Banking Fragility and the Capitalization Dilemma*

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Abstract

Bank capital plays a *dual role*: high capital amplifies TFP shocks via balance-sheet exposure; low capital amplifies financial shocks via endogenous fragility. We embed both channels in a business-cycle model with an occasionally binding leverage constraint and an empirically disciplined hazard function. A social planner internalizing the fragility externality accumulates more capital, halving output losses from financial shocks despite greater TFP exposure. We derive a leverage-rule representation of the planner's solution whose two terms move in opposite directions: tighter when bank equity is high, looser in high-TFP states. Aggregate credit-gap rules conflate these components.

Keywords: Banking fragility, bank capitalization, macroprudential policy, occasionally binding constraints, global nonlinear solution.

JEL codes: E32, E44, G21, G28

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1 Introduction

Bank capital regulation is a central question in macroeconomics and financial stability. A large literature has shown that fragile balance sheets amplify financial shocks: banks with thin equity cushions face higher failure risk, and realized failures trigger deleveraging spirals. A separate literature has established that credit exposure amplifies real shocks: well-capitalized banks lend more, and when productivity falls, the resulting contraction is proportionally larger. These two channels are individually well understood, yet the existing literature studies them in isolation. Whether they reinforce or offset each other—and what this implies for optimal policy—remains an open question. This paper builds a tractable general equilibrium model that embeds both channels simultaneously, allowing us to characterize their interaction.

The key contributions of this paper are threefold. First, by embedding the credit exposure and fragility buffer channels within a single general equilibrium framework, we reveal a *capitalization dilemma* that is invisible when the channels are studied in isolation. Well-capitalized banks extend more credit, so a negative productivity shock generates *larger* output losses when equity is high—credit booms become credit exposure. Conversely, poorly capitalized banks suffer *far larger* output losses from bank failures, because with little equity to absorb the shock, endogenous fragility amplifies the deleveraging spiral. Any level of bank equity that improves resilience to financial shocks simultaneously increases exposure to productivity shocks. This dilemma emerges only from the interaction of the two channels within a unified framework and cannot be resolved by the market alone.

Second, we compare the competitive equilibrium (CE) with a constrained social planner's problem (SPP) and show that the welfare implications of the capitalization dilemma are asymmetric. The planner, who internalizes the fragility externality, accumulates more equity and halves output losses from financial shocks. Yet the planner cannot improve upon the competitive response to TFP shocks, because productivity shocks do not activate the fragility externality. This asymmetry sharpens the scope of macroprudential policy: capital requirements are most effective against the risks that the market systematically underprices—financial distress—and offer no advantage where shocks originate in the real sector.

Third, we derive an implementable *leverage-rule representation* of the planner allocation. It is an OLS projection of the planner's implied leverage tightness onto the aggregate state (N, A). Solving the allocation globally matters: it recovers state dependence around the leverage kink that local approximations would miss. The rule also differs from existing macroprudential Taylor rules in its instrument. Debt-tax rules such as Bianchi and Mendoza (2018) and credit-gap frameworks such as the Basel III CCyB (Basel Committee on Banking Supervision, 2010) target prices or aggregates; ours is a leverage coefficient on bank equity and TFP. Its two terms move in opposite directions. The financial-cycle term tightens when bank equity is high, restraining balance-sheet expansion. The business-cycle term loosens in high-TFP states, when fundamentals reduce the marginal fragility externality. Aggregate credit-gap rules collapse both channels into a single credit-growth response and hide this structure.

The model features three key ingredients. Banks intermediate funds between households and firms through a concave lending technology, creating a wedge between the return on capital and the deposit rate. Banks face an *occasionally binding leverage constraint* that limits deposit-taking as a function of equity. Bank failures occur with an endogenous probability that decreases in the capital ratio—lower capitalization implies greater fragility. A realized bank failure, which destroys a fraction φ of bank equity, is a *hazard shock*. We characterize equilibrium dynamics through *conditional saddle paths*—the equilibrium loci of consumption and bank equity under frozen TFP regimes—and show that the occasionally binding leverage constraint creates a kink in the policy function at intermediate equity levels.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 provides the quantitative analysis, including conditional saddle paths (Section 3.3), state-dependent GIRFs (Section 3.4), and the ergodic distributions (Section 3.5). Section 4 discusses the mechanisms and derives our main policy result—the leverage-rule representation of the planner allocation and its opposite-sign financial-cycle / business-cycle decomposition (Section 4.5)—alongside comparative statics of the leverage parameters (Section 4.4) and a three-channel decomposition of the SPP–CE capitalization gap (Section 4.6). Section 5 concludes.

Related literature Our paper draws on several strands of the macro-finance literature, each of which studies one of the two channels in isolation; our contribution is to embed both within a single general equilibrium framework. First, a large body of work shows how banks' balance-sheet strength amplifies real shocks. The financial accelerator literature emphasizes intermediary net worth (Bernanke et al., 1999; Kiyotaki and Moore, 1997), while more recent models embed banks as capital-constrained intermediaries (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Intermediary asset pricing models show that net worth drives risk premia and amplification (He and Krishnamurthy, 2013, 2019; Brunnermeier and Sannikov, 2014). Our model complements these frameworks: rather than operating through the SDF or risk-price channel, the credit exposure channel works through the *real* intermediation technology. Bank equity affects output directly through lending volumes.

Second, a separate literature studies the fragility buffer channel—how bank failure or run risk arises from balance-sheet characteristics or strategic complementarities (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005). Gertler et al. (2020) embed bank runs as a self-fulfilling equilibrium in a macro model; Boissay et al. (2016) generate endogenous banking crises from the interbank market. Our approach is closest to models with smooth, state-dependent fragility measures rather than binary run equilibria (He and Xiong, 2012). By combining this channel with the credit exposure channel in a single framework, we can characterize the trade-off that neither literature addresses on its own.

Third, the paper contributes to work on occasionally binding financial constraints, which generate nonlinear amplification and kinks in policy functions (Mendoza, 2010; Bianchi, 2011; Adrian and Shin, 2010; Rampini and Viswanathan, 2019). In our model, the leverage constraint produces a kink that governs whether distress is absorbed smoothly or triggers a deleveraging spiral.

Finally, we build on the macroprudential literature comparing decentralized equilibria with constrained efficient allocations (Lorenzoni, 2008; Farhi and Werning, 2016; Van den Heuvel, 2008). Quantitative models of bank capital requirements include Begenau (2020), Corbae and D'Erasmo (2021), and Elenev et al. (2021). Because our framework nests both channels, it can speak to a question these models do not address: the planner internalizes the fragility externality, yielding sharply different responses to financial shocks but similar responses to TFP shocks. This asymmetry clarifies where capitalization policy is most effective—and where it is not.

Closest to our policy-rule exercise is the "optimized macroprudential Taylor rule" of Bianchi and Mendoza (2018), who characterize time-consistent optimal policy in a small open economy with a debt-to-output collateral constraint and a Pigouvian tax on debt. Our contribution differs along three dimensions. First, the instrument is a *leverage coefficient* on bank equity, which maps directly to the Basel III capital-requirement architecture (Basel Committee on Banking Supervision, 2010) rather than a borrowing tax. Second, the hazard function is disciplined empirically using the Jordà et al. (2017) crisis database, so the rule is quantitatively grounded in historical crisis incidence. Third, the rule admits a two-term decomposition with opposite signs on the financial-cycle and business-cycle states—a structure that collapses in rules driven by aggregate credit growth alone.

2 Model

2.1 Environment

Time is discrete and infinite. The economy consists of a representative household, a representative bank, and a representative firm. The aggregate state is $X = [N, A]$, where N is aggregate bank equity and A is aggregate TFP. We use lowercase letters (e.g., n, l, b) for individual bank variables; in the representative-bank equilibrium these coincide with their aggregate counterparts ($n = N, l = L, b = B$).

2.2 Bank

The bank maximizes its franchise value:

$$\begin{aligned} J(n; X) = \max_{n'} \; & \pi(n; X) + n - n' \\ & + (1 - \mathcal{H}(X))\, \mathbb{E}\, q(X, X')\, J(n'; X') \\ & + \mathcal{H}(X)\, \mathbb{E}\, q(X, X')\, J\big((1-\varphi)n'; X'\big) \end{aligned} \tag{1}$$

where $\pi(n; X)$ denotes bank profit, $q(X, X')$ is the stochastic discount factor, $\mathcal{H}(X)$ is the endogenous fragility hazard rate, and φ is the fraction of equity destroyed in a failure. The two continuation values embed the dual nature of the bank's environment: with probability $1 - \mathcal{H}$, the bank enters the next period intact; with probability $\mathcal{H}$, it suffers an equity loss that degrades its future intermediation capacity.

Bank profit is:

$$\pi(n; X) = \max_{l} \; \Phi(l)(R^K(X) - 1) - (R^B(X) - 1)(l - n) \tag{2}$$

where $\Phi(l) = \Phi_A l^{\Phi_B}$ is a concave intermediation technology ($\Phi_B < 1$), R^K is the gross return on capital, R^B is the gross deposit rate, and l is total lending. Deposits are $b = l - n$. The concavity captures diminishing returns in intermediation: the bank lends l units, which are transformed into $\Phi(l) \leq l$ units of productive capital. The gap $l - \Phi(l)$ represents the resource cost of intermediation.

The bank faces a leverage constraint:

$$b \leq \phi_0 + \phi_1 n \tag{3}$$

with associated multiplier $\lambda \geq 0$.

2.3 Fragility hazard

The failure probability is:

$$\mathcal{H}(X) = \sigma\left(\kappa_1 \frac{N}{L(X)} + \kappa_0\right) \tag{4}$$

where $\sigma(\cdot)$ is the logistic sigmoid, N/L is the aggregate capital ratio, and $\kappa_1 < 0$ so that lower capitalization raises fragility. Intuitively, a thinly capitalized banking sector has less capacity to absorb losses and is more vulnerable to depositor withdrawals, asset fire sales, or interbank contagion—all of which are captured in reduced form through the capital ratio. The parameter κ_1 is calibrated from a logit regression of crisis probability on the capital ratio using the Jordà et al. (2017) macrohistory database.

2.4 Firm

A representative firm operates a Cobb-Douglas technology:

$$Y = AK^{\alpha}(H^D)^{1-\alpha} \tag{5}$$

and hires capital and labor competitively. Setting $\delta = 1$ (full depreciation) eliminates capital as an endogenous state variable, leaving bank equity as the sole aggregate endogenous state. This preserves the key economic forces while maintaining tractability; we discuss relaxing this assumption in Section 5.

2.5 Household

The representative household has GHH preferences with a general CRRA parameter γ:

$$u(C, H^S) = \frac{\left(C - \eta \frac{(H^S)^{1+1/\chi}}{1+1/\chi}\right)^{1-\gamma}}{1-\gamma} \tag{6}$$

with discount factor β, Frisch elasticity χ, and labor disutility weight η. In the baseline calibration, $\gamma = 1$, so the utility function reduces to $u = \log\big(C - \eta(H^S)^{1+1/\chi}/(1+1/\chi)\big)$.

The household budget constraint is:

$$C + p'(X)\, a' + B = p(X)\, a + R^B(X)\, B + w(X)\, H^S - \xi(B) + \Psi(X) \tag{7}$$

where $\xi(B) = \zeta_1 B + \zeta_2 B^2$ is a portfolio adjustment cost for deposits, $p(X) = J(N; X)$

is the bank equity price, a is equity holdings, and $\Psi(X)$ is a distress rebate. When a failure occurs (probability $\mathcal{H}(X)$), the destroyed equity $\varphi\, n'(N;X)$ is rebated lump-sum to households; otherwise $\Psi(X) = 0$. Bank failures therefore redistribute rather than destroy aggregate resources.

2.6 Equilibrium

Definition 1 (Recursive Competitive Equilibrium). *A recursive competitive equilibrium is a collection of: (i) a bank value function $J(n;X)$ and policy functions $\{n'(n;X), l(n;X)\}$; (ii) a household value function $v(a;X)$ and policy functions $\{a'(a;X), B(a;X), H^S(a;X)\}$; (iii) pricing functions $\{R^K(X), R^B(X), w(X), p(X), p'(X), q(X,X')\}$; and (iv) an aggregate law of motion $N' = \Gamma(X, \varepsilon)$; such that: (a) the bank's and household's policy functions solve their respective Bellman equations; (b) all markets clear ($\Phi(L) = K$, $B(1;X) = l - n$, $a(1;X) = 1$, $H^S = H^D$); (c) the aggregate law of motion is consistent with individual decisions; and (d) prices are consistent with optimality.*

In equilibrium, all markets clear. The key aggregate conditions are:

- *Equity transition.* Aggregate bank equity evolves according to

$$N'(X) = \begin{cases} n'(N;X) & \text{with prob. } 1 - \mathcal{H}(X) \\ (1-\varphi)\, n'(N;X) & \text{with prob. } \mathcal{H}(X) \end{cases} \tag{8}$$

- *National accounting.* The national accounting identity is

$$Y = (C + \Psi(X)) + \xi(B) + K + (N' - N) \tag{9}$$

  where $C + \Psi(X)$ is post-distress consumption, $\xi(B)$ is a real resource cost (deadweight loss from intermediation), and bank dividends $d = \pi + n - n'$ flow to households as equity income.

- *Stochastic discount factor.* The SDF is

$$q(X, X') = \beta \frac{u_c(C', H^{S'})}{u_c(C, H^S)} = \beta \frac{\widetilde{C}(X)}{\widetilde{C}(X')} \tag{10}$$

  where $\widetilde{C} := C - \eta (H^S)^{1+1/\chi}/(1+1/\chi)$ is composite consumption and the second equality holds under log-GHH preferences ($\gamma = 1$, so $u_c = 1/\widetilde{C}$).

2.7 Social planner's problem

The constrained social planner (SPP) solves the same problem as the bank but internalizes the effect of aggregate equity on the hazard rate $\mathcal{H}(X)$. The bank's first-order condition for n' (the Euler equation) is:

$$1 = (1 - \mathcal{H})\,\mathbb{E}\left[q\,J_1(n'; X')\right] + \mathcal{H}\,(1 - \varphi)\,\mathbb{E}\left[q\,J_1((1 - \varphi)n'; X')\right] \tag{11}$$

where the envelope condition gives $J_1(n; X) = R^B(X) + \lambda(1 + \phi_1)$. The planner modifies this condition by adding a Pigouvian wedge τ that captures the systemic benefit of higher capitalization:

$$J_1^{\mathrm{SPP}}(n; X) = R^B(X) + \lambda(1 + \phi_1) + \tau(X) \tag{12}$$

where the Pigouvian tax is:

$$\tau(X) = \frac{\beta}{u_c(C, H^S)} \cdot \frac{\partial \mathcal{H}}{\partial B} \cdot \mathbb{E}\left[V(n'; X') - V\big((1 - \varphi)n'; X'\big)\right] \tag{13}$$

Here $V(N; X) := v(1; [N, A])$ is the household's equilibrium lifetime utility as a function of the aggregate state, and $\partial \mathcal{H}/\partial B > 0$ reflects that higher leverage increases the failure hazard (since $\partial(N/L)/\partial B = -N/L^2 < 0$ and $\kappa_1 < 0$). Because $V(n') > V((1 - \varphi)n')$, the tax $\tau > 0$: the planner penalizes leverage beyond what private banks choose. The underlying externality is pecuniary: each bank takes the aggregate failure probability $\mathcal{H}(X)$ as given, ignoring that its own leverage choice contributes to the sector-wide capital ratio that determines $\mathcal{H}$. The modified lending FOC becomes $\Phi'(l)(R^K - 1) - (R^B - 1) = \lambda + \tau$, inducing lower leverage and higher equity accumulation.

2.8 Analytical properties

The following propositions characterize key properties of the equilibrium. Proofs are provided in Online Appendix B.

Proposition 1 (Sign of the Pigouvian wedge)**.** *In the constrained social planner's problem, the Pigouvian wedge satisfies $\tau(X) > 0$ for all X with $\mathcal{H}(X) \in (0, 1)$. The planner accumulates strictly more equity than the competitive equilibrium: $n'^{\mathrm{SPP}}(N; X) > n'^{\mathrm{CE}}(N; X)$.*

The result follows from two observations: (i) $\partial \mathcal{H}/\partial B > 0$, since $\kappa_1 < 0$ and $\partial(N/L)/\partial B = -N/L^2 < 0$; and (ii) $V(n'; X') > V((1 - \varphi)n'; X')$ for $\varphi > 0$. Since $\tau > 0$ enters as

$J_1^{\mathrm{SPP}} = R^B + \lambda(1+\phi_1) + \tau > J_1^{\mathrm{CE}}$, the planner's Euler equation requires higher equity retention.

Proposition 2 (Conditional steady state)**.** *Fix $A \in \{A_L, A_H\}$. If the equilibrium policy function $n'(\cdot\,; A)$ is continuous with $n'(N; A) > N$ for N sufficiently small and $n'(N; A) < N$ for N sufficiently large, then a conditional steady state N_A^{CS} exists. If additionally $n'(\cdot\,; A)$ crosses the 45-degree line exactly once, the CSS is unique and globally stable under the frozen regime.*

The CSS captures the equity level at which dividend payouts exactly offset the bank's retained earnings motive: below N_A^{CS}, equity is scarce and intermediation profits induce accumulation; above it, the marginal return falls below the household's required rate of return and equity is paid out.

Proposition 3 (Planner's CSS ordering)**.** *Under the conditions of Proposition 2, $N_A^{\mathrm{CS,SPP}} > N_A^{\mathrm{CS,CE}}$ for each TFP regime A.*

This follows directly from Proposition 1: since the planner retains more equity at every N, the policy function $n'^{\mathrm{SPP}}(\cdot; A)$ lies above $n'^{\mathrm{CE}}(\cdot; A)$, shifting the fixed point rightward.

3 Quantitative analysis

3.1 Calibration

Table 1 summarizes the baseline calibration. Household preferences follow standard values: discount factor $\beta = 0.96$ (annual), log utility ($\gamma = 1$), unit Frisch elasticity ($\chi = 1$), and labor disutility $\eta = 0.65$ to target steady-state labor supply near one-third. The capital share is $\alpha = 0.33$.

For banking parameters, the intermediation curvature $\Phi_B = 0.9$ generates moderate diminishing returns. The quadratic deposit cost $\zeta_2 = 0.5$ pins down the deposit rate spread; the linear component $\zeta_1 = 0$ is set to zero for parsimony. The leverage parameters $\phi_0 = 0.01$ and $\phi_1 = 1.5$ are chosen so that the constraint binds approximately 20% of the time in the CE.[1]

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[1] The implied steady-state capital ratio $N/L \approx 0.50$ is higher than typical bank leverage ratios (5–10%). This is a direct consequence of full depreciation ($\delta = 1$): the entire capital stock is rebuilt each period, so the bank's balance sheet L reflects only one period's lending flow. Under realistic annual depreciation ($\delta \approx 0.10$), the steady-state capital stock $K_{\mathrm{ss}} = I_{\mathrm{ss}}/\delta$ is an order of magnitude larger for the same output level. Since the intermediation technology $K = \Phi(L)$ links capital to lending, a larger durable K requires a proportionally larger L, inflating the bank's balance sheet. Bank equity N, however, is determined by the intertemporal optimization (the Euler equation) and does not scale with the capital stock. The result is a much lower N/L under partial depreciation. The leverage parameters are calibrated to match a binding frequency target (20%) rather than a capital ratio level.

The hazard slope $\kappa_1 = -6.86$ is estimated from a logit regression of annual crisis probability on the capital ratio using the Jordà et al. (2017) macrohistory database; $\kappa_0 = 0.47$ pins down the unconditional failure frequency. The distress loss $\varphi = 0.60$ implies 60% equity destruction upon failure.

Table 1: Baseline Calibration

Parameter	Description	Value
Households		
β	Discount factor	0.960
γ	CRRA coefficient	1.000
η	Labor disutility weight	0.650
χ	Frisch elasticity	1.000
Firms		
α	Capital share	0.330
δ	Depreciation rate	1.000
Banking		
Φ_A	Intermediation scale	1.000
Φ_B	Intermediation curvature	0.900
ζ_1	Deposit cost (linear)	0.000
ζ_2	Deposit cost (quadratic)	0.500
ϕ_0	Leverage intercept	0.010
ϕ_1	Leverage slope	1.500
Fragility		
κ_1	Hazard slope (capital ratio)	-6.86
κ_0	Hazard intercept	0.47
φ	Distress recovery loss	0.60
Aggregate TFP shock		
A	TFP grid	$\{0.98,\ 1.02\}$
Π_A	TFP transition	$\begin{pmatrix} 0.90 & 0.10 \\ 0.10 & 0.90 \end{pmatrix}$

Notes: Lending technology: $\Phi(l) = \Phi_A\, l^{\Phi_B}$. Deposit cost: $\xi(B) = \zeta_1 B + \zeta_2 B^2$. Leverage: $B \leq \phi_0 + \phi_1 N$. Hazard: $\mathcal{H} = \sigma(\kappa_1\, N/L + \kappa_0)$, where $\sigma(\cdot)$ is the logistic sigmoid and $N/L = N/(N+B)$. κ_1 from Jordà et al. (2017) logit (annual, capital ratio).

3.2 Solution method

We solve the model using the Repeated Transition Method (RTM) of Lee (2025). The key computational challenge is tracing counterfactual paths to evaluate the conditional expectations in the Euler equation (11) accurately. The algorithm proceeds as follows:

1. *Backward step.* Given aggregate allocations, the bank's Euler equation (11) is solved for the optimal equity choice $n'(N; X)$ using iso-shock matching, simultaneously determining the household's labor supply and the deposit market equilibrium.

2. *Forward step.* The economy is simulated using the resulting policy functions and aggregate allocations are updated via damped iteration.

3. *Convergence.* The two steps alternate until the mean squared error between successive aggregate allocations falls below 10^{-8}.

The simulation uses $T = 5{,}001$ periods with a burn-in of 500. The deterministic steady state (computed under $\mathcal{H} = 0$) serves as the initial guess; the stochastic equilibrium endogenously determines positive hazard rates.

3.3 Conditional saddle paths

Following Lee (2026), we analyze the model's equilibrium dynamics through *conditional saddle paths*—the stochastic analogue of deterministic phase diagrams. Fix a TFP regime $A \in \{A_L, A_H\}$ and an initial equity level N_0. The *conditional saddle path* $\mathcal{M}(A; N_0) := \{(N_t, C_t)\}_{t\geq 0}$ is the orbit generated by the policy function $n'(N_t; A)$ and the consumption mapping $C(N_t, A)$ under the frozen regime A—no TFP switches, no bank failures. Crucially, these objects are computed from the decision rules of the *stochastic* equilibrium: agents correctly anticipate future regime switches and bank failures, so the saddle paths embed forward-looking expectations even though the exogenous state is held fixed. The limit point $(N_A^{\text{CS}}, C_A^{\text{CS}})$, the fixed point of $n'(\cdot\,; A)$, is the *conditional steady state* (CSS). Table 2 reports CSS values; Figure 1 plots the saddle paths.

Table 2: Conditional Steady States

		N_{css}	C_{css}
CE (Competitive)	$A = A_L$ (Recession)	0.0305	0.2128
	$A = A_H$ (Expansion)	0.0424	0.2400
SPP (Social Planner)	$A = A_L$ (Recession)	0.0537	0.2130
	$A = A_H$ (Expansion)	0.0721	0.2429

The conditional saddle paths decompose all equilibrium fluctuations into two types of movements. *Across-saddle* jumps occur when the exogenous state switches: since bank equity N is predetermined, a TFP switch from A_H to A_L corresponds to a vertical jump

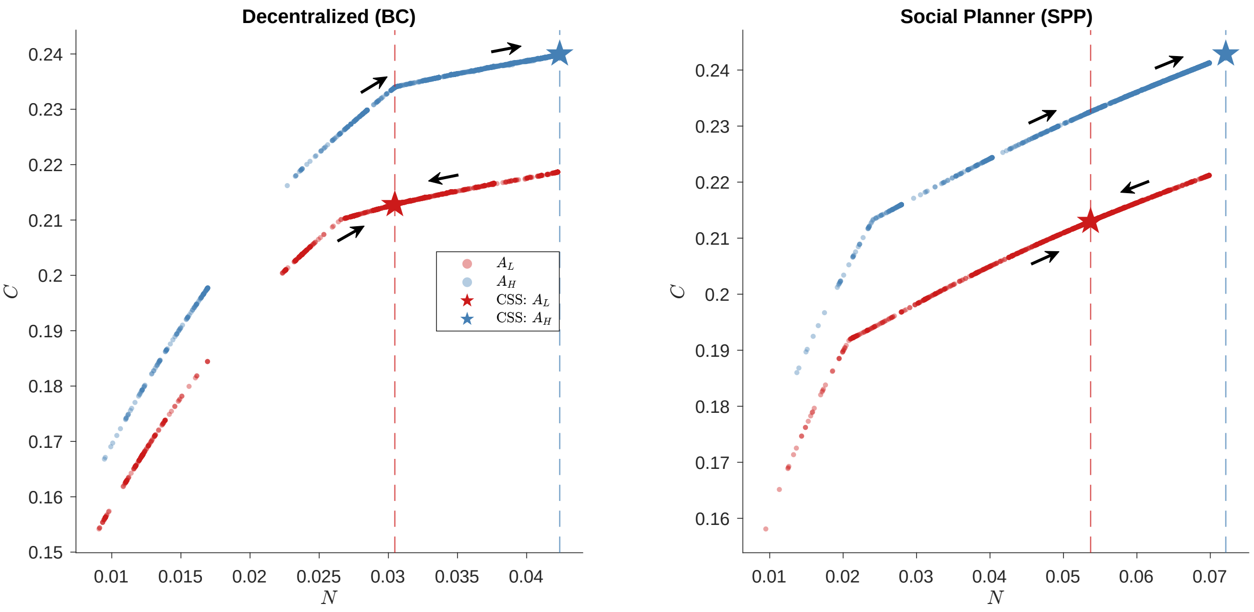

Figure 1: Conditional saddle paths: CE vs. SPP.

Notes: Conditional saddle paths under frozen TFP regimes, computed from the globally solved model. Stars mark conditional steady states. Panel (a): CE. Panel (b): SPP. In the SPP, the planner's recession CSS ($N = 0.0537$) exceeds the CE expansion CSS ($N = 0.0424$).

from the expansion saddle to the recession saddle at the same N. *Along-saddle* movements capture the endogenous propagation that follows: once on a given saddle, the economy converges toward the corresponding CSS through the equilibrium policy function. Together, these two forces—exogenous transitions across saddles and endogenous propagation along them—trace out the full stochastic equilibrium path in the (N, C) plane.

The across-saddle gap measures the *credit exposure channel*. The vertical distance

$$\Delta C(N) \;:=\; C(N, A_H) - C(N, A_L) \tag{14}$$

gives the impact effect of a TFP switch on consumption. Because the expansion saddle is steeper—higher TFP amplifies the marginal return to bank-intermediated credit—$\Delta C(N)$ widens with N. This generates both size and duration dependence: longer booms push N toward $N^{\text{CS}}_{A_H}$, widening the gap at the point of reversal. Geometrically, the slope difference between the two saddles is the primitive source of state-dependent amplification: if the saddles were parallel, TFP shocks would produce identical consumption drops regardless of capitalization.

The along-saddle dynamics encode the *fragility buffer channel*. A bank failure displaces the economy leftward along the active saddle path by destroying a fraction φ of equity. How much this displacement costs depends on the saddle's local curvature. The occasionally binding leverage constraint (3) creates a kink at intermediate equity levels, dividing the state space into two regimes. Above the kink, equity is abundant relative to the constraint, deposits adjust freely, and losses are absorbed smoothly—the economy operates as if unconstrained. Below the kink, the constraint binds, deposits are rationed, and each unit of lost equity forces a contraction in lending that propagates to output: a credit crunch. The kink is the economy's "fault line"—crossing it transforms a manageable equity loss into a self-reinforcing deleveraging spiral. In the CE, the constraint binds roughly 20% of the time. The planner shifts both saddle paths rightward (Table 2), keeping the economy above the kink for the vast majority of periods (binding only 2% of the time) and neutralizing this amplification channel.

3.4 Generalized impulse response functions

We compute state-dependent generalized impulse response functions (GIRFs; Koop et al., 1996):

$$\text{GIRF}_t = \mathbb{E}[\text{path}_t \mid \text{shock},\ N_0] - \mathbb{E}[\text{path}_t \mid \text{no shock},\ N_0] \tag{15}$$

expressed as a percentage of steady-state values, with expectations over 1,000 Monte Carlo paths. We condition on N_0 at the 10th percentile (fragile) and 90th percentile (well-capitalized) of the ergodic distribution.

3.4.1 TFP shock

The TFP shock GIRF compares paths starting in recession (A_L) against expansion (A_H), expressed as the percentage difference relative to the counterfactual (no-shock) path: $100 \times (\text{shocked} - \text{unshocked})/\text{unshocked}$.[2] Figure 2 confirms that well-capitalized banks amplify TFP shocks: starting from high N, the proportional output decline is roughly twice as large as from low N (-10% vs. -6% at impact). The mechanism is a credit multiplier operating in reverse—the very lending that boosts output in good times magnifies the contraction in bad times.

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[2] The two paths draw independently from the Markov transition matrix for future TFP but share uniform draws for run realizations to reduce Monte Carlo variance. The counterfactual normalization ensures that the GIRF captures the *proportional* impact of the shock, removing level differences across conditioning states.

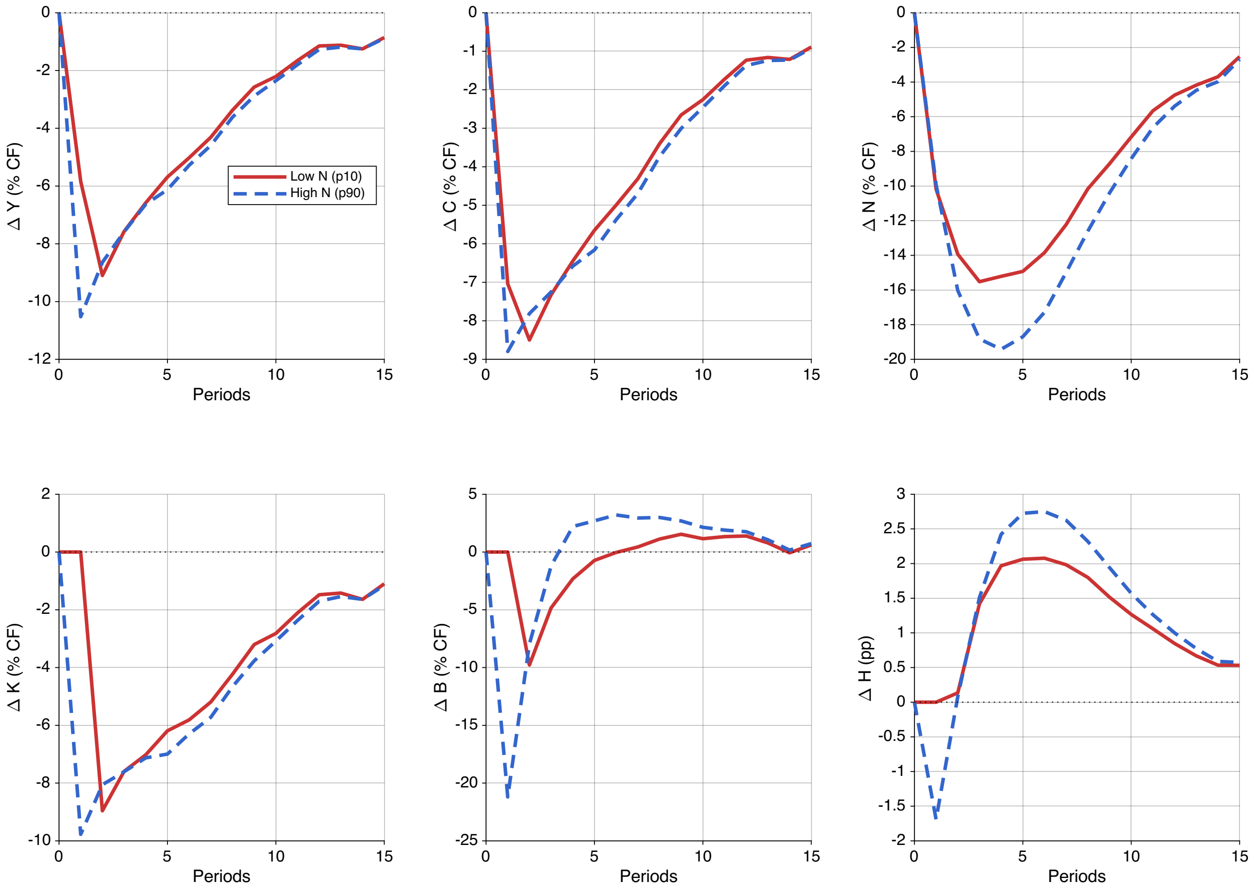

Figure 2: State-dependent GIRFs: negative TFP shock.

Notes: GIRF expressed as percentage difference relative to the counterfactual (no-shock) path. Red solid: low N_0 (p10); blue dashed: high N_0 (p90).

3.4.2 Bank hazard shock

The hazard shock GIRF isolates the financial channel. Both paths share the same TFP sequence, but the shocked path forces a bank failure at $t = 1$ ($\varphi = 60\%$ equity loss). From $t \geq 2$, failures are drawn endogenously with shared uniform draws. Under the counterfactual normalization, the equity panel shows a uniform -60% initial drop across all conditioning states—the mechanical destruction is identical—isolating the *recovery dynamics* as the economically informative comparison. Figure 3 shows fragility amplification during expansion: starting from low N, the proportional output decline (-30%) is far deeper than from high N (-5%), and the recovery is slow because the equity loss pushes the economy below the leverage kink, triggering a credit crunch. Starting from high N, the same proportional equity loss is absorbed within the unconstrained region, and the economy recovers within a few periods. Figure 4 shows that damage is smaller in recession, as banks are already partially deleveraged and closer to the kink regardless of initial capitalization.

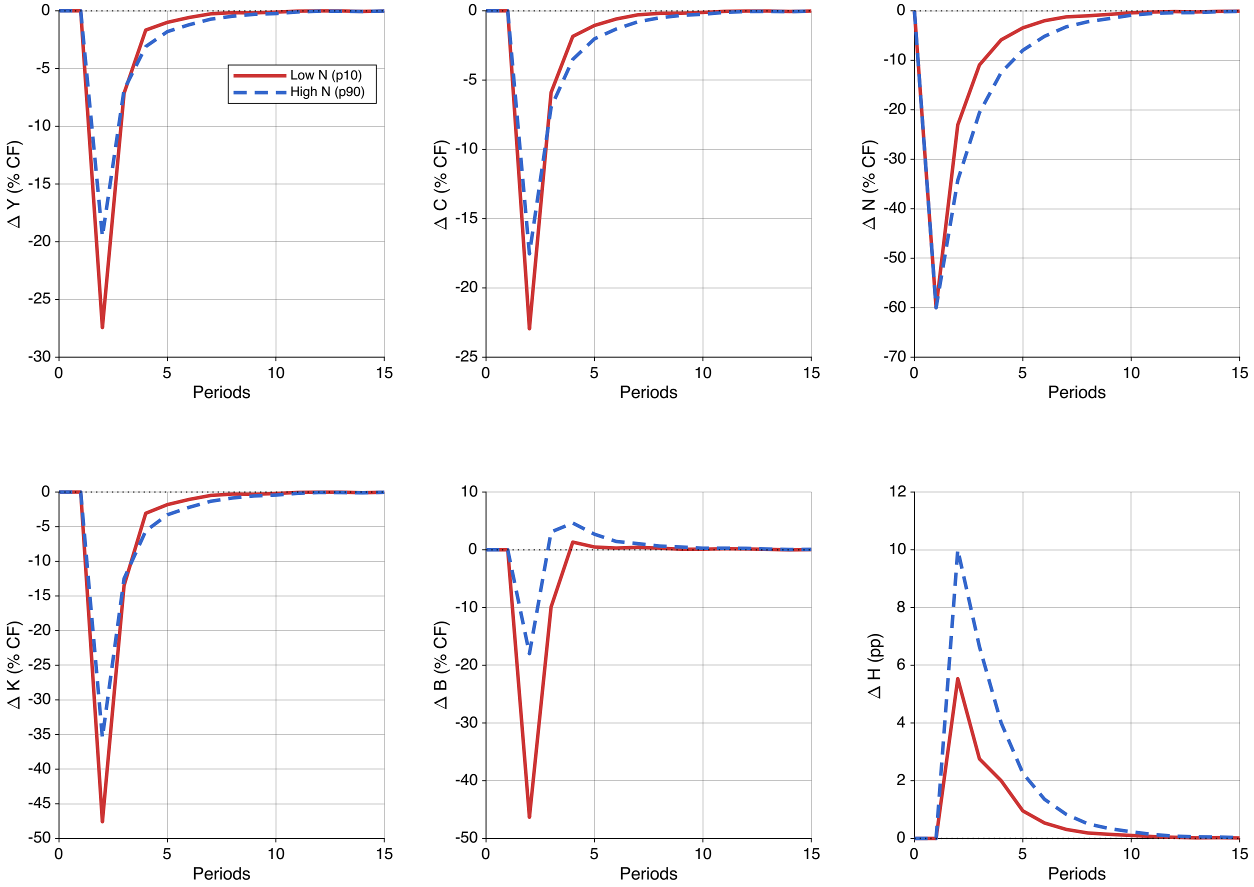

Figure 3: State-dependent GIRFs: bank hazard shock (expansion).

Notes: Both paths share the same TFP sequence starting in expansion. The shock forces a bank failure at $t = 1$. Red solid: low N_0 (p10); blue dashed: high N_0 (p90). GIRF expressed as percentage difference relative to the counterfactual (no-shock) path.

3.4.3 CE vs. SPP

We now overlay the CE and SPP impulse responses to isolate the planner's fragility correction.

Figure 5 compares TFP shock responses, conditioning both CE and SPP on the *same* initial equity levels (the CE's 10th and 90th percentiles) to ensure a like-for-like comparison.[3] Two features emerge. First, in output and consumption (Y, C panels), the CE and SPP responses are similar but not identical: at low N, the SPP's proportional output drop

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[3] If each economy is conditioned on its own percentiles, level differences between CE and SPP confound the comparison. Conditioning on common initial N isolates the effect of the planner's policy function at the same starting point.

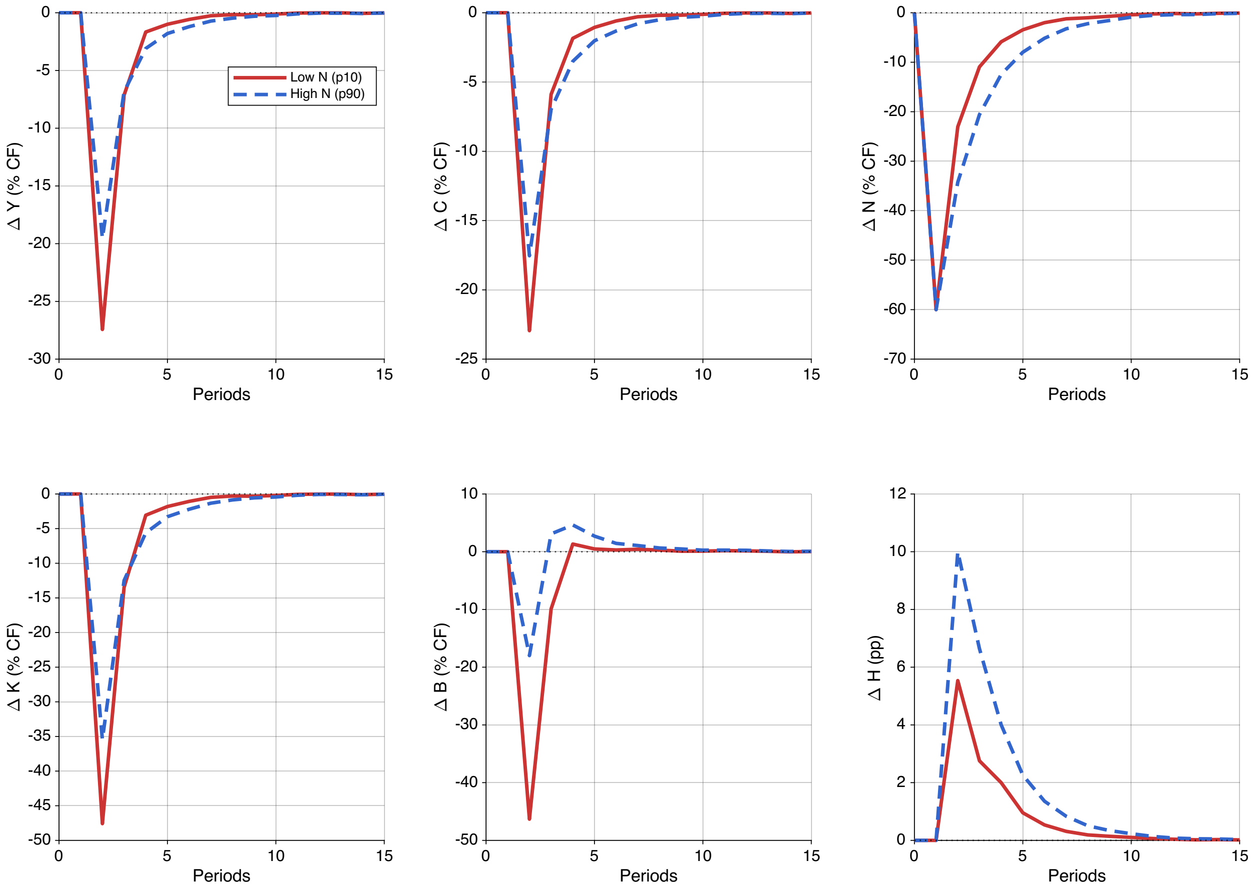

Figure 4: State-dependent GIRFs: bank hazard shock (recession).

Notes: Same construction as Figure 3, with both paths starting in recession.

($\approx -9\%$) is slightly larger than the CE's ($\approx -6\%$). This is the capitalization dilemma in its precise form: at the same equity level, the planner's allocation creates *more* proportional TFP exposure. The mechanism is not that the SPP intermediates more—it actually intermediates slightly less (lower B, lower L)—but that the planner's Pigouvian tax replaces the binding leverage constraint with a smooth price, giving the bank more freedom to adjust its balance sheet in response to TFP. In the CE, the binding constraint acts as a straitjacket that limits both upside and downside; the SPP's voluntary restraint unravels during stress, producing wider swings.

Second, the deposit panel (ΔB) reveals a sharp CE–SPP asymmetry. At both conditioning states, the SPP's deposits respond more dramatically than the CE's—rising in relative terms as the planner's unconstrained bank adjusts freely, while the CE bank's deposits are pinned by the binding constraint. This flexibility is the mirror image of the fragility benefit: the same freedom that allows the planner to stabilize fragility makes the economy more responsive to TFP.

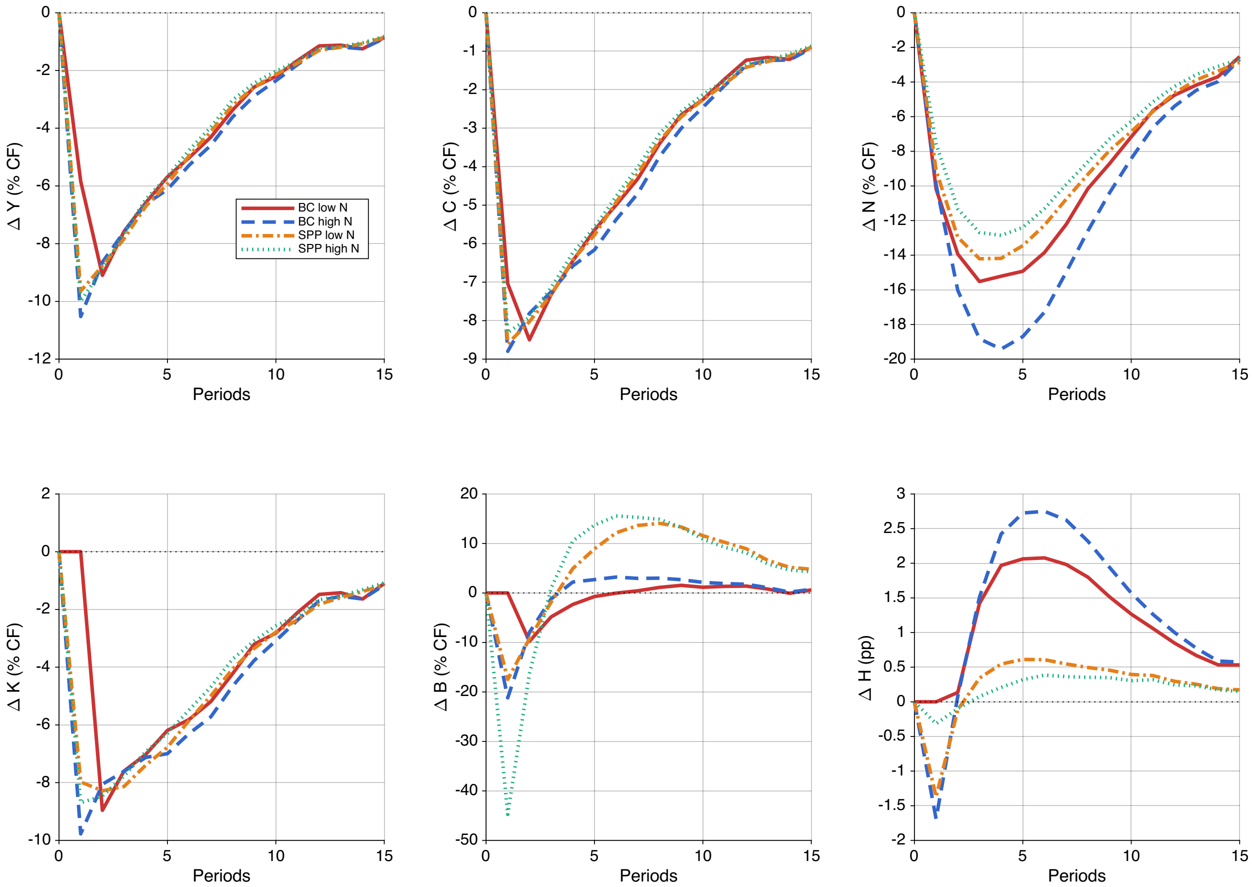

Figure 5: TFP shock GIRFs: CE vs. SPP.

Notes: CE low N_0 (red solid), CE high N_0 (blue dashed), SPP low N_0 (orange dash-dot), SPP high N_0 (teal dotted). Both CE and SPP are conditioned on the CE's 10th and 90th percentiles of N. GIRF expressed as percentage difference relative to the counterfactual (no-shock) path.

The contrast is stark for financial shocks. Figure 6 overlays the expansion-state hazard shock responses. Under the counterfactual normalization, the equity panel shows a uniform -60% initial drop for all four lines—the mechanical destruction is identical—so the figure isolates the recovery dynamics. In the CE, the fragile bank (low N_0) suffers a deep, persistent proportional output contraction (-30%) as the leverage constraint binds and deleveraging amplifies the initial equity destruction. The planner's economy, starting from the same N_0, absorbs the same proportional equity loss but recovers much faster: the constraint remains slack (because the Pigouvian tax has already induced lower leverage), so the bank can rebuild its balance sheet without hitting the hard cap. The deposit panel is particularly striking: CE deposits collapse (-70%), while SPP deposits *increase* ($+80\%$ at high N) as the unconstrained bank expands intermediation to exploit the post-crisis recovery.

Figure 7 shows that this pattern persists in recession, though the CE–SPP gap nar-

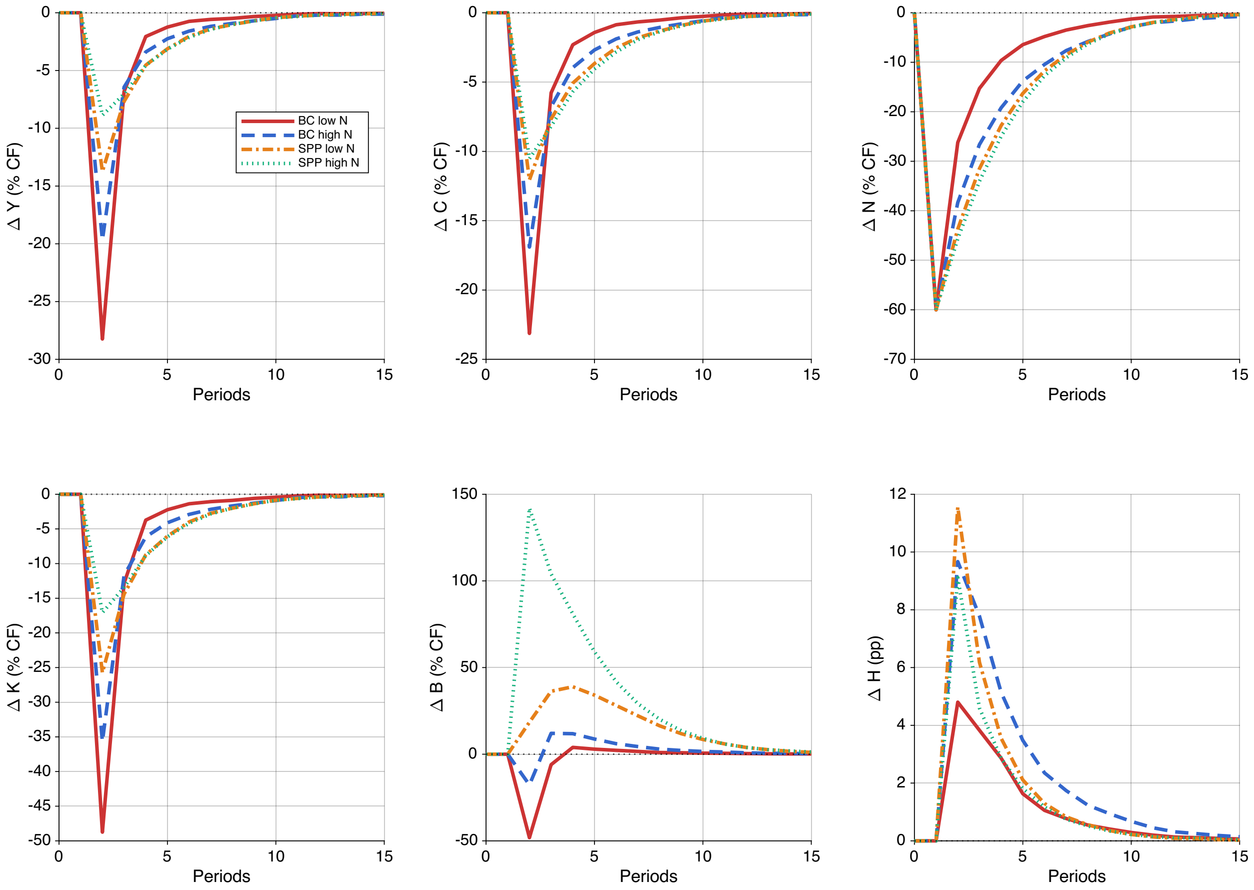

Figure 6: Hazard shock GIRFs: CE vs. SPP (expansion).

Notes: CE low N_0 (red solid), CE high N_0 (blue dashed), SPP low N_0 (orange dash-dot), SPP high N_0 (teal dotted). GIRF expressed as percentage difference relative to the counterfactual (no-shock) path.

rows. In recession, lower TFP compresses intermediation profits and both economies have drawn down equity, so even the CE bank is closer to the constrained region—the planner's insurance margin shrinks. Taken together, the three comparisons reveal the capitalization dilemma in its full form. The planner's Pigouvian correction replaces the rigid leverage constraint with a flexible price, which is decisive for financial shocks (the unconstrained bank recovers faster and avoids the credit crunch) but creates wider proportional swings in response to TFP shocks (the unconstrained bank adjusts more freely in both directions). This is the prices-versus-quantities trade-off of Weitzman (1974) applied to macroprudential policy: the price instrument is welfare-superior for the fragility margin but sacrifices the incidental TFP stabilization that the binding quantity constraint provided. Section 4 discusses the mechanisms further.

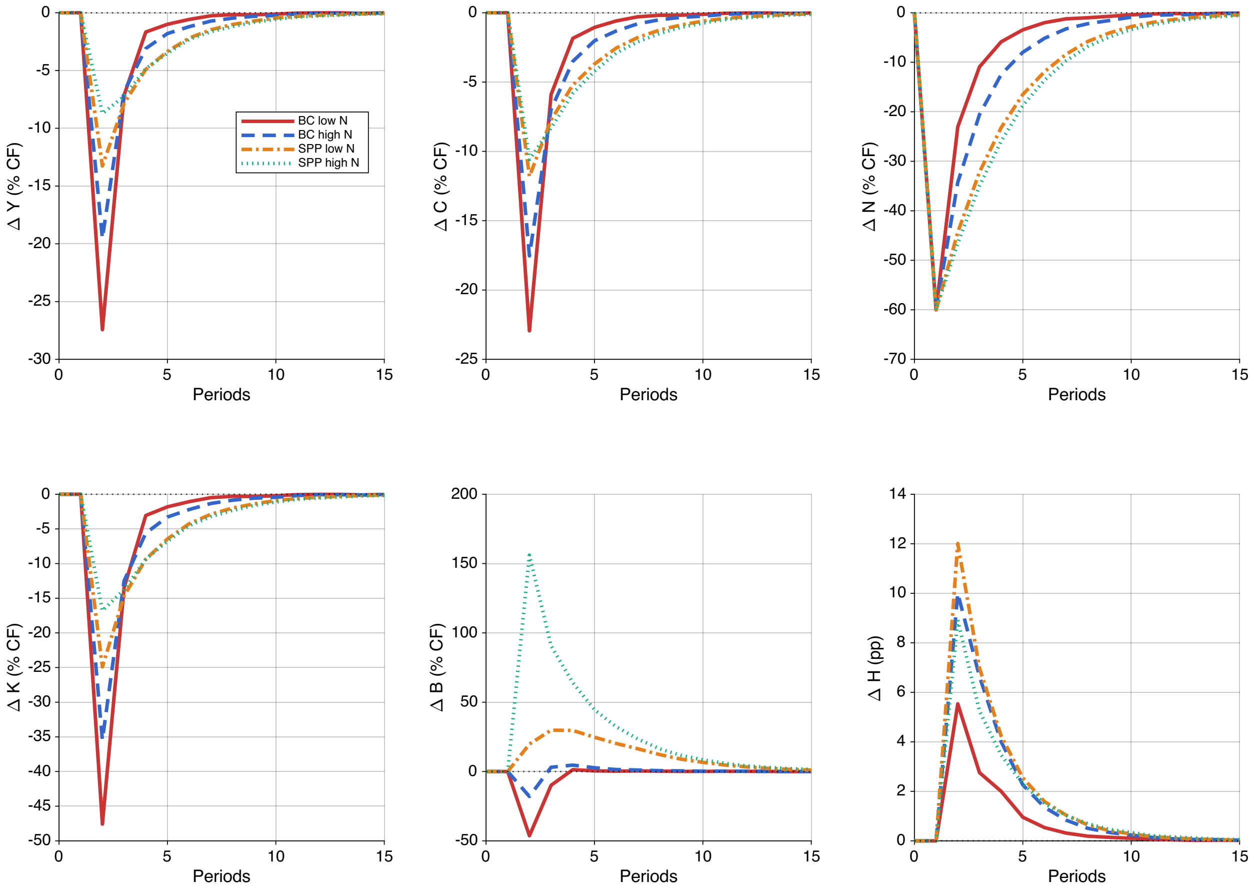

Figure 7: Hazard shock GIRFs: CE vs. SPP (recession).

Notes: Same construction as Figure 6; both paths start in recession.

3.5 Ergodic distributions and welfare

The ergodic distributions are sharply separated: the CE occupies $N \in [0.009, 0.042]$, the SPP occupies $N \in [0.021, 0.072]$. The two economies effectively inhabit different regions of the state space. The planner's recession floor ($N \approx 0.021$) exceeds the CE's typical recession equity, keeping the economy above the leverage kink. Both economies achieve similar mean consumption; the welfare gain comes not from higher average output but from eliminating the left tail—the rare but costly episodes in which the leverage constraint binds and deleveraging spirals amplify distress.

Table 3 reports business cycle moments. The planner maintains higher mean equity, lower mean hazard ($\bar{\mathcal{H}}$), and lower crisis frequency (2.1% vs. 8.9%), while mean output and consumption are similar. A telling difference is in the higher moments: the CE displays strongly negative skewness in K, N, and H (reflecting occasional sharp contractions when the leverage constraint binds), whereas the SPP substantially compresses these left tails. The planner trades a small reduction in average dividends for a large reduction in downside risk—a classic insurance motive. The consumption-equivalent variation (CEV) is the permanent proportional increase in composite consumption $\widetilde{C}_t = C_t - \eta(H_t^S)^{1+1/\chi}/(1+1/\chi)$ that makes the CE household indifferent to the SPP allocation:

$$\lambda_{\text{CEV}} = \exp(\bar{u}_{\text{SPP}} - \bar{u}_{\text{CE}}) - 1 \tag{16}$$

where $\bar{u} = T^{-1} \sum_t \log \widetilde{C}_t$ is the average per-period utility over the ergodic path.

Table 3: Business Cycle Moments: CE vs. SPP

	Competitive Equilibrium			Social Planner		
Variable	Mean	Std. Dev.	Skewness	Mean	Std. Dev.	Skewness
Y	0.3255	0.0311	-1.362	0.3290	0.0222	-0.333
C	0.2216	0.0190	-1.205	0.2239	0.0145	-0.328
K	0.1012	0.0146	-2.124	0.1030	0.0087	-1.004
H	0.5785	0.0288	-1.532	0.5820	0.0198	-0.407
N	0.0334	0.0085	-0.909	0.0559	0.0125	-1.144
B	0.0452	0.0059	-1.492	0.0241	0.0069	+1.104
N/L	0.4184	0.0611	-0.131	0.6909	0.1160	-1.545
R^K	1.0743	0.1008	+3.103	1.0563	0.0380	+4.402
R^B	1.0452	0.0059	-1.492	1.0241	0.0069	+1.104
$\mathcal{H}$	0.0888	0.0344	+0.772	0.0208	0.0282	+3.156
Crisis frequency (%)		8.88			2.08	
CEV (%)		—			+1.0819	

Notes: Statistics computed over the ergodic distribution ($T = 5001$, burn-in $= 500$). CEV: consumption-equivalent variation of moving from CE to SPP. See Table 4 for the implied Pigouvian tax.

4 Discussion

4.1 The capitalization dilemma

The saddle path decomposition in Section 3.3 maps the two individually well-known channels into a single geometric framework, making their interaction transparent.

The *credit exposure channel*—well documented in the financial accelerator literature—operates through the across-saddle gap $\Delta C(N)$. Higher bank equity supports more intermediation, making the economy more productive but also more exposed to TFP fluctuations. This is a "sowing the seeds" mechanism: credit booms during expansions simultaneously raise output and build vulnerability to productivity reversals. Since $\Delta C(N)$ widens with N, longer booms generate mechanically deeper busts. This channel is symmetric across the CE and SPP because both economies face the same intermediation technology.

The *fragility buffer channel*—studied in the banking crisis literature—operates through along-saddle displacement and the leverage kink. Lower initial equity means a distress event pushes the economy into the steep, constrained region of the saddle path, where deleveraging amplifies the initial equity loss. This channel is asymmetric across regimes because it depends on the *distance* from the kink, which the planner—but not the decentralized bank—can control. The market failure is a pecuniary externality: each bank's leverage choice is privately optimal, but the resulting aggregate capital ratio determines the system-wide failure probability that all banks face.

The unified framework reveals a tension that is invisible when the channels are studied separately: bank equity is the single state variable through which both channels operate, and any policy that adjusts it necessarily affects both. This has a direct implication for capital regulation. Tightening leverage requirements (raising ϕ_1) reduces credit exposure by compressing balance sheets and lowering $\Delta C(N)$, but also limits equity accumulation during expansions, potentially leaving the economy more exposed to distress. The regulator faces a frontier, not a free lunch: moving the economy rightward along the saddle path (more equity, less fragility) necessarily steepens the across-saddle gap (more TFP exposure).

The nature of this trade-off, however, depends critically on the *instrument* used to correct the fragility externality. The SPP internalizes the externality through a state-contingent Pigouvian tax $\tau(X)$ that raises the marginal cost of deposits. This tax *substitutes* for the leverage constraint: at the planner's allocation, the constraint shadow value λ falls sharply (from 0.066 in the CE to 0.003 in the SPP at the same equity level N), because the tax has already induced the bank to reduce leverage voluntarily. The bank no longer needs the hard cap to restrain its deposit-taking—the price signal does the work.

This substitution creates a subtle but genuine version of the capitalization dilemma. In the CE, the binding leverage constraint acts as a *straitjacket*: it prevents the bank from expanding in good times, but it also limits how far the bank can contract in bad times. The constraint inadvertently stabilizes the economy's response to TFP fluctuations by compressing the range of intermediation volumes. The planner's Pigouvian tax removes this rigidity. With the constraint nearly slack, the bank is free to adjust its balance sheet in response to TFP—expanding more aggressively in expansions, but also contracting more sharply in recessions. The result is a wider across-saddle gap for the SPP than for the CE at the same equity level: the proportional output decline from a negative TFP shock is larger under the planner's allocation, not because the planner intermediates more at a given N, but because the bank's *flexibility to respond* is greater when the hard constraint is replaced by a smooth price.

This mechanism connects to a classic insight in the regulation literature. Weitzman (1974) showed that price and quantity instruments yield different welfare outcomes when the relevant functions are nonlinear. Here, the nonlinearity is in the hazard function (4): the marginal cost of leverage rises steeply as the capital ratio falls. The Pigouvian tax—a price instrument—tracks this nonlinearity precisely, providing the optimal correction at every state. The leverage constraint—a quantity instrument—provides a cruder but more rigid correction that happens to stabilize TFP responses as a by-product. The capitalization dilemma is the cost of moving from the rigid instrument to the precise one: the planner gains fragility protection but loses the incidental TFP stabilization that the binding constraint provided.

The dilemma has an important temporal dimension. Ex ante, the planner's allocation is welfare-maximizing: no alternative policy can deliver higher expected lifetime utility. But ex post—conditional on the realized sequence of shocks—the competitive equilibrium can outperform the planner along particular sample paths. During prolonged expansions without bank failures, the CE economy produces more output and higher consumption because it is not holding the extra equity buffer that the planner demands. The planner's higher capital requirement acts as an insurance premium: it depresses dividends and credit intermediation in normal times, and its payoff materializes only when crises actually occur. In a fortunate history where no distress events strike, the premium is paid but never collected. This ex-post reversal is not a deficiency of the planner's policy but a defining feature of insurance: any policy that reduces tail risk must sacrifice expected payoffs in the states where the insured event does not occur. For the central banker, this creates a political economy challenge—the costs of higher capital requirements are visible in every period, while the benefits are concentrated in rare crises that may not materialize during a policymaker's tenure.

4.2 Precautionary savings as macroprudential policy

The planner's first-order condition differs from the bank's by the Pigouvian wedge $\tau(X)$ in (13), which reflects the marginal effect of leverage on failure probability. This wedge generates three reinforcing consequences. First, SPP conditional steady states exceed CE values by 76% in recession (0.0537 vs. 0.0305) and 70% in expansion (0.0721 vs. 0.0424), reflecting substantially higher equity targets. Second, these higher equity buffers prevent distress events from triggering the leverage constraint; deposits remain stable (ΔB GIRF ≈ 0), averting the credit contraction that drives CE output losses. Third, the SPP's post-distress equity floor ($N \approx 0.021$) exceeds the CE floor ($N \approx 0.009$) by a factor of 2.3, placing the economy above the leverage kink even after distress and enabling recovery along the flat portion of the saddle path without the amplification that prolongs CE recessions.

These mechanisms are mutually reinforcing: higher equity targets enable deposit stabilization, which prevents the constraint from binding and ensures fast recovery. In the CE, the externality operates through a tragedy-of-the-commons logic: each bank rationally minimizes its own equity to maximize dividends, but the collective outcome is a fragile system in which every bank is exposed to crises that none would individually choose.

The calibrated model pins down the implied Pigouvian tax. The wedge $\tau(X)$ enters the deposit margin in the same units as a net interest rate: it represents the per-unit tax on deposits that would induce banks to replicate the planner's equity choice.

Table 4: Implied Pigouvian Tax

	τ
Mean	0.0394
$A = A_L$ (Recession)	0.0412
$A = A_H$ (Expansion)	0.0378
Std. Dev.	0.0277

Notes: The Pigouvian wedge $\tau(X)$ from (13), in net-rate units (same as $R^B - 1$). Statistics over the ergodic distribution.

The wedge is countercyclical, rising in recessions—when the capital ratio is low and the hazard function is steep—and falling in expansions. The key insight is what this tax *buys* in terms of the saddle-path geometry. The CE's recession CSS ($N = 0.0305$) sits right at the leverage kink, so a single distress event pushes the economy into the steep, constrained region where deleveraging spirals take hold. The planner's tax shifts both conditional steady states well above this zone: the SPP recession CSS ($N = 0.0537$) exceeds even the CE *expansion* CSS ($N = 0.0424$). The planner's worst-case anchor point already lies beyond the best-case anchor of the unregulated economy.

This separation from the kink is not a knife-edge property of the calibration but arises from a self-reinforcing mechanism in the structure of τ itself. The Pigouvian wedge is *endogenously amplifying*: it rises sharply as the economy approaches the constrained region, because proximity to the kink simultaneously steepens the hazard function $\partial\mathcal{H}/\partial B$ and widens the value gap $V(n') - V((1-\varphi)n')$ in (13). In the calibrated model, τ quadruples from its ergodic mean of 0.04 to roughly 0.15 when the leverage constraint binds. This sharp increase in the cost of deposits near the kink acts as an endogenous "guardrail": the closer the economy drifts toward the constrained region, the more aggressively the planner's wedge pushes back by penalizing leverage, creating a restoring force that is absent in the CE.

That said, the guardrail does not provide absolute protection. Sufficiently large or repeated distress events can still drive the SPP economy into the constrained region—the constraint binds approximately 2% of the time even under the planner's policy, compared with 20% in the CE. But when it does bind, the starting point matters enormously. In the CE, the economy enters the constrained zone from a low equity base ($N \approx 0.009$), deep in the steep portion of the saddle path where each unit of equity loss triggers further deleveraging. The planner's economy enters the same zone from a much higher base ($N \approx 0.021$), close to the kink's boundary rather than deep within it, enabling a faster exit. The constraint binds *briefly and mildly* rather than *persistently and severely*: the planner converts what are prolonged credit crunches in the CE into transient episodes that resolve within a few periods.

The magnitude of τ reflects this geometric logic. The wedge is large—comparable in size to the CE deposit spread $R^B - 1$—because the externality it corrects is severe: in the CE, the leverage constraint binds frequently, amplifying every distress event into a prolonged contraction. The standard deviation of τ (Table 4) is large relative to its mean, reflecting the highly nonlinear nature of the hazard function: the marginal value of equity as a crisis buffer rises sharply as the capital ratio declines toward the steep region of the sigmoid (4).

In a decentralized implementation, a regulator could replicate the planner's allocation by levying a state-contingent tax $\tau(X)$ on deposit issuance and rebating the proceeds lump-sum to households. The tax raises the effective cost of deposits from $R^B - 1$ to $R^B - 1 + \tau$, inducing banks to substitute toward equity financing and thereby internalizing the fragility externality. Because the wedge varies with the aggregate state $X = (N, A)$, a fixed capital surcharge would only approximate the planner's optimum; the time variation in τ provides a quantitative benchmark for the state-contingency that countercyclical capital buffers should exhibit.

For TFP shocks, however, the planner faces the same technology constraint. Productivity shocks do not trigger failures, so the fragility externality is not activated and the planner cannot improve upon the market outcome. This clarifies the scope of macroprudential policy: it is effective where the market failure lies—in the systemic undervaluation of equity as a buffer against financial distress—and offers no advantage where shocks originate in the real sector.

4.3 Leverage limits vs. Pigouvian taxation

The Pigouvian tax $\tau(X)$ is a state-contingent instrument that is informationally demanding: it requires the regulator to observe the aggregate state and compute the marginal externality in real time. A simpler alternative is to tighten the leverage constraint directly by lowering ϕ_1. This exercise isolates what a blunt, time-invariant capital requirement can and cannot achieve.

Figure 8 compares the conditional saddle paths under three regimes: the baseline CE ($\phi_1 = 1.5$), a tighter leverage constraint ($\phi_1 = 0.8$), and the SPP. Tightening the constraint shifts both saddle paths rightward, raising the conditional steady states and moving the economy away from the kink—qualitatively similar to the Pigouvian tax. However, the mechanism differs in two important respects.

First, the tight constraint operates by *compressing credit across all states*. By capping deposits at a lower multiple of equity, it forces banks to shrink their balance sheets even when the economy is well-capitalized and the fragility externality is negligible. This flattens the saddle paths, reducing the across-saddle gap $\Delta C(N)$ and dampening output in normal times. The SPP, by contrast, achieves its equity targets through a state-contingent wedge that is small when the economy is healthy and large only near the kink. The planner's saddle paths retain their slope—credit intermediation is unimpaired in good states—while selectively steepening the penalty for leverage when it matters.

Second, the tight constraint cannot fully replicate the planner's allocation because it addresses the *symptom* (excessive leverage) rather than the *cause* (the uninternalized hazard externality). The constraint binds 13% of the time under the tight policy, versus 2% under the SPP. When it binds, it prevents deleveraging spirals—but it also prevents the economy from reaching the efficient level of intermediation. The SPP achieves lower binding frequency with higher mean output, revealing the efficiency cost of the blunt instrument.

The comparison highlights a general principle: leverage limits and Pigouvian taxes are complementary rather than substitutable instruments. Leverage limits provide a ro-

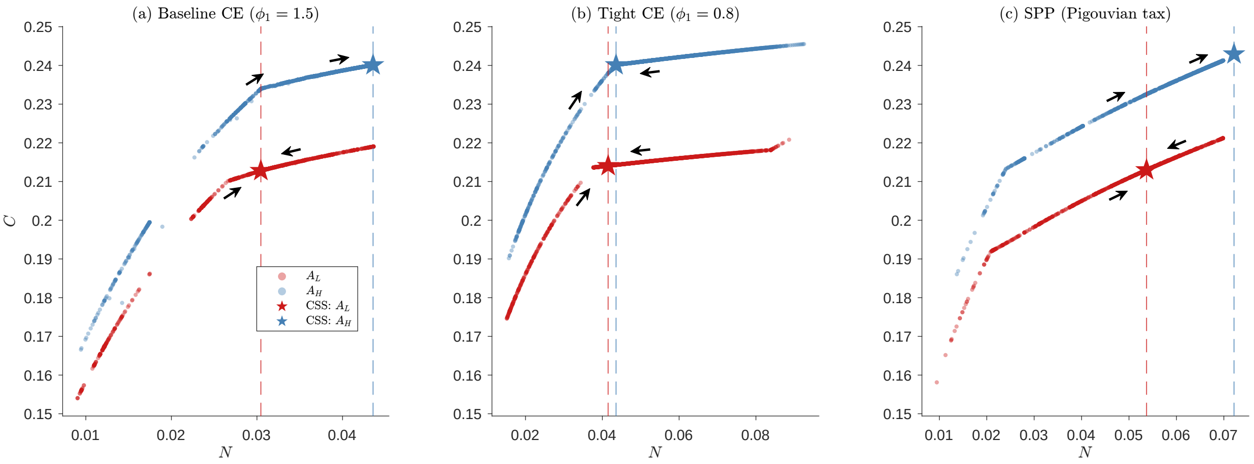

Figure 8: Conditional saddle paths: baseline CE, tight leverage constraint, and SPP.

Notes: Saddle paths under frozen TFP regimes. Stars mark conditional steady states. Panel (a): baseline CE ($\phi_1 = 1.5$). Panel (b): tight CE ($\phi_1 = 0.8$). Panel (c): SPP. All panels share common axes for direct comparison. The tight constraint shifts saddle paths rightward but also compresses them vertically, reflecting reduced credit intermediation.

bust floor that does not require real-time state observation, but at the cost of compressing credit intermediation uniformly. The Pigouvian wedge targets the externality precisely but requires the regulator to track the aggregate state. In practice, a well-designed macroprudential framework would combine a time-invariant leverage floor (a relaxed version of the tight constraint) with a countercyclical buffer (approximating the state-contingent wedge) that activates as the system approaches the fragile region. The remainder of this section formalizes both ideas: Section 4.4 traces the policy frontier as ϕ_1 varies, and Section 4.5 derives a macroprudential rule from the planner's solution and interprets its structure.

4.4 Comparative statics of leverage policy

Before turning to state-dependent instruments, we trace the policy frontier over the *level* of the leverage constraint. Table 5 reports steady-state and ergodic outcomes as ϕ_1 varies from 0.50 (tight) to 2.00 (loose), holding all other parameters at their baseline values. Three regularities emerge.

First, tightening the constraint monotonically reduces fragility. The mean hazard rate falls from roughly 10% at $\phi_1 = 2.00$ to below 3% at $\phi_1 = 0.50$, and crisis frequency declines proportionally. The mechanism is direct: a lower ϕ_1 caps deposits at a tighter multiple of equity, raising the equilibrium capital ratio N/L and pushing the economy into the flat region of the sigmoid hazard function (4).

Second, the binding frequency of the constraint is highly nonlinear in ϕ_1. At $\phi_1 = 0.50$ the constraint binds roughly two-thirds of the time, acting as a near-permanent cap on intermediation. At the baseline $\phi_1 = 1.50$ it binds 20% of the time—only during distress episodes. This nonlinearity implies that a regulator setting a constant leverage limit faces a steep trade-off: small reductions in ϕ_1 from the baseline dramatically increase the fraction of periods with binding credit rationing.

Third, welfare improves monotonically as ϕ_1 falls within the range considered (CEV reaches +1.6% at $\phi_1 = 0.50$), suggesting that the baseline leverage limit is too permissive from a welfare perspective. However, the welfare gains from a tighter constant ϕ_1 come from a fundamentally different source than the SPP's Pigouvian correction: the constant constraint compresses credit *uniformly* across all states, whereas the SPP targets the externality *selectively*. A constant ϕ_1 that binds two-thirds of the time reduces the hazard but also prevents the economy from reaching efficient intermediation levels in states where fragility is low. This distinction motivates the search for a state-dependent instrument.

Table 5: Comparative Statics: Leverage Slope ϕ_1

ϕ_1	N_{ss}/L_{ss}	$\bar{\mathcal{H}}$	Binding (%)	$\sigma(Y)$	$\bar{Y}$	CEV (%)
0.50	0.4975	0.0277	66.8	0.0258	0.3310	+1.5559
0.75	0.4975	0.0437	27.3	0.0281	0.3286	+0.8478
1.00	0.4975	0.0582	20.3	0.0294	0.3274	+0.5096
1.25	0.4975	0.0732	19.8	0.0304	0.3265	+0.2532
1.50†	0.4975	0.0887	17.8	0.0311	0.3255	+0.0024
1.75	0.4975	0.1033	18.1	0.0314	0.3246	-0.2417
2.00	0.4975	0.1190	20.0	0.0316	0.3236	-0.4844

Notes: †Baseline calibration. $\bar{\mathcal{H}}$: mean ergodic hazard. Binding: fraction of periods with active leverage constraint. CEV: consumption-equivalent welfare change relative to baseline.

4.5 A macroprudential rule from the planner's solution

We now translate the globally-solved planner allocation into an implementable leverage rule. The rule separates two policy motives. The financial-cycle term tightens when bank equity is high. This restrains balance-sheet expansion that erodes sector-wide capitalization. The business-cycle term loosens in high-TFP states, when strong fundamentals reduce the fragility externality. Aggregate credit-gap rules such as the Basel III CCyB (Basel Committee on Banking Supervision, 2010) collapse these two channels into a single credit-growth response and hide the structure. The closest antecedent is the optimized macroprudential Taylor rule of Bianchi and Mendoza (2018), which targets a Pigouvian tax on debt in a small open economy. Our rule differs on two dimensions. The instrument is a leverage coefficient on bank equity rather than a debt tax, mapping directly to Basel-style capital requirements. The hazard function is calibrated empirically using the Jordà et al. (2017) crisis database. Separately from these differences, the global solution also matters: it estimates the rule over the nonlinear region around the leverage kink, where local approximations would understate the state dependence.

4.5.1 From the Pigouvian tax to a leverage instrument

The SPP's Pigouvian tax $\tau(X)$ corrects the fragility externality by entering two margins simultaneously: the lending FOC ($\Phi'(l)(R^K - 1) - (R^B - 1) = \lambda + \tau$) and the equity envelope condition ($J_1 = R^B + \lambda(1 + \phi_1) + \tau$). A leverage constraint, by contrast, operates through a single margin—it caps deposits—and cannot independently influence the equity retention decision. This "one instrument, two margins" limitation means that the leverage constraint generally cannot replicate the SPP allocation.

Nevertheless, the leverage constraint is the instrument available to regulators. A natural approach is to ask what leverage rule would *replicate the planner's deposit choice*. We proceed in two steps: first, extract the SPP-implied leverage tightness $\phi_1^*(N, A)$—the ϕ_1 that would make the constraint bind at exactly the SPP's deposit level; second, fit a parsimonious linear rule to ϕ_1^* and interpret its structure.

4.5.2 SPP-implied leverage tightness

From the SPP solution, define the state-contingent leverage tightness:

$$\phi_1^*(N, A) \ := \ \frac{B^{\text{SPP}}(N, A) - \phi_0}{N} \tag{17}$$

This quantity answers the following question: if the CE bank faced a leverage constraint with slope ϕ_1^* at state (N, A), at what level would the constraint bind to exactly match the SPP's deposit choice?

The ergodic distribution of ϕ_1^* reveals the gap between the CE and SPP in terms of the leverage instrument. At the SPP's ergodic distribution, ϕ_1^* averages 0.32—roughly one-fifth of the baseline CE value of 1.50. The standard deviation is 0.31, reflecting substantial state dependence. ϕ_1^* is near zero in recessions and rises to the CE cap of 1.50 in high-TFP states. This does not mean that low equity tightens the rule. Holding TFP fixed, low equity in fact *loosens* it via $\gamma_N < 0$. The tight recession values arise because the low-TFP channel (via $\gamma_A > 0$) dominates the offsetting low-equity channel. Interpreting γ_N therefore requires conditioning on TFP: it is a *partial* effect. Crucially, $\phi_1^* > 0$ everywhere—the planner never requires $B < \phi_0$—so the one-sided nature of the constraint is not a binding limitation.

4.5.3 A linear macroprudential rule

The SPP-implied tightness $\phi_1^*(N, A)$ varies smoothly with the aggregate state, suggesting that a simple rule can approximate it well. We fit a linear macroprudential rule by OLS:

$$\phi_1^*(N, A) \;\approx\; \bar{\phi}_1 + \gamma_N\left(\frac{N}{N_{\text{ss}}} - 1\right) + \gamma_A(A - 1) \tag{18}$$

where the regressors are the fractional deviation of equity from its steady state and the TFP deviation. The estimation uses observations from the SPP's post-burn-in ergodic simulation ($T = 5{,}001$ periods).

Table 6 reports the results. The OLS fit achieves an R^2 of 0.927; adding an interaction term raises it only modestly to 0.931, confirming that the linear specification captures the essential structure. The high explanatory power is itself a finding: the planner's complex, forward-looking policy—which solves a dynamic programming problem over the full state space—can be well approximated by a regulator who observes only two aggregate indicators and adjusts the leverage limit according to a fixed linear formula.

Table 6: SPP-Implied Macroprudential Rule

	Estimate
$\bar{\phi}_1$ (intercept)	0.687
γ_N (financial cycle: equity response)	-1.055
γ_A (business cycle: TFP response)	$+5.358$
R^2	0.927
R^2 (with interaction)	0.931

Notes: OLS regression of the SPP-implied leverage tightness $\phi_1^*(N, A) := (B^{\text{SPP}} - \phi_0)/N$ on $(N/N_{\text{ss}} - 1)$ and $(A - 1)$, using the SPP's ergodic simulation ($T = 5{,}001$). The interaction specification adds $(N/N_{\text{ss}} - 1)(A - 1)$ as a regressor. The negative γ_N tightens leverage when credit expands; the positive γ_A loosens it when fundamentals are strong.

4.5.4 Separating the financial cycle from the business cycle

The estimated coefficients reveal a fundamental decomposition of the planner's prudential stance into two orthogonal components:

- **Financial cycle** ($\gamma_N = -1.06$): The rule *tightens* the constraint when bank equity rises above its steady-state level and *loosens* it when equity falls below—a "lean against the credit cycle" motive. The sign may appear counter-intuitive, since well-capitalized banks are individually less fragile, but it captures a compositional externality. When aggregate equity N rises, each bank's balance sheet expands: lending $L = N + B$ grows, and since the leverage constraint permits deposits up to $\phi_0 + \phi_1 N$, the *absolute volume* of deposits rises even if the leverage ratio B/N remains constant. The resulting decline in the aggregate capital ratio $N/L = N/(N+B)$ pushes the economy into the steep region of the hazard function (4). The planner sees what individual banks do not: the collective buildup of balance sheets during credit booms systematically erodes the sector-wide capital ratio, raising fragility even as each bank appears well-capitalized in isolation. The negative γ_N is the planner's response—tighten as the credit cycle inflates, imposing a *speed limit* on aggregate balance sheet expansion.

- **Business cycle** ($\gamma_A = 5.36$): The rule *loosens* the constraint in expansions ($A > 1$) and tightens it in recessions ($A < 1$). This sign appears pro-cyclical, seemingly at odds with counter-cyclical capital buffer (CCyB) logic. The resolution lies in the structure of the Pigouvian tax (13). The tax $\tau \propto \frac{\partial \mathcal{H}}{\partial B} \cdot \mathbb{E}[V(n') - V((1-\varphi)n')]$ depends on two factors: the sensitivity of the hazard to leverage ($\partial\mathcal{H}/\partial B$) and the welfare cost of failure ($V(n') - V((1-\varphi)n')$). In expansions, both factors are small. Higher TFP raises intermediation profits, pushing the economy into the flat region of the hazard sigmoid where $\partial\mathcal{H}/\partial B$ is low; simultaneously, the value gap $V(n') - V((1-\varphi)n')$ narrows because the economy can recover quickly from distress when fundamentals are strong. The planner can therefore afford to loosen the constraint, allowing banks to intermediate more when doing so is least dangerous. In recessions, both factors rise sharply: the hazard function steepens as the economy approaches the kink, and the welfare cost of failure increases because recovery is slow when profits are compressed. The planner tightens accordingly.

The main implication is the *separation*: optimal macroprudential policy should track the financial cycle and the business cycle independently, because the two cycles call for opposite responses. A regulator who observes rising output and rising credit simultaneously receives conflicting signals. Output growth says "loosen—fundamentals are strong and the hazard is low"; credit growth says "tighten—balance sheets are expanding and systemic leverage is building." The rule (18) resolves this conflict quantitatively: each coefficient responds to its respective state variable with its own sign and magnitude.

This separation has direct implications for the design of the Basel III counter-cyclical capital buffer (CCyB). The CCyB responds to the credit-to-GDP gap. This gap is a single composite indicator that conflates the financial and business cycles.

Consider a recession with sticky credit. Credit-to-GDP rises mechanically as GDP falls, and the CCyB tightens. Our rule's TFP margin ($\gamma_A > 0$, $A < 1$) also tightens, because the marginal fragility externality is elevated when fundamentals are weak. But whether the *net* response is a tightening depends on the equity state. With $\gamma_N < 0$, the equity margin loosens when N falls below steady state. The composite CCyB cannot tell which channel is driving the signal.

Consider now a high-TFP expansion with ordinary balance-sheet growth. The CCyB triggers a tightening on credit-to-GDP. Our rule's TFP term pushes toward loosening, because strong fundamentals reduce the hazard externality. The equity term pushes toward tightening. The net response depends on which channel dominates—a distinction the composite cannot make.

By conditioning on N and A separately, the rule (18) lets the regulator distinguish credit growth driven by balance-sheet expansion (tighten via γ_N) from credit growth driven by strong fundamentals (loosen via γ_A).

4.5.5 The intercept: the planner's baseline stance

The intercept $\bar{\phi}_1 = 0.69$ deserves separate discussion. It is less than half the baseline CE value of 1.50, implying that the planner's *unconditional* leverage limit is substantially tighter than in the decentralized equilibrium. In the baseline calibration, $\phi_1 = 1.50$ is chosen so that the constraint binds approximately 20% of the time—a target motivated by the observation that financial constraints are not permanently binding. The planner's implied intercept of 0.69 would, if applied as a constant rule, cause the constraint to bind far more frequently.

This gap reflects the wedge between private and social returns to leverage. In the CE, banks choose leverage to maximize private franchise value, ignoring its effect on the aggregate hazard rate. The planner internalizes this externality and demands a tighter baseline—not because individual banks are imprudent, but because their collectively rational choices produce a systemically fragile outcome. The magnitude of the gap ($0.69/1.50 = 46\%$) measures the severity of the externality at the calibrated parameters: the planner would roughly halve the permitted leverage ratio.

4.5.6 Limitations of the leverage instrument

While the linear rule achieves a high R^2, it is important to emphasize what it does *not* accomplish. The leverage constraint controls deposits—one of two margins that the Pigouvian tax corrects. The other is equity retention: the choice of n'. In the SPP, the tax enters the envelope condition as $J_1^{\text{SPP}} = R^B + \lambda(1+\phi_1) + \tau$, raising the marginal value of equity and inducing the bank to retain more. A binding leverage constraint provides an indirect bonus to equity retention ($\lambda > 0$ raises $J_1 = R^B + \lambda(1+\phi_1)$), but this bonus is determined by the constraint's shadow value, not by the externality itself. Consequently, the bank's equity choice under the leverage rule differs from the planner's, and the resulting allocation does not replicate the SPP.

This "one instrument, two margins" limitation implies that no leverage rule—however carefully calibrated—can achieve the same outcome as the Pigouvian tax. The leverage instrument controls the *quantity* of deposits but not the *price* of the externality. The linear rule (18) is therefore best understood not as a substitute for the Pigouvian tax but as a *characterization* of what the planner would want the leverage instrument to do: how it should move with bank equity, how it should respond to fundamentals, and where the baseline should be set. It provides a structural benchmark against which actual regulatory rules can be evaluated.

4.5.7 The value of making the rule common knowledge

Having characterized the rule, a natural question is how much of its welfare content depends on agents *anticipating* its state-contingent structure—as opposed to merely facing its realized values. We compare two rational-expectations equilibria in which the same rule $\phi_1(N, A)$ is enforced but the bank's Euler equation treats ϕ_1 differently. In the *anticipated* economy, the bank knows the rule and uses $\mathbb{E}[\phi_1(N', A') \mid (N, A)]$ correctly when pricing next-period equity. In the *unanticipated* economy, the bank believes ϕ_1 is an iid draw around the unconditional mean $\overline{\phi}_1$. Its forecast of the *level* of ϕ_1 is unbiased by construction, but it fails to exploit the conditional dependence on (N, A). The difference isolates the welfare value of making the rule's *structure* common knowledge, holding the realized policy path fixed.

Table 7 reports the comparison. The anticipated economy delivers a welfare gain of $+0.163\%$ in consumption-equivalent terms relative to the unanticipated one, with a 95% confidence interval of $[+0.140\%, +0.186\%]$ across $n = 10$ independent simulation seeds. The interval is bounded away from zero, so the welfare ordering is robust to the realized shock path: under a fixed enforceable policy, correct conditional beliefs weakly dominate misspecified ones at the bank's own optimization, and in our calibration the dominance carries through to household welfare. The mechanism is visible in the allocations. The unanticipated bank systematically *over-borrows*—mean B is 22% higher than under anticipation—because it expects a looser constraint than is realized in tight states. The resulting higher leverage pushes the capital ratio into the steeper region of the hazard function, raising the mean failure probability from 2.0% to 3.0%.

Table 7: Welfare gain from making the state-contingent rule common knowledge

	Anticipated *rule known*	Unanticipated *iid belief*
ϕ_1 policy (enforced)	$\phi_1(N, A)$	$\phi_1(N, A)$
ϕ_1 belief (Euler)	$\phi_1(N, A)$	iid, mean $\overline{\phi}_1$
Mean ϕ_1	0.302	0.446
Binding share	86.4%	85.2%
Mean C	0.2253	0.2248
Mean B	0.0238	0.0290
Mean hazard (%)	2.01	2.98
CEV (Anticipated vs. Unanticipated)	+0.163%	
95% CI, $n = 10$ seeds	[+0.140%, +0.186%]	

Notes: Both economies enforce the same linear rule $\phi_1(N, A) = 0.687 - 1.055\,(N/N_{ss} - 1) + 5.358\,(A - 1)$, floored at 0.10. The *anticipated* economy uses $\phi_1(N', A')$ in the bank's Euler equation; the *unanticipated* economy treats ϕ_1 as an iid draw with mean $\overline{\phi}_1$—unbiased unconditionally but missing the conditional dependence on (N, A). The reported CEV is the consumption-equivalent welfare of the anticipated economy relative to the unanticipated one, with mean and 95% confidence interval taken over $n = 10$ independent simulation seeds; each seed draws a 5,001-period TFP path with the first 500 periods dropped as burn-in. The unanticipated bank over-borrows on average (mean B is 22% higher) because it expects looser constraints than are realized in tight states. Means in the upper rows are post-burn-in averages from a representative seed; cross-seed dispersion is negligible at the displayed precision.

The economic content is that publishing a macroprudential rule is not merely a commitment device for its average stringency: communicating the conditional dependence on (N, A) has independent welfare value. A regulator who imposes the same rule without explaining its state-contingent structure leaves banks to form beliefs about ϕ_1 that are unbiased on average but wrong state by state, inducing a systematic over-borrowing bias. The welfare cost of that bias is modest in consumption-equivalent terms (about one sixth of a percent) but striking in its financial-stability dimension: making the rule's conditional structure common knowledge lowers the ergodic crisis frequency from 3.0% to 2.0%—a one-third proportional reduction in banking-crisis incidence. The lesson for macroprudential design is that transparent communication of the rule's structure, not just its level, is itself part of the rule's value.

4.6 Anatomy of the SPP–CE capitalization gap

Proposition 1 establishes that the planner *always* accumulates more equity than the competitive equilibrium: $N^{\text{SPP}} > N^{\text{CE}}$. Because TFP shocks are exogenous and symmetric across agents, no individual bank's actions can alter them, so there is no externality to correct on that margin. The failure hazard, by contrast, is an endogenous aggregate outcome: each bank takes $\mathcal{H}$ as given, yet its leverage choice contributes to the sector-wide capital ratio that determines $\mathcal{H}$. The Pigouvian wedge $\tau > 0$ prices exactly this missing feedback. Consequently, the *direction* of optimal policy is unambiguous—more equity is always warranted—and the operative question is *how much*. This section decomposes the quantitative magnitude of the SPP–CE gap into three structural channels and shows, via a counterfactual exercise, that the gap is highly sensitive to two primitive parameters: the severity of bank failure (φ) and the volatility of TFP shocks (σ_A).

4.6.1 Three-channel decomposition

The envelope conditions for the CE and SPP are:

$$J_1^{\text{CE}}(N;X) = R^B + \lambda\,(1+\phi_1), \tag{19}$$

$$J_1^{\text{SPP}}(N;X) = R^B + \lambda\,(1+\phi_1) + \tau. \tag{20}$$

At each economy's respective ergodic distribution, the Euler equation pins down the average marginal value of equity. The gap $J_1^{\text{SPP}} - J_1^{\text{CE}}$ can be decomposed into three channels evaluated at the ergodic means:

$$\underbrace{\mathbb{E}\left[J_1^{\text{SPP}}\right] - \mathbb{E}\left[J_1^{\text{CE}}\right]}_{\text{total gap}} = \underbrace{\Delta R^B}_{\text{deposit rate}} + \underbrace{\Delta[\lambda\,(1+\phi_1)]}_{\text{shadow value}} + \underbrace{\mathbb{E}[\tau]}_{\text{Pigouvian wedge}}, \tag{21}$$

where $\Delta R^B \equiv \mathbb{E}[R^{B,\text{SPP}}] - \mathbb{E}[R^{B,\text{CE}}]$ and $\Delta[\lambda(1+\phi_1)]$ is defined analogously. Although Proposition 1 guarantees $\tau > 0$, the other two channels are *negative* in equilibrium, partially offsetting the Pigouvian wedge:

- **[+] Pigouvian wedge** ($\tau > 0$): The planner internalizes that higher equity reduces the failure hazard, creating a positive marginal value absent in the CE. This channel pushes the SPP toward *more* equity.

- **[−] Deposit rate gap** ($\Delta R^B < 0$): The SPP restricts deposits to reduce leverage. With lower B, the deposit cost $\zeta'(B) = 2\zeta_2 B$ falls, so $R^B = 1 + \zeta'(B)$ is lower for the SPP. A lower deposit rate reduces the return on equity, pushing the SPP toward *less* equity.

- **[−] Shadow value gap** ($\Delta[\lambda(1+\phi_1)] < 0$): The SPP's lower leverage means its constraint binds far less frequently (2% vs. 20% at baseline). The resulting lower average λ reduces the equity return, again pushing the SPP toward *less* equity.

Table 8 reports the decomposition at baseline calibration. The Pigouvian wedge ($\tau = 0.040$) is positive but *smaller* than the combined offset from the deposit rate and shadow value channels ($-0.021 - 0.103 = -0.124$). The net gap is negative: $\mathbb{E}[J_1^{\text{SPP}}] < \mathbb{E}[J_1^{\text{CE}}]$. This is *consistent* with Proposition 1: the SPP holds more equity, and at higher N the marginal return to equity is lower due to diminishing returns from intermediation. The planner accumulates equity precisely *until* the combined return—inclusive of the Pigouvian bonus—falls to the household's required rate. The negative net gap reflects the fact that the SPP's equilibrium equity is high enough to exhaust the excess returns that the CE leaves on the table.

Table 8: Decomposition of the SPP–CE Return Gap

	Baseline ($\sigma_A = \pm 2\%, \varphi = 0.60$)	Counterfactual ($\sigma_A = \pm 5\%, \varphi = 0.20$)	Direction
Pigouvian wedge $\mathbb{E}[\tau]$	$+0.040$	$+0.014$	[+] more N
Deposit rate gap ΔR^B	-0.021	-0.006	[−] less N
Shadow value gap $\Delta[\lambda(1+\phi_1)]$	-0.103	-0.009	[−] less N
Net gap	-0.084	-0.001	
$N^{\text{SPP}}/N^{\text{CE}} - 1$	+67%	+14%	

Notes: Ergodic means from the baseline ($\sigma_A = \pm 2\%$, $\varphi = 0.60$) and counterfactual ($\sigma_A = \pm 5\%$, $\varphi = 0.20$) calibrations. The Pigouvian wedge τ scales approximately linearly with φ; the shadow value gap depends on TFP volatility through the frequency with which the leverage constraint binds.

4.6.2 Counterfactual: high TFP volatility, low failure severity

To assess how much the SPP–CE gap depends on primitives, we re-solve the model under a counterfactual calibration that widens TFP volatility ($\sigma_A = \pm 5\%$ vs. baseline $\pm 2\%$) while reducing the bank failure severity ($\varphi = 0.20$ vs. baseline 0.60). These changes move the two offsetting forces in opposite directions: lower φ shrinks τ (bank failures are less costly, so the externality weakens), while wider TFP shocks increase the frequency with which the CE leverage constraint binds (amplifying the CE's shadow value).

As the right column of Table 8 reports, the SPP–CE equity gap shrinks dramatically in the counterfactual: from 67% at baseline to 14%. The Pigouvian wedge falls from $\tau = 0.040$ to 0.014—close to the combined deposit rate and shadow value offsets—so the SPP's additional equity retention is modest.

The mechanism operates through the interaction of the three channels. In the counterfactual, the Pigouvian wedge τ falls to approximately one-third of its baseline value, since $\tau \propto \varphi$: for small φ, $V(n') - V((1-\varphi)n') \approx \varphi \cdot V'(n') \cdot n'$, so τ scales nearly linearly with the failure severity parameter. Meanwhile, the CE leverage constraint binds less frequently (8% vs. 18%) despite wider TFP, because the CE itself accumulates more equity as a buffer against larger shocks. The SPP constraint still binds 4% of the time (vs. 2% at baseline), so the shadow value gap between the two economies shrinks dramatically (from -0.103 to -0.009). Finally, the deposit rate gap narrows because the SPP, facing a smaller externality, restricts deposits less aggressively.

In the limit $\varphi \to 0$, bank failures become costless, $\tau \to 0$, and the SPP collapses to the CE. The Pigouvian wedge is the single structural force separating the two economies, and its magnitude is governed by the severity of bank failure.

Because the Pigouvian wedge is strictly positive, the *direction* of optimal policy is never in doubt: the planner always wants more equity than the market provides. The economically important question is *how much more*. The three-channel decomposition (21) provides a structural answer. The case for large macroprudential capital buffers is strongest when bank failures are costly and TFP volatility is moderate—the configuration in which the Pigouvian wedge dominates the offsetting deposit rate and shadow value channels. When failure costs are low (e.g., due to effective resolution regimes or deposit insurance), the Pigouvian wedge shrinks and the two offsetting channels nearly neutralize it, so the optimal buffer narrows from 67% to 14% of CE equity. The decomposition thus serves as a diagnostic for calibrating countercyclical capital buffers: the buffer should be large enough to close the Pigouvian gap but not so large as to overshoot the externality it corrects.

5 Conclusion

This paper develops a general equilibrium model that nests the credit exposure and fragility buffer channels—individually well understood in the literature—within a single framework. Doing so reveals a capitalization dilemma: bank equity amplifies TFP shocks through the credit exposure channel but mitigates financial shocks through the fragility buffer channel. Because both channels operate through the same state variable, a social planner who internalizes the fragility externality halves the damage from financial shocks while gaining no advantage for productivity shocks. This asymmetry delineates the scope of macroprudential policy: it is most valuable for financial fragility rather than real-sector fluctuations.

Several simplifying assumptions merit discussion. First, the representative bank abstracts from interbank markets and balance-sheet heterogeneity, which matter for contagion and the cross-sectional distribution of fragility. Second, the two-state TFP process suffices for the dual-role mechanism but limits the model's ability to match business cycle moments. Third, full depreciation ($\delta = 1$) eliminates physical capital dynamics; relaxing it would add capital as an endogenous state and enrich the interaction between bank equity and real investment. Fourth, the reduced-form hazard (4) does not microfound the failure mechanism; a strategic complementarity structure à la Goldstein and Pauzner (2005) would endogenize the hazard at the cost of additional complexity.

These limitations suggest natural extensions. Heterogeneous banks would allow study of how cross-sectional variation in capitalization affects systemic risk and bank-specific capital requirements. A richer shock process (e.g., persistent TFP, time-varying financial shock intensity) would improve quantitative realism. Combining the model with nominal rigidities would permit study of macroprudential–monetary policy interactions—in particular, whether the dual role extends to environments where the central bank faces a trade-off between output stabilization and financial stability.

From a policy perspective, we derive a linear macroprudential rule from the planner's solution: $\phi_1(N, A) = \bar{\phi}_1 + \gamma_N(N/N_{\text{ss}} - 1) + \gamma_A(A - 1)$, which approximates the SPP-implied leverage tightness with an R^2 of 0.93. The rule reveals a fundamental separation in the prudential stance. The financial cycle component ($\gamma_N < 0$) tightens leverage limits when aggregate bank equity rises, leaning against credit expansion that erodes the sector-wide capital ratio. The business cycle component ($\gamma_A > 0$) loosens the constraint in expansions, when strong fundamentals naturally push the economy away from the fragile region. These two responses carry opposite signs because the financial and business cycles move the fragility externality in opposite directions—a distinction that is lost in composite indicators such as the credit-to-GDP gap used in the Basel III CCyB framework. The rule provides a structural benchmark against which actual regulatory instruments can be evaluated, identifying where existing frameworks conflate distinct sources of risk and where the planner would prescribe differentiated responses.

The three-channel decomposition (Section 4.6) shows that the magnitude of the optimal buffer is sensitive to the severity of bank failure: when resolution regimes effectively limit failure costs, the Pigouvian wedge shrinks and the SPP–CE gap narrows from 67% to 14%, tempering the case for aggressive capital surcharges while preserving the qualitative direction of the policy prescription.

References

Adrian, T. and H. S. Shin (2010). Liquidity and leverage. *Journal of Financial Intermediation 19*(3), 418–437.

Basel Committee on Banking Supervision (2010). Guidance for national authorities operating the countercyclical capital buffer. Bcbs publication, Bank for International Settlements.

Begenau, J. (2020). Capital requirements, risk choice, and liquidity provision in a business-cycle model. *Journal of Financial Economics 136*(2), 355–378.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1, Chapter 21, pp. 1341–1393.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Bianchi, J. and E. G. Mendoza (2018). Optimal time-consistent macroprudential policy. *Journal of Political Economy 126*(2), 588–634.

Boissay, F., F. Collard, and F. Smets (2016). Booms and banking crises. *Journal of Political Economy 124*(2), 489–538.

Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *American Economic Review 104*(2), 379–421.

Corbae, D. and P. D'Erasmo (2021). Capital buffers in a quantitative model of banking industry dynamics. *Econometrica 89*(6), 2975–3023.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2021). A macroeconomic model with financially constrained producers and intermediaries. *Econometrica 89*(3), 1361–1418.

Farhi, E. and I. Werning (2016). A theory of macroprudential policies in the presence of nominal rigidities. *Econometrica 84*(5), 1645–1704.

Gertler, M. and P. Karadi (2011). A model of unconventional monetary policy. *Journal of Monetary Economics 58*(1), 17–34.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3, Chapter 11, pp. 547–599.

Gertler, M., N. Kiyotaki, and A. Prestipino (2020). A macroeconomic model with financial panics. *The Review of Economic Studies 87*(1), 240–288.

Goldstein, I. and A. Pauzner (2005). Demand–deposit contracts and the probability of bank runs. *The Journal of Finance 60*(3), 1293–1327.

He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. *American Economic Review 103*(2), 732–770.

He, Z. and A. Krishnamurthy (2019). A macroeconomic framework for quantifying systemic risk. *American Economic Journal: Macroeconomics 11*(4), 1–37.

He, Z. and W. Xiong (2012). Rollover risk and credit risk. *The Journal of Finance 67*(2), 391–430.

Jordà, O., M. Schularick, and A. M. Taylor (2017). Macrofinancial history and the new business cycle facts. *NBER Macroeconomics Annual 31*(1), 213–263.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147.

Lee, H. (2025). A global dynamic nonlinear solution framework and the repeated transition method. *The Review of Economic Studies*. Forthcoming.

Lee, H. (2026). Dancing on the saddles: A geometric framework for stochastic equilibrium dynamics. Working paper, University of Cambridge.

Lorenzoni, G. (2008). Inefficient credit booms. *The Review of Economic Studies 75*(3), 809–833.

Mendoza, E. G. (2010). Sudden stops, financial crises, and leverage. *American Economic Review 100*(5), 1941–1966.

Rampini, A. A. and S. Viswanathan (2019). Financial intermediary capital. *The Review of Economic Studies 86*(1), 413–455.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics 55*(2), 298–320.

Weitzman, M. L. (1974). Prices vs. quantities. *Review of Economic Studies 41*(4), 477–491.