Computational Methods for Macroeconomics (Part II)
Lecture 2: A heterogeneous household model with aggregate uncertainty

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Background: Heterogeneous agent model with aggregate uncertainty

- Prior to the paper, heterogeneous agents had not been often considered in the business cycle literature.
  - Theoretical reason: the irrelevance result in Caplin and Spulber (1987)
  - Computation reason: there was no good method to track the dynamics of the infinite-dimensional object.
- Heterogeneous agent models were being actively developed in the stationary environment.
  - Bewley-Huggett-Aiyagari models: Heterogeneous household models under the incomplete market
  - Hopenhayn (1992): A heterogeneous firm model with endogenous exit/entry/size

Background: Krusell and Smith (1998)

- (Bewley-Huggett-)Aiyagari model + aggregate uncertainty
- Borrowing constraint: two implications
- Incomplete market: two types of incompleteness
  - Incomplete insurance for idiosyncratic shocks
  - Incomplete insurance for aggregate shocks (by definition)
- Aggregate uncertainty:
  - Stochastic fluctuations in the productivity (c.f., Aiyagari (1994))
  - Stochastic fluctuations in the price
  - Stochastic fluctuations in the distribution
- If the distribution fluctuates (fluctuations in the infinite-dimensional object), how do we solve the problem? What do we mean by rational expectation?

Model

Environment

- Preference:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

- Budget constraint:

$$c_{it} + a_{it+1} = w_t z_{it} + (1 + r_t) a_{it}$$

- Initial condition:

$$z_{i0}, a_{i0} \geq 0.$$

- Borrowing constraint (precautionary motivation ↑):

$$a_{t+1} \geq \underline{a}$$

Income process

- Stochastic income process for each household i: $\{z_{it}\}_{t=0}^{\infty}$

$$z_{it} \in \mathcal{L} = \{z_1, z_2, \ldots, z_N\}$$

- A Markov process with transitions: $\Gamma_{z,z'} \geq 0$
- Krusell and Smith (1998) divides the type of states into aggregate and idiosyncratic states: S and ϵ
  - $S \in \{G, B\}$ and $\epsilon \in \{U, E\}$ (employed/unemployed): $2 \times 2 = 4$ idiosyncratic states
- These two types are correlated: (un-)employment in Bad and Good are different!
  - Let's denote the four possible idiosyncratic states as $\{BU, BE, GU, GE\}$
  - GU and BU are different in terms of transition to the other state. (next slide)
  - GU and BU are not different in terms of productivity: $z_{GU} = z_{BU} = 0.25$ and $z_{GE} = z_{BE} = 1$ (Unemployment benefit that is 1/4 of employed income).
  - $a' \geq 0$ is a meaningful constraint: the natural borrowing limit¡0 as $z_min < 0$.
- c.f.) Independent case: idiosyncratic shock process stays the same regardless of the aggregate state of the economy (many models are in this setup)

Income process (cont'd)

$$\Gamma = \begin{bmatrix} \gamma_{BU|BU} & \gamma_{BE|BU} & \gamma_{GU|BU} & \gamma_{GE|BU} \\ \gamma_{BU|BE} & \gamma_{BE|BE} & \gamma_{GU|BE} & \gamma_{GE|BE} \\ \gamma_{BU|GU} & \gamma_{BE|GU} & \gamma_{GU|GU} & \gamma_{GE|GU} \\ \gamma_{BU|GE} & \gamma_{BE|GE} & \gamma_{GU|GE} & \gamma_{GE|GE} \end{bmatrix}$$

- $G0$ and $B0$ are different in terms of transition to the other state.
- The state grid points: $\mathbb{Z} = [z_{BU}, z_{BE}, z_{GU}, z_{GE}]' = [0.25, 1; 0.25, 1]'$
- Can we get an idiosyncratic state transition matrix (without aggregate states)?
- This transition matrix will be calibrated (in two slides).

Production sector

- A representative firm (perfect competition + frictionless) with Cobb-Douglas production function:

$$\max_{K_t, L_t} \quad F(K_t, L_t; A_t) - w_t L_t - (r_t + \delta) K_t$$

$$\text{where } F(K_t, L_t; A_t) = A_t {K_t}^{\alpha} {L_t}^{1-\alpha}$$

  - A_t follows a Markov chain.
  - In Aiyagari (1994), $A_t = 1$.
  - Zero profit (dividend) to be distributed.
- The first-order conditions lead to the following characterization:

$$r_t = MPK_t - \delta = \alpha A_t = r(K_t, A_t) \left(\frac{K_t}{L_t} \right)^{\alpha - 1} - \delta = r(K_t, A_t)$$

$$w_t = MLK_t = (1-\alpha) A_t \left(\frac{K_t}{L_t} \right)^{\alpha} = A_t^{\frac{1}{1-\alpha}} (r_t + \delta)^{\frac{\alpha}{\alpha - 1}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) = w(K_t, A_t)$$

Calibration (external)

- Unit period = a quarter ($\beta = 0.99$).
- CRRA utility with $\sigma = 1$.
- Capital share $\alpha = 0.36$.
- Quarterly depreciation rate = 0.025.
- Productivity process: $A_t \in \{0.99, 1.01\}$
- The income process is also externally calibrated:
  - The target moments are separate from endogenous equilibrium outcomes.
  - First, assume there exists an aggregate transition matrix Γ_S, and then, we will split this matrix into a finer idiosyncratic transition matrix.

$$\Gamma_S = \begin{bmatrix} \gamma_{BB} & \gamma_{BG} \\ \gamma_{GB} & \gamma_{GG} \end{bmatrix}$$

Calibration (external)

- Expected time from $S = G$ to $S = B$: 8 quarters:

$$8 = \sum 1 * (1 - \gamma_{GG}) + 2 * \gamma_{GG}(1 - \gamma_{GG}) + 3 * \gamma_{GG}^2(1 - \gamma_{GG}) + \dots$$
$$= \frac{1}{1 - \gamma_{GG}} \implies \gamma_{GG} = \gamma_{BB} = \frac{7}{8} \implies \Gamma_S = \begin{bmatrix} 7/8 & 1/8 \\ 1/8 & 7/8 \end{bmatrix}$$

- Now, we calibrate the conditional idiosyncratic transition probabilities:
  - Expansion: average time from U to E: 1.5 quarters

$$1.5 = \sum 1 * (1 - \gamma_{UU|G}) + 2 * \gamma_{UU|G}(1 - \gamma_{UU|G}) + 3 * \gamma_{UU|G}^2(1 - \gamma_{UU|G}) + \dots$$
$$= \frac{1}{1 - \gamma_{UU|G}} \implies \gamma_{UU|G} = \frac{1}{3} \implies e.g.)\ \gamma_{GU|GU} = \gamma_{GG} * \gamma_{UU|G} = \frac{7}{24}$$

  - Recession: average time from E to U: 2.5 quarters

$$2.5 = \sum 1 * (1 - \gamma_{UU|B}) + 2 * \gamma_{UU|B}(1 - \gamma_{UU|B}) + 3 * \gamma_{UU|B}^2(1 - \gamma_{UU|B}) + \dots$$
$$= \frac{1}{1 - \gamma_{UU|B}} \implies \gamma_{UU|B} = \frac{3}{5} \implies e.g.)\ \gamma_{BU|BU} = \gamma_{BB} * \gamma_{UU|B} = \frac{21}{40}$$

Calibration (external) (cont'd)

- γ_{UU} when the economy switches from G to B is 25% greater than γ_{UU} when the economy stays in B.

$$\frac{\gamma_{BU|GU}}{\gamma_{BU|BU}} = 1.25$$

- γ_{UU} when the economy switches from B to G is 25% smaller than γ_{UU} when the economy stays in G.

$$\frac{\gamma_{GU|BU}}{\gamma_{GU|GU}} = 0.75$$

- Aggregate labor supply fluctuate (exogenously)
  - Expansion: $\Phi_G(\epsilon = u) = 0.04$
  - Recession: $\Phi_B(\epsilon = u) = 0.1$
  - No distinction between non-participation and unemployment

Calibrated transition matrix

$$\Gamma = \begin{bmatrix} 0.525 & 0.350 & 0.03125 & 0.09375 \\ 0.035 & 0.84 & 0.0025 & 0.1225 \\ 0.09375 & 0.03125 & 0.292 & 0.583 \\ 0.0099 & 0.1151 & 0.0245 & 0.8505 \end{bmatrix}$$

- The state grid points: $\mathbb{Z} = [z_{BU}, z_{BE}, z_{GU}, z_{GE}]' = [0.25, 1; 0.25, 1]'$
- The transition matrix of aggregate state only:

$$\Gamma_S = \begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix}$$

Recursive form

A household begins a period with wealth a and labor productvity z given.

$$v(a,z;S,\Phi) = \max_{c,a'} u(c) + \beta \sum_{z'} \Gamma_{z,z'} v(a,z';S',\Phi')$$

$$\begin{aligned} \text{s.t.} \quad & c + a' = w(S,\Phi)z + a(1 + r(S,\Phi)) \\ & a' \geq \underline{a} = 0 \\ & z' \sim \Gamma(z'|S',z) \quad \text{(Markov chain)} \\ & S' \sim \Gamma(S'|S) \quad \text{(Markov chain)} \\ & \Phi' = G(S,S',\Phi) \end{aligned}$$

- Q) Why do households need to understand Φ? Why Φ'? What if *iid* S'?
- Q) Why does S' show up in the law of motion, G?
- cf.) Arrow-Debreu economy:

$$c + \sum_{z'} \Gamma_{z,z'} q(z',\Phi') a'(a,z,z';S,S'\Phi') = w(S,\Phi)z + a(1 + r(S,\Phi))$$

Recursive competitive equilibrium

$(g_a, g_c, v, G, g_K, g_L, r, w)$ are recursive competitive equilibrium if

- (g_a, g_c, v) solves household's problem.
- (g_K, g_L) solves a representative firms' problem.
- (r, w) clears capital and labor markets.

$$\text{(Capital market)} \quad g_K(S', G(S, S', \Phi)) = \int g_a(a, z; S, \Phi) d\Phi \quad \text{at } r = r(S, \Phi)$$

$$\text{(Labor market)} \quad g_L(S, \Phi) = \sum_{i=1}^{2} \Phi_S(z_i) z_i \quad \text{at } w = w(S, \Phi)$$

- $\int (g_c(a, z; S, \Phi) + g_a(a, z; S, \Phi)) \, d\Phi = F(g_K(S, \Phi), g_L(S, \Phi)) + (1-\delta) g_K(S, \Phi)$
- $G(S, S', \Phi) = \Phi'$ holds, where for $\forall \mathcal{A}' \subseteq \mathbb{A}$,
  $\Phi'(\mathcal{A}', z') = \int \Gamma_{z,z'} \mathbb{I}\{z' \in \{z_{S'U}, z_{S'E}\}\} \mathbb{I}\{g_a(a, z; S, \Phi) \in \mathcal{A}\} d\Phi$
  where $\mathbb{A}$ is the Borel σ-algebra generated from $[\underline{a}, \infty)$.

Note: $g_K(S', G(S, S', \Phi))$ is invariant over S'. Why? Hint: Φ_S varies over S.

Conceptual hurdle: a counter-factual realization

- Suppose an aggregate state at period t is G.
- A rational agent should rationally expect the future period $t+1$:
  - The future realization with the state G
  - The future realization with the state B
- If we simulate an aggregate shock, only a single state is realized at $t+1$. (Say $S_{t+1} = B$)
- No one can observe the counter factual observation: the world with $S_{t+1} = G$.
- Then, how can we specify the mysterious counter-factual world as a possible future outcome? (Specifically, the value function)
- It is like Marvel's multiverse where the world is diverging to the different universe with different outcomes at each second: With Thanos' mistake, a world still has Iron man alive. But this is not observable to an econometrician (only to Dr. Strange).
- In Aiyagari (1994), if a person is given an idiosyncratic shock z, it was not a problem: we will always find another guy exactly the same as the person while only the shock realization is different.

Comptational hurdle

- Suppose we can somehow specify the counter-factual future.
- Still, how can we track the dynamics of Φ, which is the infinite-dimensional object?
- In Aiyagari (1994), we did not have to track it as it will stay the same in SRCE, reducing the problem to find the proper $K \in \mathbb{R}$. (Huge dimensional reduction)
- Will there be such a nice dimensional reduction in the end?
- Yes: Sufficient statistic approach

Computation

Roadmap

- The following is the roadmap for computation section.
  1. Set the parametric law of motion: assumption
  2. Guess the paramters
  3. Solution (optimization)
     - VFI/PFI/EGM/Projection method
     - Interpolation
  4. Simulation
     - Stochastic simulation
     - Non-stochastic simulation: Iteration method
  5. Aggregation
  6. Update the parameters
  7. After convergence, verify the assumption
- After this, we will talk about more recent developments.

Basic setup

The codes start from the usual steps.

- Set directories
- Set parameters (might be a function argument)
- Setting grid points
  - Individual wealth grid
  - Aggregate wealth grid: this grid can be sparse (5∼10 grids)
- Specifying idiosyncratic and aggregate shock process (Markov chain)
  - Krusell and Smith (1998) suggests a specific Markov chain.
  - Tauchen/Rouwenhorst method
- Compute the state-dependent labor supply
  - Not to make the unemployed get the benefit out of nowhere, let's assume they also supply labor by 0.25.

Parametric law of motion

There are two layers of choices:

- First, we need to set what are **sufficient statistics** to characterize the dynamics of the individual state distribution.
  - A good candidate is the first moment of the endogenous individual state.
  - In Krusell and Smith (1998), by tracking only K_t, the aggregate prices are also characterized as in Aiyagari (1994).
  - Not many models are in this ideal setup: then, two laws of motions are necessary (Khan and Thomas, 2008)
- Then, we need to decide the **parametric form** of the law of motion.
  - A good candidate is the log-linear form: because we know if heterogeneity does not matter, the dynamics will be similar to log-linearized dynamics in the representative-agent world.
- So start from four parameter guesses $(\alpha_B, \beta_B; \alpha_G, \beta_G)$

$$log(K_{t+1}) = \alpha_S + \beta_S log(K_t) \quad \text{when } S_t = S \in \{B, G\}$$

- How does this help the concerns we had?

How to address the concerns

- The concern on the mysterious counter-factual future world:
  - We assume that the counter-factual world can be linearly approximated.
  - Suppose we know $V(a_{t+1}, z_{t+1}; S_{t+1} = G, \Phi_{t+1}) \approxeq \widehat{V}(a_{t+1}, z_{t+1}; S_{t+1} = G, \widetilde{K}_{t+1})$.
  - Suppose we do not know $V(a_{t+1}, z_{t+1}; S_{t+1} = B, \Phi_{t+1}) \approxeq \widehat{V}(a_{t+1}, z_{t+1}; S_{t+1} = B, \widetilde{K}_{t+1})$.
  - We approximate $V(a_{t+1}, z_{t+1}; S_{t+1} = B, \Phi_{t+1})$ by $\widehat{V}(a_{t+1}, z_{t+1}; S_{t+1} = B, \ \cdot \ )$ using interpolation: interpolate the value on $\widetilde{K}_{t+1}$.
  - The same logic applies to the policy function iteration.
- The concern on the computational cost:
  - Dramatic cost reduction: only the dynamics of K_t needs to be tracked.

Computation - GE: Solution

Policy function iteration

- We will use the policy function iteration to compute the optimal household policy.
- Another name of this method is time iteration.
- Given the law of motion $G(S, K)$, it utilizes the following relationship:

$$u'(g_c(a, z; S, K)) = \beta \sum \Gamma_{S,S'} \left[u'(g_c(g_a(a, z; S, K), z'; S', K'))(1 + r(S', K')) \right]$$

- Note that this relationship holds only when the borrowing constraint does not bind.
- How can we use PFI under the occasionally binding constraint?
- Maliar, Maliar, and Valli (2010): excellent explanation with public codes for the replication
- The detailed steps are in the next slide.

Policy function iteration

$$u'(g_c(a,z;S,K)) = \beta \sum \Gamma_{S,S'} \left[u'(g_c(g_a(a,z;S,K),z';S',K'))(1+r(S',K')) \right]$$

The pseudo code is as follows:

1. Given $(\alpha_B, \beta_B; \alpha_G, \beta_G)$, guess $g_a^{(n)}$.
2. For given shock S and K, compute the future capital stock K'.
3. Compute $g_a^{(n)}(g_a^{(n)}, z'; S', K')$ using two-dimensional interpolation.
4. If $g_a^{(n)}(g_a^{(n)}, z'; S', K') < 0$, then enforce $g_a^{(n)}(g_a^{(n)}, z'; S', K') = 0$. (Mechanically, $\mu \geq 0$)
5. Compute $g_c(g_a^{(n)}, z'; S', K') = budget(g_a^{(n)}, z'; S', K') - g_a^{(n)}(g_a^{(n)}, z'; S', K')$
6. Compute RHS$(a,z;S,K)$: $\beta \sum \Gamma_{S,S'} \cdot \left[u'(g_c(g_a^{(n)}(a,z;S,K), z'; S', K'))(1+r(S',K')) \right]$.
7. Obtain $\hat{g}_c = u^{-1}(RHS)$.
8. Obtain $g_a^{(n+1)} = a(1+r(S,K)) + w(S,K)z - \hat{g}_c$.
9. Check if $||g_a^{(n+1)} - g_a^{(n)}||_p < Tol$
   - If yes, the solution converged.
   - If no, go back to step 2. (The policy function is already updated)

Obtaining RHS with *Interpn*

```
mPolaprimeprime =
    interpn(vGrida',vGridK,squeeze(mPolaprime(:,:,izprime,iAprime)),...
           mPolaprime,Kprime,"spline");
mPolaprimeprime(mPolaprimeprime<0) = 0;

K2Lprime = Kprime/supplyL;
rprime = vGridA(iAprime).*pAalpha*K2Lprime.^(pAalpha-1)-pDdelta;
mmuprime = rprime+pDdelta;
wprime = (1*pAalpha)^(1/(1-pAalpha))*((1-pAalpha)/pAalpha)...
       * mmuprime.^(pAalpha/(pAalpha-1));
cprime = wprime.*zprime + (1+rprime).*mPolaprime - mPolaprimeprime;

muprime = 1./cprime;
mExpectation = mExpectation +
    mTransz(iz,izprime)*mTransA(iA,iAprime).*(1+rprime).*muprime;
```

Computation - GE: Simulation

Non-stochastic simulation

We use a non-stochastic iteration method. (Eigenvalue method is not feasible).

- Simulate a long enough aggregate shock path using Γ_S. (Not idiosyncratic shocks)
- Start from an initial guess $\mathcal{H}_0$. Compute the corresponding K_0.
- Given (S_0, K_0), compute the optimal policy $g_{a,0}$.
- Let $\mathcal{H}_0$ evolve to $\mathcal{H}_1$ using $g_{a,0}$, and compute the corresponding K_1.
- Given (S_1, K_1), compute the optimal policy $g_{a,1}$.
- Let $\mathcal{H}_1$ evolve to $\mathcal{H}_2$ using $g_{a,1}$, and compute the corresponding K_2.
- ...
- By repeating this process, we obtain $\{K_t\}_{t=0}^{T}$.
- As it takes some time for the simulation to be on the regular dynamics, discard the early part of the simulation: the burn-in period.
- We obtain $\{K_t\}_{t=burnIn}^{T}$.

Note: How does the initial guess on the histogram converges to the regular stochastic path? Path stability. (c.f. a stable equilibrium in the stationary case)

Computation - GE: Updating the parameters

Updating the parameters

- We have $\{K_t\}_{t=burnIn}^{T}$ and $\{S_t\}_{t=burnIn}^{T}$.
- Fit the time-series into the parametric form of the law of motion to estimate the parameters: $(\widehat{\alpha}_B, \widehat{\beta}_B; \widehat{\alpha}_G, \widehat{\beta}_G)$

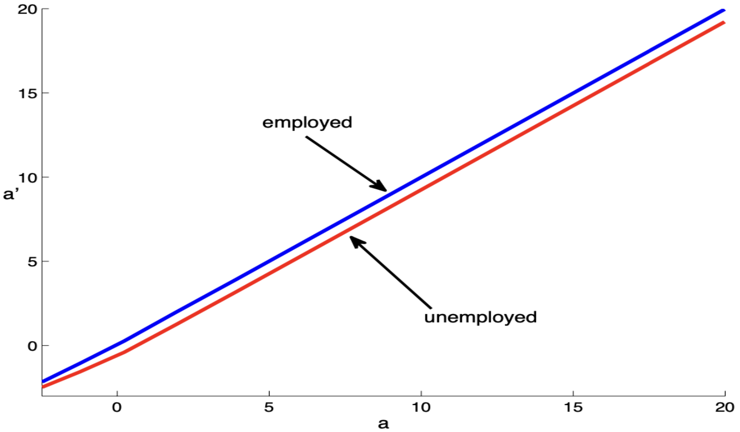
$$log(K_{t+1}) = \widehat{\alpha}_S + \widehat{\beta}_S log(K_t) + \epsilon_{t+1} \quad \text{when } S_t = S \in \{B, G\}$$

- If the parameter estimates are not close to the guess, return to the initial step.
- Otherwise, the solution is converged.
- Check R^2 as the first check for the validity of the parametric form.
- In Krusell and Smith (1998), the solution is as follows:

$$\begin{aligned}
[S_t = G] \quad logK_{t+1} &= 0.095 + 0.962 logK_t + \epsilon_{t+1} \\
R^2 &= 0.999998, \quad \widehat{\sigma} = 0.00028\% \\
[S_t = B] \quad logK_{t+1} &= 0.085 + 0.965 logK_t + \epsilon_{t+1} \\
R^2 &= 0.999998, \quad \widehat{\sigma} = 0.00036\%
\end{aligned}$$

Discussions

Almost perfect insurance in the incomplete economy



Notes: The figure is from Krusell and Smith (1998).

Accuracy test

The dynamics in the equilibrium allocations, including the infinite-dimensional object, are perfectly governed by 4 parameters. How do we know it's really enough? R^2 and $s.e.$ enough?

- Den Haan (2010) show that R^2 and $s.e.$ not enough:
  - R^2 and $s.e.$ only check the one-period ahead forecast error.
  - R^2 and $s.e.$ captures only average, and hide infrequent large errors.
  - "R^2 scales the errors by the variance of the dependent variable.": R^2 might increase if simply the variance of aggregate shock increases.
  - Alternative: "fundamental accuracy plot"
    - Plot the equilibrium allocations.
    - Plot the law of motion (independent from the equilibrium allocations).
    - Compare the levels at each time t.

Accuracy test (cont'd)

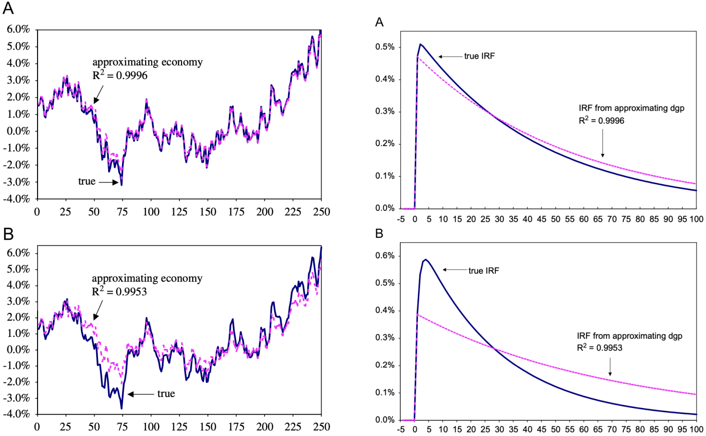


Figure: The figure is from Den Hann (2010)

Applications

How does micro-level heterogeneity affect the macro-level dynamics?

- In the stationary environment, it is difficult to see how micro-level heterogeneity affects the macro. Macro to micro is possible.
- Transition dynamics: Does a micro-level heterogeneity affect the growth?
- Impulse response: Does a micro-level heterogeneity affect the responsiveness?
- Business cycle: Does a micro-level heterogeneity affect the business cycle?

Depending on the research interest, multiple variants have been developed:

- Wealth distribution (Thicker tail): Krusell and Smith (1998) adopted idiosyncratic discount factor shock (β shock)
- Labor supply: Chang and Kim (2007) incorporated extensive-margin labor supply decision.
- Occupation choice: Kwark and Ma (2021) and Lee (2021) study how a decision to become an entrepreneur interacts with the business cycle.

Limitations

Done : Non-trivial market clearing conditions

- This concern often happens for heterogeneous firm models.
- Recall that each loop starts from getting the price out of K_t. However, there might be no explicit MPK_t or MPL_t: we cannot determine the price even if we know K_t.
- Krusell and Smith (1997): a variant of Krusell and Smith (1998) that adopts an additional law of motion between price and the sufficient statistics (K_t). Then this law of motion is updated by computing the accurate price at each time t in the simulation.
- Computationally very costly.
- e.g., Khan and Thomas (2008), Chang and Kim (2007), Kwark and Ma (2021)

-ing : Parametric form of the law of motion

- Why log linear?
- Which allocations to include?

-ing : History dependence (state dependence)

-ing : Uncertainty shock

- The second moment shock generates nonlinear dynamics
- Fernandez Villaverde et al. (2011) and Bloom et al. (2018)

Recent developments

- Reiter (2009): Linearizing around the steady-state.
  - Solve SRCE first. The first-order perturbation around the SRCE $\rightarrow$ the LoM.
- Boppart et. al (2018)
  - Solve SRCE first. Investigate impulse responses $\rightarrow$ the LoM.
- Winberry (2018)
  - Perturbation of parametrized distribution functions using Dynare and Bayesian estimation.
- Childers (2018)
  - Linearization in the function valued state $\rightarrow$ the LoM is represented by a combination of basis functions.
- Auclert et al. (2021)
  - Perturbation of Reiter (2009) on the sequence space instead of state space.
  - In other words, linearization + Boppart et al. (2018): very fast algorithm
- Kahou et. al (2021)
  - Deep learning on the LoM using finite agent's dynamics under permutation-invariance.
- Lee (2022)
  - Interpolating missing counter-factual realizations using closest counter parts on the single path of aggregate shock: Ergodic theorem

Summary

- Krusell and Smith (1998) studies how heterogeneous households interact with the TFP-driven aggregate fluctuations.
- The log linear law of motion works extremely well.
- In this model, heterogeneity does not matter much: a large fraction of households behave as if there is perfect insurance about the idiosyncratic shocks. (That's why log-linear law of motion works well.)
- The methodology has limitations to be applied for a broader class of models.
- Such limitations include a difficulty to specify non-linear law of motions.
- The non-trivial market-clearing condition is tricky to handle: we will study this in the last lecture (heterogeneous firm models).

Homework

Do it yourself, and compare it with your colleagues. The sample replication code will be provided in the following week (not an answer key).

1. Replicate the Krusell-Smith (1998) economy with idiosyncratic and aggregate productivity process discretized by Tauchen method under the following parametrization:
   - log utility and the discount factor of 0.96
   - log income persistence 0.9 and volatility 0.1 (AR(1) process)
   - log aggregate productivity persistence 0.9 and volatility 0.1 (AR(1) process)
   - Tauchen: 7 grid points with a three-standard deviation range covered.
   - Production: capital share 0.36 and depreciation 0.08
   - Borrowing constraint at 0
2. Compute the time-series of the wealth portions of top 0.1%/1%/10%/20%/50%.
3. Compute the time-series of consumption and compare the average with the previous homework's consumption level at the stationary equilibrium.