z, \tilde{s} - 1 \pmod{\overline{s}}), large; \Phi_t, A), \text{ for } \forall \tilde{s} \in \mathbb{S}$$

$\Phi_t \in \mathbb{D}_0$ implies $\exists s^* \in \mathbb{S}$ such that,

$$\int_{\mathbb{K}\times\mathbb{Z}\times\{s^*\}\times\{y,o\}} d\Phi_t\left(k, z, s^*, j; \Phi_t, A\right) \neq \frac{1}{\overline{s}}$$

So,

$$\int_{\mathbb{K}\times\mathbb{Z}\times\{s^*+1\}\times\{y,o\}} d\Phi_{t+1}\left(k,z,s^*+1,j;\Phi_{t+1},A\right) \neq \frac{1}{\overline{s}}$$

Therefore, $\Phi_t \in \mathbb{D}_0 \implies \Phi_{t+1} \in \mathbb{D}_0$. ■

K.3 Proof for Corollary 1

Corollary 1 (Aggregate endogenous cycle under the persistent shock)**.**
Given $\mu^a_{large} \geq \overline{\mu}_{G_z}$ and the initial distribution $\Phi_0 \in \mathbb{D}_0$, for $\forall\ \epsilon > 0$, there is a sufficiently large $\overline{\tau} \in \{1,2,3,...\}$ such that the implied sequence of distributions $\{\Phi_\tau\}_{\tau=0}^{\infty}$ satisfies following property:

$$||\Phi_{\tau+\overline{s}} - \Phi_\tau||_{\sup} < \epsilon,\ \textit{for}\ \forall\tau > \overline{\tau}$$

Proof.

The strategy of proof is to utilize the law of large numbers that gives convergence of conditional joint distribution of (k,z) given s. For the notational brevity, the aggregate state variables are now omitted. By the isolation propositions,

$$(\Phi_{\tau+1})(k,z,s) = \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'}dz'\right) \mathbb{I}\{\hat{k}'(k,z,s)\in K\}\mathbb{I}\{\hat{s}'(s)\in S\}d\Phi_\tau(k,z,s)$$

Let $\phi_{s,\tau}$ denote the marginal density of s for the distribution Φ_τ.

$$\phi_{s,\tau}(\tilde{s}) := \int_{\mathbb{K}\times\mathbb{Z}\times\{\tilde{s}\}\times\{y,o\}} d\Phi_\tau\left(k,z,\tilde{s},j;\Phi_\tau,A\right)$$

From the exactly same derivation as the equations (1), the marginal density of s satisfies the following property:

$$\phi_{s,\tau}(s) = \phi_{s,\tau+\overline{s}}(s) \tag{2}$$

i) for $S = \tilde{s} \in 1 = 2,3,...,\overline{s}$,

$$\begin{aligned}(\Phi_{\tau+1})(K,Z,\tilde{s}) &= \int_{\mathbb{K}\times\mathbb{Z}\times\mathbb{S}} \left(\int_Z \Gamma_{z,z'}dz'\right) \mathbb{I}\{\hat{k}'(k,z,s)\in K\}\mathbb{I}\{\tilde{s} = s+1 \ (\mathrm{mod}\ \overline{s})\}d\Phi_\tau(k,z,s)\\ &= \int_{\mathbb{K}\times\mathbb{Z}} \left(\int_Z \Gamma_{z,z'}dz'\right) \mathbb{I}\{\hat{k}'(k,z,\tilde{s}-1\ (\mathrm{mod}\ \overline{s}))\in K\}d\Phi_\tau(k,z,\tilde{s}-1\ (\mathrm{mod}\ \overline{s}))\end{aligned} \tag{3}$$

and it is known that

$$(\Phi_{\tau+1})(K, Z|\tilde{s}) * \phi_{s,\tau+1}(\tilde{s}) = (\Phi_{\tau+1})(K, Z, \tilde{s})$$
$$\phi_{s,\tau+1}(\tilde{s}) = \phi_{s,\tau}(\tilde{s} - 1 \pmod{\overline{s}})$$

Dividing both sides of the equation (3) by $\phi_{s,\tau+1}(\tilde{s})$,

$$(\Phi_{\tau+1})(K, Z|\tilde{s}) = \int_{\mathbb{K}\times\mathbb{Z}} \left(\int_Z \Gamma_{z,z'} dz' \right) \mathbb{I}\{\hat{k}'(k, z, \tilde{s} - 1 \pmod{\overline{s}}) \in K\} d\Phi_\tau(k, z|\tilde{s} - 1 \pmod{\overline{s}})$$

This is equivalent to

$$(\Phi_{\tau+1})(K, Z|s + 1 \pmod{\overline{s}}) = \int_{\mathbb{K}\times\mathbb{Z}} \left(\int_Z \Gamma_{z,z'} dz' \right) \mathbb{I}\{\hat{k}'(k, z, s) \in K\} d\Phi_\tau(k, z|s)$$

Let $\Phi_\tau|_s$ denote the joint distribution of (k, z) conditional on s.
Define a transition operator Λ_s such that

$$\Lambda_s(\Psi)(K, Z) := \int_{\mathbb{K}\times\mathbb{Z}} \left(\int_Z \Gamma_{z,z'} dz' \right) \mathbb{I}\{\hat{k}'(k, z, s) \in K\} d\Psi(k, z), \text{ for } \forall\Psi \text{ measure on } \mathbb{K} \times \mathbb{Z}$$

Then,

$$(\Phi_{\tau+1}|_{s+1 \pmod{\overline{s}}})(K, Z) = \Lambda_s(\Phi_\tau|_s)(K, Z)$$

By applying the transition $\overline{s} - 1$ times additionally,

$$(\Phi_{\tau+\overline{s}}|_s)(K, Z) = \Lambda_s^{(\overline{s})}(\Phi_\tau|_s)(K, Z) \tag{4}$$

The equation above holds for $\forall s \in \mathbb{S}$.
Define a transition operator T_s as

$$T_s(\Psi)(K, Z) := \Lambda_s^{(\overline{s})}(\Psi)(k, z, s), \text{ for } \forall\Psi \text{ measure on } \mathbb{K} \times \mathbb{Z}$$

Hence,

$$(\Phi_{\tau+T}|_s)(K, Z) = T_s(\Phi_\tau|_s)(K, Z) \tag{5}$$

Note that this transition preserves the conditioning state variable s. By infinitely applying the transition T_s to $\Phi_\tau|_s$, under the mild regularity conditions, the law of large numbers gives the following result:[51]

$$\exists \Phi_s^* \text{ such that } (\Phi_s^*)(K,Z) = \lim_{n\to\infty} (T_s)^n(\Phi_\tau|_s(K,Z)), \text{ for } \forall (K,Z) \in (\mathcal{K}\times\mathcal{Z})$$

Then, Φ_s^* is a fixed point of the transition T_s such that

$$(\Phi_s^*)(K,Z) = T_s(\Phi_s^*)(K,Z)$$

All the convergent sequence is Cauchy sequence in metric space. Thus, we can find τ_s^* such that for $\forall \tau > \tau_s^*$

$$||\Phi_{\tau+T}|_s(K,Z) - \Phi_\tau|_s(K,Z)||_{\text{sup}} < \epsilon, \text{ for } \forall (K,Z) \in (\mathbb{K}\times\mathbb{Z}) \tag{6}$$

Then, define

$$\tau^* = \sup_{s\in\mathbb{S}} \tau_s^*$$

For $\forall (k,z,s) \in (\mathbb{K}\times\mathbb{Z}\times\mathbb{S})$ and $\forall \tau > \tau^*$,

$$\begin{aligned}
&||\Phi_{\tau+T}(k,z,s) - \Phi_\tau(k,z,s)||_{\text{sup}} \\
&= ||\int_s \Phi_{\tau+T}|_s(K,Z)\phi_{s,\tau+T}(s)ds - \int_s \Phi_\tau|_s(K,Z)\phi_{s,\tau}(s)ds||_{\text{sup}} \\
&= ||\int_s \Phi_{\tau+T}|_s(K,Z)\phi_{s,\tau}(s)ds - \int_s \Phi_\tau|_s(K,Z)\phi_{s,\tau}(s)ds||_{\text{sup}} \quad \text{, from (2)} \\
&\leq \int_s ||\Phi_{\tau+T}|_s(K,Z) - \Phi_\tau|_s(K,Z)||_{\text{sup}}\phi_{s,\tau}(s)ds \\
&< \int_s \epsilon\phi_{s,\tau}(s)ds \\
&\leq \epsilon
\end{aligned}$$

Therefore, the proof is completed. ■

K.4 Proof for Corollary 2

Corollary 2 (Commonness of aggregate cycles)**.**
Consider a non-degenerate atomless distribution Ψ defined on σ-algebra $\mathcal{D}$ generated from $\mathbb{D}$, where $\mathbb{D}$ is the support of Ψ. Then, $\Psi(\mathbb{D}_1) = 0$, and $\Psi(\mathbb{D}_0) = \Psi(\mathbb{D}) = 1$.

[51] As the transition relies on capital policy that only depends on the stochastic process z, and k, convergence of distribution of z to the ergodic distribution makes the whole joint distribution of (k,z) converges as well. This is the result coming from the law of large numbers, but I do not directly prove the convergence in this paper.

Proof.

$\mathbb{D}_1$ is a set of all distributions of which marginal distribution of x is a uniform distribution. Out of all possible marginal distribution of x, $\mathbb{D}_1$ represents a singleton. Therefore, $\Psi(\mathbb{D}_1) = 0$. Because $\mathbb{D}_0 = \mathbb{D} \setminus \mathbb{D}_0$, $\Psi(\mathbb{D}_0) = 1$ ■