

# Bridging Firm-Level and State-Level Input Elasticities: The Fiscal Multiplier of Infrastructure Investment\*

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## Abstract

This paper investigates the fiscal multiplier of infrastructure investment using an estimated heterogeneous-firm general equilibrium model. The analysis centers on a firm-level production function that incorporates the non-rivalrous nature of the public capital stock and its utilization by individual firms. The crucial determinant of the fiscal multiplier through the firm-level investment channel is the elasticity of substitution between private and public capital stocks. We provide both theoretical and quantitative analysis revealing a significant discrepancy between the estimated input elasticities at the firm level and the state level when non-rivalry is considered. The quantitative findings indicate a fiscal multiplier of approximately 1.04 over a 2-year horizon, suggesting a moderate net economic benefit from infrastructure investment. Notably, the implementation of infrastructure investment leads to crowding out of the aggregate investment, primarily driven by the general equilibrium effect.

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# 1 Introduction

Infrastructure spending and its effect on output and welfare have become one of the central issues in recent policy discussions. Especially the Infrastructure Investment and Jobs Act includes more than \$1.2 trillion in transportation and other physical infrastructure spending over a decade. How much economic benefit can we expect from the infrastructure investment?

To answer this question, we quantify the fiscal multiplier of infrastructure investment based on a structural model. We consider the firm-level production function that incorporates a significant aspect of infrastructure spending: the non-rivalrous nature of the public capital stock for the utilization by individual firms. We present a theoretical and quantitative analysis demonstrating that the inclusion of non-rivalry in the firm-level production function leads to a notable disparity between the elasticities of substitution estimated at the firm-level and the state-level.<sup>1</sup> To the best of our knowledge, our paper is the first one to bridge the firm-level and state-level elasticity differences and provide the firm-level elasticity estimated from a heterogeneous firm general equilibrium model. Moreover, our paper fills a gap in the literature by presenting the fiscal multiplier of infrastructure spending, which takes into account the endogenous investment and production by firms.

Our model incorporates a firm-level CES production function with private capital, public capital, and labor input. Public capital enters firms' production function in a non-rivalrous manner as in [Glomm and Ravikumar \(1994\)](#).<sup>2</sup> Subject to idiosyncratic productivity shocks, firms make lumpy investment decisions with both fixed and convex adjustment costs ([Cooper and Haltiwanger, 2006](#); [Winberry, 2021](#)). Using revenue financed from household income tax and corporate tax, the govern-

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<sup>1</sup>We aggregate the firm-level equilibrium allocations up to the state level and compare the elasticities. However, the theoretical implications of this aggregation are not limited to a particular level of aggregation.

<sup>2</sup>In contrast to [Glomm and Ravikumar \(1994\)](#), we incorporate firm-level heterogeneity in the model.

ment spends through infrastructure investment, lump-sum subsidy, and public employment. The infrastructure evolves with an exogenous law of motion subject to convex adjustment costs similar to private investment.

To capture state-level variations based on micro data, our model incorporates two regions distinguished by their infrastructure levels: one with poor infrastructure and another with good infrastructure.<sup>3</sup> Motivated by the cross-state variations in the infrastructure spending that has stayed almost invariant over the sample period, we assume the allocation of expenditures between these regions is exogenously given. In our model, the elasticity of substitution between private and public capital in the firm's production function serves as a critical parameter. If private and public capital exhibit a stronger degree of complementarity, it is anticipated that the region with high infrastructure will possess a greater proportion of private capital stock. Unlike the commonly employed approach of utilizing time-series variations, our study offers a novel approach by utilizing cross-sectional variations at the state level to identify and estimate the input elasticity parameter at the firm level.

Estimating a general equilibrium model with heterogeneous agents, such as our model, is widely recognized as computationally challenging due to the need to solve for market clearing prices for every potential value of the model parameters. However, we introduce a novel extension to an existing estimation method that significantly reduces computational costs. Our method is closely related to estimation method to match the model-simulated moments to the data moments. To handle the general equilibrium, we extend this method by including market clearing conditions as additional moments. In this paper, we employ the multi-block Metropolis Hastings algorithm, which involves dividing the parameter space into two blocks: one for the price block and another for the model parameters. From running this algorithm, we generate draws that bring the market clearing condi-

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<sup>3</sup>In the model, these two states are also assumed to feature different state-specific TFP levels. These heterogeneous TFP levels are also estimated.

tions closer to zero, while ensuring a closer fit to the empirical moments.

Our estimates indicate that the elasticity of substitution is around 1.19, suggesting gross substitutability between private and public capital inputs. To validate our model, we compute the elasticity at the state level from our model and compare it to the empirical elasticity derived from U.S. state-level data. In the model, the state-level elasticity is computed by aggregating firms' behaviors in two regions and estimating the state-level production functions. Our estimation yields a state-level elasticity of 0.48. In comparison, by estimating the state-level production function following [An, Kangur, and Papageorgiou \(2019\)](#), we obtain an empirical counterpart of 0.45. Our findings suggest that public and private capital inputs exhibit gross complementarity at the state level, while demonstrating substitutability at the firm level. This observation aligns with our theoretical result, where we establish that the nature of substitution can change as the micro-level (firm-level) input elasticity is aggregated to the state-level counterpart.

Given our estimated model, we conduct the quantitative analysis to compute fiscal multipliers with one-time unexpected infrastructure spending shock whose magnitude is 1% of steady-state GDP value. We assume that the fiscal policy shock is financed by a lump-sum tax on impact. As our model has two regions of different infrastructure level, we take the weighted average of fiscal multipliers to obtain the aggregate multipliers. The substantial increase in the public capital leads to a boost in output. However, the fiscal policy shock causes an increase in the interest rate due to lump-sum financing which initially reduces consumption. Consequently, these general equilibrium effects result in crowding out of private investment on impact. Accounting for these opposing forces, the short-run aggregate fiscal multiplier over a two-year period is estimated to be 1.04, while the short-run multiplier in the partial equilibrium is estimated to be 1.86.

Furthermore, our analysis reveals that varying micro-level elasticities of substitution yields significantly different fiscal multipliers. A lower elasticity corre-

sponds to a larger fiscal multiplier, as the private investment is crowded out less. This emphasizes the importance of sharply estimating the input elasticity to quantify fiscal multipliers while considering firms' investment. We also find that the fiscal multipliers vary with the inclusion of time-to-build assumption.<sup>4</sup> The time-to-build assumption impacts fiscal multipliers through two key channels. First, there is a news effect where individuals adjust their behaviors as they expect a future increase in the infrastructure. Second, there is a general equilibrium effect endogenously stemming from the news effect. Consistent with [Ramey \(2020\)](#), we find that the aggregate fiscal multiplier decreases when compared to scenarios without the extended time-to-build assumption.

Our paper contributes to several strands of existing literature. First, it is closely connected to the literature on government spending multipliers ([Ramey and Zubairy, 2018](#); [Chodorow-Reich, 2019](#); [Hagedorn, Manovskii, and Mitman, 2019](#); [Auerbach, Gorodnichenko, and Murphy, 2020](#); [Ramey, 2020](#); [Hasna, 2021](#)). Our focus is specifically on quantifying the multipliers associated with infrastructure spending, which represents a distinct category of public investment. In empirical research, there have been attempts to estimate the output elasticity of the public investment ([An, Kangur, and Papageorgiou, 2019](#); [Espinoza, Gamboa-Arbelaes, and Sy, 2020](#); [Ramey, 2020](#), [An, Zhang, and Li, 2022](#)). Alongside empirical analysis using the data on the American Recovery and Reinvestment Act (ARRA), [Ramey \(2020\)](#) analyzes the impacts of government investment using a stylized neoclassical and New Keynesian models. Our contribution lies in quantifying the infrastructure spending multipliers based on a heterogeneous firm model, incorporating firm-level investment, which has not been addressed in the aforementioned papers.

Second, we contribute to the literature that bridges the gap between aggregate estimates and micro-level estimates using a structural model. Similar to [Nakamura](#)

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<sup>4</sup>[Suárez Serrato and Zidar \(2016\)](#) identify local incidence of corporate taxation using a spatial model where firms are heterogeneous in productivity and imperfectly mobile. If we had firms' location choice in our model, time-to-build assumption would generate imperfectly mobile firms as firms cannot make decisions that perfectly insure themselves against idiosyncratic shocks.

and Steinsson (2018) and Oberfield and Raval (2021), we estimate the elasticity of substitution between private and public capital stocks at the firm level using cross-state variations. We obtain the firm-level elasticity of substitution of 1.19. Then we aggregate the capital stock within states to measure the state-level elasticity of substitution based on our model, which yields an estimate of 0.48. This estimate closely aligns with the empirical estimate of 0.45 obtained from the data. Our results suggest that private capital and public capital are gross substitutes at the firm level, while they act as gross complements at the state level. We provide theoretical support for this relations by demonstrating the non-rivalry effect of infrastructure in the production function.

Lastly, our paper is related to the literature that studies firm-level investments. This literature has empirically and theoretically investigated the firm-level lumpy investment patterns and their macroeconomic implications (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006; Abel and Eberly, 2002; Khan and Thomas, 2008; Winberry, 2021). Based on the literature, we incorporate the convex and fixed adjustment cost for the firm-level capital adjustment and estimate the cost parameters to capture the observed investment dynamics at the firm level. Our goal is to establish a micro-level foundation for analyzing the fiscal multiplier of infrastructure spending, specifically taking into account firm-level investment and incorporating the non-rivalrous nature of the public capital stock.

The rest of this paper proceeds as follows. Section 2 presents a theory showing that the nature of substitution between private and public capital flips with the aggregation from firm-level to state-level. Section 3 presents the model. Section 4 presents the estimation results and validates the model using the state-level data. Section 5 presents a comprehensive quantitative analysis to compute infrastructure spending multipliers. Section 6 concludes.

## 2 A simple theory on the firm-level and state-level elasticity estimates

In this section, we examine the theoretical relationship between the elasticities of substitution between private and public capital at both the firm-level and state-level. Our analysis will focus on non-rivalrous public capital, such as infrastructure, where multiple firms within a state can use the public capital simultaneously without competition.

Consider a CES production function  $F(K, N, L; \lambda, z)$  with CRS:<sup>5</sup>

$$F(K, N, L; \lambda, z) = z(\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}\alpha} L^{1-\alpha}$$

where  $\lambda$  is the elasticity of substitution between private and public capital;  $K$  is the private capital input;  $N$  is the public capital input,  $L$  is the labor input;  $z$  is the productivity level. Then, we consider a static labor demand problem:

$$\max_L F(K, N, L; \lambda, z) - wL$$

Using the solution of this problem  $L^* = z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} (\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$ , we can rewrite the production function with the implicit labor demand:

$$F(K, N, L(K, N; \lambda, z); \lambda, z) = f(K, N; \lambda, z) := z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} (\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$$

Then, we consider estimation of the elasticity  $\lambda$  at the firm level and at the state level using the production function  $f$ . Suppose we use a dataset that contains firm-

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<sup>5</sup>When  $\lambda \rightarrow 1$ , the production function takes the following form:

$$F(K, N, L; \lambda, z) = z(K^{\theta\alpha})(N^{(1-\theta)\alpha})L^{1-\alpha}.$$

When the sum of the weights between the private and public capital stocks is not normalized by unity, the production function follows an IRS form in [Baxter and King \(1993\)](#).

level observations  $(k_1, k_2, y_1, y_2, N)$ , where the subscript  $i \in \{1, 2\}$  represents two different firms in the same state. It's important to note that the state-level capital stock  $N$  is shared among all firms in the same state. In the firm-level estimation, we estimate the firm-level elasticity and the productivity  $(z, \lambda)$  that satisfy

$$\begin{aligned} f(k_1, N; \lambda, z) &= y_1 \\ f(k_2, N; \lambda, 1) &= y_2 \end{aligned}$$

where the second firm's productivity is normalized to be unity.

In the state-level estimation, we estimate the state-level elasticity  $\xi$  that satisfies

$$f(k_1 + k_2, N; \xi, 1) = y_1 + y_2.$$

where the state-level productivity is normalized to be unity.

In this estimation, due to the non-rivalrous nature of public capital, firm-level estimate  $\lambda$  and state-level estimate  $\xi$  can be starkly different. Specifically, under the commonly observed conditions, which will be formally specified later, the private and public capitals are gross substitutes at the firm level, even if private and public capitals are gross complements at the state level.

The intuition behind the logic is that when the elasticity is estimated at the aggregated level, the non-rivalry of public capital stock is missing in the estimation. Therefore, in our paper's context, the state-level estimate supports a substantially stronger complementarity between private and public capital stocks than the firm-level estimate does. The following proposition formally states and proves this discrepancy in the firm-level and state-level estimates.

**Proposition 1.** *Suppose we are given the micro-level data set  $(k_1, k_2, y_1, y_2, N)$  s.t.*

$$\exists i \in \{1, 2\} \text{ s.t. } k_i < N, \quad N \leq k_1 + k_2, \quad \frac{y_1}{k_1} = \frac{y_2}{k_2}.$$



Suppose the micro-level estimates  $(z, \lambda)$  and the aggregate-level estimate  $\xi$  are exactly identified by fitting the data with the production functions as follows:

$$\begin{aligned} f(k_1, N; \lambda, z) &= y_1 \\ f(k_2, N; \lambda, 1) &= y_2. \\ f(k_1 + k_2, N; \xi, 1) &= y_1 + y_2 \end{aligned}$$

Then, if the micro-level input elasticity satisfies  $\lambda \geq 1$ , the aggregate-level input elasticity satisfies  $\xi < 1$ .

*Proof.* Without loss of generality suppose  $k_1 > k_2$ ,  $z > 1$ , and let  $k_2 < N$ . From the production functions, we have

$$\begin{aligned} y_1 &= z^{\frac{1}{\alpha}} B(\theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\ y_2 &= B(\theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\ y_1 + y_2 &= B(\theta(k_1 + k_2)^{\frac{\xi-1}{\xi}} + (1-\theta)N^{\frac{\xi-1}{\xi}})^{\frac{\xi}{\xi-1}} \end{aligned}$$

where  $B := \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$ . Therefore, the following relationships hold (from the second and the third equations above):

$$\begin{aligned} \left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} &= \theta + (1-\theta) \left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} \\ \left(\frac{y_1 + y_2}{B(k_1 + k_2)}\right)^{\frac{\xi-1}{\xi}} &= \theta + (1-\theta) \left(\frac{N}{k_1 + k_2}\right)^{\frac{\xi-1}{\xi}}. \end{aligned}$$

Suppose we are given  $\lambda \geq 1$ . We will prove the proposition by contradiction, so we assume  $\xi \geq 1$ . As  $N > k_2$ ,  $\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1$ . Thus,

$$\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta) \left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1.$$

Hence,  $\frac{y_2}{Bk_2} > 1$ . From the condition  $\frac{y_1}{k_1} = \frac{y_2}{k_2}$ ,

$$1 < \frac{y_2}{Bk_2} = \frac{y_1 + y_2}{B(k_1 + k_2)}.$$

As  $\xi \geq 1$ , we have

$$1 < \left( \frac{y_1 + y_2}{B(k_1 + k_2)} \right)^{\frac{\xi-1}{\xi}} = \theta + (1 - \theta) \left( \frac{N}{k_1 + k_2} \right)^{\frac{\xi-1}{\xi}}.$$

However,  $N \leq k_1 + k_2$ . Thus,  $\left( \frac{N}{k_1 + k_2} \right)^{\frac{\xi-1}{\xi}} \leq 1$ . This leads to

$$\theta + (1 - \theta) \left( \frac{N}{k_1 + k_2} \right)^{\frac{\xi-1}{\xi}} \leq 1,$$

which is a contradiction. Therefore, if the micro-level input elasticity satisfies  $\lambda \geq 1$ , then the aggregate-level input elasticity satisfies  $\xi < 1$ . ■

Our goal is to measure the infrastructure investment multipliers, taking into account both the endogenous investment decisions of heterogeneous firms and the non-rivalrous nature of public capital. To achieve this, we need a reliable estimate of the firm-level substitutability between public and private capital. However, it is infeasible to obtain this input elasticity from directly estimating the production function as the firm-level data on the usage of public infrastructure is barely available. Although one can find the state-level data on inputs of interest, Proposition 1 indicates that blindly employing the state-level input elasticity estimates from the literature for the firm-level production function would not be appropriate. Overcoming this hurdle, we build a structural model in Section 3 and provide a novel way in Section 4 to estimate the firm-level input elasticity through the lens of our model.

## 3 Model

### 3.1 Production technology

Time is discrete and lasts forever. A measure one of ex-ante homogenous firms are considered. Each firm owns capital. It produces a unit of goods from the inputs of labor and capital. The production technology of a firm  $i$  located at a region  $j$  follows a CES form as specified below:<sup>6</sup>

$$z_{i,t}x_{j,t}f(k_{i,t}, l_{i,t}, \mathcal{N}_{j,t}) = z_{i,t}x_{j,t} \left( \theta(k_{i,t})^{\frac{\lambda-1}{\lambda}} + (1-\theta)\mathcal{N}_{j,t}^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}\alpha} l_{i,t}^{\gamma}$$

where  $k_{i,t}$  is capital input,  $l_{i,t}$  is labor input, and  $\mathcal{N}_{j,t}$  is a region-specific infrastructure stock.  $z_{i,t}$  is idiosyncratic productivity and  $x_{j,t}$  is a region-specific productivity shock.  $\lambda > 0$  is the elasticity of substitution between private capital and the infrastructure.  $\theta \in (0, 1)$  is the weight parameter between the private and public capital.  $\alpha$  is capital share, and  $\gamma$  is labor share such that  $\alpha + \gamma < 1$ .<sup>7</sup>

Idiosyncratic productivity  $z_{i,t}$  is specified as below:

$$\ln(z_{i,t+1}) = \rho_z \ln(z_{i,t}) + \epsilon_{z,i,t+1}, \quad \epsilon_{z,i,t+1} \sim_{iid} N(0, \sigma_z)$$

where  $\rho_z$  and  $\sigma_z$  are persistence and standard deviation of *i.i.d* innovation in the process. The idiosyncratic shock process is discretized using the Tauchen method for computation.

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<sup>6</sup>In the baseline specification, we normalize the aggregate productivity as unity, as our estimation and the fiscal multiplier analysis are based on the stationary recursive competitive equilibrium. The extension of including the stochastic aggregate productivity process would allow the state-dependent fiscal multiplier analysis, which we leave for future research.

<sup>7</sup>Proposition 1 is based on the production function with constant returns to scale. This assumption is intended for the theoretical clarity of the statement. For example, with decreasing returns to scale, additional boundary conditions of parameters are necessary for the proposition. In the baseline model, we assume the decreasing returns to scale production function to capture the empirically-supported dividend stream level used in the literature. In the quantitative analysis, we show that the theoretical implications of Proposition 1 is unaffected by this assumption.

In the economy, there are two regions  $j \in \{P, G\}$  of which infrastructure levels and productivity levels are different from each other. We denote the poor infrastructure region as  $P$  and the good infrastructure region as  $G$ :  $N_G > N_P$ . Firms switch from one region to another following an exogenous Markov process:

$$\begin{bmatrix} p_{t+1}^P \\ p_{t+1}^G \end{bmatrix} = \begin{bmatrix} \pi_{PP} & \pi_{PG} \\ \pi_{GP} & \pi_{GG} \end{bmatrix}' \begin{bmatrix} p_t^P \\ p_t^G \end{bmatrix}$$

Using the production function, firms at a region  $j$  earn operating profit in each period by solving the following problem:

$$\pi(z_{i,t}, k_{i,t}, j, \mathcal{N}_t, w_t, r_t) = \max_{l_{i,t}} z_{i,t} x_{j,t} f(k_{i,t}, l_{i,t}, \mathcal{N}_{j,t}) - w_t l_{i,t}$$

where  $w_t$  is the real wage.

### 3.2 Firm-level investment

Firms make an investment decision as in [Khan and Thomas \(2008\)](#). A small-scale capital adjustment is specified as  $\Omega(k_{i,t}) := [-\nu k_{i,t}, \nu k_{i,t}]$ . When they make a large-scale capital adjustment,  $I_{i,t} \notin \Omega(k_{i,t})$ , they need to pay a fixed adjustment cost  $\zeta_{i,t}$ , where  $\zeta_{i,t} \sim_{iid} Unif[0, \bar{\zeta}]$ . This cost is regarded as a labor overhead cost, so the actual cost is  $w_t \zeta_{i,t}$ , where  $w_t$  is the real wage. If a firm makes a small-scale capital adjustment,  $I_{i,t} \in \Omega(k_{i,t})$ , it does not need to pay a fixed adjustment cost.<sup>8</sup>

Following [Cooper and Haltiwanger \(2006\)](#) and [Winberry \(2021\)](#), we assume all investments are subject to a convex adjustment cost,  $C(I_{i,t}, k_{i,t}) = \frac{\mu}{2} \left(\frac{I_{i,t}}{k_{i,t}}\right)^2 k_{i,t}$ . The convex adjustment cost plays an essential role in this paper, as it helps to capture the realistic sensitivity of aggregate investment to the general equilibrium effect driven by the exogenous shocks such as fiscal policy shocks ([Zwick and Mahon,](#)

<sup>8</sup>As in [Khan and Thomas \(2008\)](#), there exists a threshold rule  $\zeta^* = \zeta^*(z, k, j; \mathcal{S})$  for the fixed cost shock  $\zeta$  realization in the large-scale investment. For the brevity, we omit the detailed description.

2017; Koby and Wolf, 2020; Lee, 2022).

### 3.3 Government

The government collects income tax from households at the rate of  $\tau^h$  and corporate tax  $\tau^c$ . Household income is the sum of labor income  $w_t l_t$  and dividend income  $D_t$ . The tax rates are exogenously determined. Government issues a bond  $B_{t+1}$  which matures in one period and is discounted by the gross bond return,  $1 + r_t^B$  and pays back the maturing bond,  $B_t$ . Using the revenue  $\mathcal{G}_t$  financed from the taxation and the net debt issuance, the government spends through three channels: infrastructure investment  $\mathcal{F}_t$ , public employment  $w_t \mathcal{E}_t$ , and lump-sum subsidy  $T_t$ :

$$\begin{aligned} \mathcal{G}_t &= \tau^h(w_t l_t + D_t) + \int \tau^c \pi(z_{i,t}, k_{i,t}, j; \mathcal{N}_t, w_t, r_t) d\Phi_t + \frac{B_{t+1}}{1 + r_t^B} - B_t & \text{[Revenue]} \\ &= \mathcal{F}_t + w_t \mathcal{E}_t + T_t & \text{[Spending]} \end{aligned}$$

We assume the overhead fixed cost of infrastructure investment is covered by public sector workers,  $\mathcal{E}_t$ , without an extra cost. The public employment  $\mathcal{E}_t = \mathcal{E}$  is exogenously determined. The split between the lump-sum subsidy and the infrastructure investment is determined exogenously by  $\varphi$ . To be specific, for  $\varphi > 0$ ,  $\mathcal{F}_t = \varphi(\mathcal{G}_t - w_t \mathcal{E}_t)$ , and  $T_t = (1 - \varphi)(\mathcal{G}_t - w_t \mathcal{E}_t)$ .

The country-level infrastructure  $\mathcal{N}_{A,t}$  and state-level infrastructure  $\mathcal{N}_{j,t}$  ( $j \in \{P, G\}$ ) evolve according to the following law of motion:

$$\begin{aligned} \mathcal{N}_{A,t+s} &= \mathcal{N}_{A,t+s-1}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2} \left( \frac{\mathcal{F}_t}{\mathcal{N}_{A,t+s-1}} \right)^2 \mathcal{N}_{A,t+s-1} \\ \mathcal{N}_{j,t} &= \zeta_j \mathcal{N}_{A,t} \quad \text{for } j \in \{P, G\} \end{aligned}$$

where the aggregate infrastructure  $\mathcal{N}_{A,t}$  satisfies  $\mathcal{N}_{A,t} = \mathcal{N}_{P,t} + \mathcal{N}_{G,t}$ . The split between the poor infrastructure region and the good infrastructure region is ex-

ogenously determined by  $\zeta_j$ , which is calibrated to match the distribution of infrastructures described in Table 2. A positive integer  $s$  represents time to build for the infrastructure investment. Infrastructure investment is subject to the same convex capital adjustment cost as private investment.

To summarize the state variables, the individual state variables are idiosyncratic productivity shock,  $z_{i,t}$ , and individual capital stock,  $k_{i,t}$ . The aggregate state variables are the tuple of each region's infrastructure stocks,  $\mathcal{N}_t = (\mathcal{N}_{P,t}, \mathcal{N}_{G,t})$ , infrastructure spending history and plan,  $\mathbb{F}_t = (\mathcal{F}_{t+\bar{s}})_{\bar{s}=-s}^{\infty}$ , and the distribution of individual state variables,  $\Phi_t$ .

### 3.4 Recursive formulation

For the brevity of notation, we drop the time subscripts for each allocation from this point on. A representative household consumes, saves, and supplies labor. We define the collection of aggregate state variables  $\mathcal{S} := (B, \Phi, \mathcal{N}, \mathbb{F})$ . The recursive formulation of the household problem is as follows:

$$V(a; \mathcal{S}) = \max_{c, a', L, B'} \log(c) - \frac{\eta}{1 + \frac{1}{\chi}} L^{1 + \frac{1}{\chi}} + \beta V(a'; \mathcal{S}')$$

s.t.

$$c + \frac{a'}{1 + r(\mathcal{S})} + \frac{B'}{1 + r^B(\mathcal{S})} = w(\mathcal{S})L(1 - \tau^h) + a + T(\mathcal{S}) + B$$

where  $c$  is consumption;  $a'$  and  $a$  are future and current wealth;  $L$  is labor supply;  $B'$  is savings in government bonds;  $J$  is the individual firm value after the dividend tax;  $T$  is the lump-sum subsidy.

In the recursive formulation, a firm's problem is as follows:

$$\begin{aligned}
J(z, k, j; \mathcal{S}) &= \max_{I, I^c} \pi(z, k, j; \mathcal{S})(1 - \tau^c)(1 - \tau^h) \\
&+ \int_0^{\bar{\xi}} \max\{(-I - w(\mathcal{S})\xi - C(I, k))(1 - \tau^h) + \frac{1}{1 + r(\mathcal{S})} \mathbb{E}J(z', k', j'; \mathcal{S}'), \\
&(-I^c - C(I^c, k))(1 - \tau^h) + \frac{1}{1 + r(\mathcal{S})} \mathbb{E}J(z', k^c, j'; \mathcal{S}')\} dG(\xi) \\
\text{s.t.} \quad k' &= (1 - \delta)k + I, \quad I \notin \Omega(k_t) = [-vk_t, vk_t] \\
k^c &= (1 - \delta)k + I^c, \quad I^c \in \Omega(k_t) \\
\mathcal{S}' &= G^{ALM}(\mathcal{S}) \\
dG(\xi) &= \frac{1}{\bar{\xi}} d\xi \quad (\text{Uniform dist.}) \\
\pi(z, k, j; \mathcal{S}) &= \max_n z x_j f(k, n, N_j) - w(\mathcal{S})n \\
C(I, k) &= \frac{\mu}{2} \left(\frac{I}{k}\right)^2 k
\end{aligned}$$

We assume the optimal dividend payout policy fully internalizes the income tax of households,  $\tau^h$ . Without this assumption, there would be an inefficient allocation of dividends, which is beyond the scope of this paper.<sup>9</sup> By allowing the fixed cost  $\xi$  to follow the *i.i.d* shock process, the value function becomes smooth without a kink.  $G^{ALM}$  is the aggregate law of motion that reflects the rational expectation for the future aggregate state allocations.

### 3.5 Equilibrium

In the stationary recursive competitive equilibrium, the interest rate and the wage are determined in the competitive market. Specifically, the following market clear-

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<sup>9</sup>Without this assumption, the firm's profit maximization would not take into account the household's income tax. This contrasts with the household's saving decision, which is based on the future after-income-tax dividend, leading to a distortionary effect of the corporate tax. Analyzing this distortionary effect is beyond the scope of this paper.

ing conditions determine each price.<sup>10</sup>

$$\begin{aligned}
\text{[Capital]} \quad & \int \underbrace{\mathbb{E}J(z', k'(z, k); \mathcal{S}) d\Phi}_{\text{Capital Demand}} = \underbrace{a'(a; \mathcal{S})}_{\text{Capital Supply}} \\
\text{[Labor]} \quad & \int \underbrace{\left( n(z, k, j; \mathcal{S}) + \left( \frac{\min\{\zeta^*, \bar{\zeta}\}^2}{2\bar{\zeta}} \right) \right)}_{\text{Private Labor Demand}} d\Phi + \underbrace{\mathcal{E}}_{\text{Public Labor Demand}} = \underbrace{L(a; \mathcal{S})}_{\text{Labor Supply}}
\end{aligned}$$

The aggregate dividend is a sum of individual after-corporate-tax operating profits net of investment, and the ex-dividend portfolio value  $P(\mathcal{S})$  is a sum of all the firms' values after the dividend payout:

$$\begin{aligned}
\text{[Aggregate Dividend]} \quad D(\mathcal{S}) &= \int \left( \pi(z, k, j; \mathcal{S})(1 - \tau^c) \right. \\
&\quad \left. - I^*(z, k, j; \mathcal{S}) - C(I^*(z, k, j; \mathcal{S}), k) - \mathbb{I}\{I^* \notin \Omega(k)\} w(\mathcal{S}) \bar{\zeta} \right) d\Phi
\end{aligned}$$

$$\text{[Ex-dividend Portfolio Value]} \quad P(\mathcal{S}) = \int J(z, k, j; \mathcal{S}) d\Phi - D(\mathcal{S})$$

And the government budget constraint and the spending constraint clear:

$$\begin{aligned}
\text{[Government Budget]} \quad \mathcal{G}(\mathcal{S}) &= \tau^h(w(\mathcal{S})L(a; \mathcal{S}) + D(\mathcal{S})) \\
&\quad + \int \tau^c \pi(z, k, j; \mathcal{S}) d\Phi + \frac{B'}{1 + r^B(\mathcal{S})} - B
\end{aligned}$$

$$\text{[Infrastructure Investment]} \quad \mathcal{F}(\mathcal{S}) = \varphi(\mathcal{G}(\mathcal{S}) - w(\mathcal{S})\mathcal{E})$$

$$\text{[Lump-sum Subsidy]} \quad \mathcal{T}(\mathcal{S}) = (1 - \varphi)(\mathcal{G}(\mathcal{S}) - w(\mathcal{S})\mathcal{E})$$

From the law of motion of the infrastructure, the stationary infrastructure stock is obtained.<sup>11</sup>

<sup>10</sup>On the private labor demand side, overhead labor demand is computed by multiplication of the probability of implementing lumpy investment  $\frac{\min\{\zeta^*, \bar{\zeta}\}}{\bar{\zeta}}$  and the conditional expectation  $\frac{\min\{\zeta^*, \bar{\zeta}\}}{2}$ , where  $\zeta^* = \zeta^*(z, k, j; \mathcal{S})$  is the threshold rule for making lumpy investments, as in [Khan and Thomas \(2008\)](#).

<sup>11</sup>There are two fixed points for the stationary infrastructure stock. We focus only on the greater one, which is a stable fixed point.



$$\text{[Infrastructure]} \quad \mathcal{N}_A = \frac{1 + \sqrt{1 - 2\mu\delta_N}}{2\delta_N} \mathcal{F}(\mathcal{S}), \quad \mathcal{N}_j = \zeta_j \mathcal{N}_A \quad \text{for } j \in \{P, G\}$$

Lastly, there is no arbitrage between the wealth return and the bond return.

$$\text{[No Arbitrage]} \quad r(\mathcal{S}) = r^B(\mathcal{S})$$

## 4 Estimation

We postulate how we estimate the parameters of our general-equilibrium model with heterogeneous firms. This has been a computationally demanding task than estimating a partial-equilibrium model since the market clearing prices have to be solved for each candidate value for the model parameters. We provide a novel way to bypass this bottleneck by estimating market clearing prices simultaneously with the model parameters.

We first illustrate how we choose the values of externally calibrated parameters. We then provide a brief summary of the limited-information Bayesian method that we extend to estimate a general-equilibrium model. Our novelty lies in that we augment general equilibrium conditions as additional moments and estimate market clearing prices together with the model parameters. After we describe the estimation method, we provide identification arguments with our targeted moments and report the estimation results.

### 4.1 External calibration

We first fix a few parameters at the common level in the literature:  $\beta = 0.96$  (annual frequency),  $\alpha = 0.28$ , and  $\gamma = 0.64$ . Some parameters are externally calibrated outside of the model, and their values are reported in Table 1.

For the average of household income tax rate, we use 0.15 as in [Krueger and Wu \(2021\)](#) where they compute the tax rate with the data from [Blundell, Pistaferri, and](#)

Parameter	Description	Value
$\tau^h$	household income tax rate (average)	0.15
$\tau^c$	corporate tax rate	0.27
$\mathcal{E}$	public employment	0.05
$\varphi$	infrastructure spending	0.09
$s$	time to build	1
$\chi$	Frisch elasticity	4
$\delta$	depreciation rate of private capital	0.09
$\delta_{\mathcal{N}}$	depreciation rate of public capital	0.02
$\rho_z$	idiosyncratic shock persistence	0.75
$\sigma_z$	idiosyncratic shock volatility	0.13

Table 1: Externally calibrated parameters

*Notes:* Each period in the model corresponds to one year in the data.

Saporta-Eksten (2016). For corporate tax rate, we use 0.27 from Gravelle (2014) that is the effective tax paid after deductions and credits. We use 0.05 for the fraction of public employment, using the data on the government employees (*USGOVT*) and the private employees (*USPRIV*). We use 0.09 for the infrastructure spending out of tax revenue. This comes from the fact that the infrastructure spending as share of GDP is 2.4% and the tax revenue as share of GDP is 27.1%. We assume one year of time-to-build for the baseline analysis. We set Frisch elasticity to be 4 as in Ramey (2020). We use 0.09 for the private capital depreciation rate, and 0.02 for the public capital depreciation rate from the BEA depreciation data. Following Lee (2022), we use the Compustat estimates of the persistence and volatility of the idiosyncratic productivity shocks.<sup>12</sup>

Furthermore, our model captures state-level variations by including two regions  $P, G$  that differ in infrastructure levels. To map this to the data pooled across years after detrending, we divide states into two groups by the median infrastructure level. Table 2 show some summary statistics between poor and good infrastructure groups. The transition probabilities are set to be persistent

<sup>12</sup>Lee (2022) uses the methodology of Akerberg, Caves, and Frazer (2015) to estimate the firm-level TFP shock process.

$(\pi_{PP} = 0.90, \pi_{GG} = 0.98)$ .<sup>13</sup>

	Poor infrastructure	Good infrastructure
Infrastructure portion	0.19 ( 0.001 )	0.81 ( 0.001 )
Establishment (#) portion	0.17 ( 0.005 )	0.83 ( 0.005 )
Firm (#) portion	0.173 ( 0.006 )	0.827 ( 0.006 )
GDP (\$) portion	0.151 ( 0.005 )	0.849 ( 0.005 )

Table 2: Comparison of two states: regions with good vs. poor infrastructure

*Notes:* Standard errors are in parentheses. # stands for the number of observations.

## 4.2 Estimation method

We estimate the remaining key parameters through a limited-information Bayesian approach augmented with general equilibrium conditions. We first explain the limited-information Bayesian method that uses a set of moments from the data for estimation. Then we illustrate our idea on augmenting general equilibrium conditions as additional moments.

### 4.2.1 The limited-information Bayesian method

The limited-information Bayesian method, as described in [Kim \(2002\)](#) and later advocated by [Christiano, Trabandt, and Walentin \(2010\)](#) and [Fernández-Villaverde, Rubio-Ramírez, and Schorfheide \(2016\)](#) among others, can be viewed as the Bayesian version of the generalized method of moments (GMM). Similar to GMM, the limited-information Bayesian method only uses a set of moments from the data for parameter inference.

<sup>13</sup>Transition probabilities are constructed using the state-level data in Table B.1 in the appendix.

Let  $\Theta$  denote the parameters of interest and  $\hat{\mathbf{m}}$  denote the vector of  $M$  empirical moments from the data for estimation. The likelihood of  $\hat{\mathbf{m}}$  conditional on  $\Theta$  is approximately

$$f(\hat{\mathbf{m}}|\Theta) = (2\pi)^{-\frac{M}{2}} |S|^{-\frac{1}{2}} \exp \left[ -\frac{1}{2} (\hat{\mathbf{m}} - \mathbf{m}(\Theta))' S^{-1} (\hat{\mathbf{m}} - \mathbf{m}(\Theta)) \right], \quad (1)$$

where  $\mathbf{m}(\Theta)$  is the model's prediction for the moments under parameter  $\Theta$ , and  $S$  is the covariance matrix of  $\hat{\mathbf{m}}$ . The covariance matrix  $S$  is often unknown but can be replaced by a consistent estimator of it, which can be obtained through bootstrap. Bayes' theorem tells us that the posterior density  $f(\Theta|\hat{\mathbf{m}})$  is proportional to the product of the likelihood  $f(\hat{\mathbf{m}}|\Theta)$  and the prior density  $p(\Theta)$ :

$$f(\Theta|\hat{\mathbf{m}}) \propto f(\hat{\mathbf{m}}|\Theta)p(\Theta), \quad (2)$$

and we can then apply the standard Markov Chain Monte Carlo (MCMC) techniques such as the Random-Walk Metropolis-Hastings (RWMH) algorithm to obtain a sequence of random samples from the posterior distribution.

Suppose we estimate parameters of the model in which market clearing conditions need to be satisfied as general equilibrium conditions. Given each candidate parameter vector, the model is solved with an additional loop that makes sure the market clearing conditions become zero with numerical precision. This additional layer regarding general equilibrium conditions is likely to result in prohibitively high computational costs.

#### 4.2.2 The limited-information Bayesian method augmented with general equilibrium conditions

In order to make the estimation procedure computationally feasible, we extend the limited-information Bayesian method by augmenting data moments with market clearing conditions. In other words, we treat market clearing prices as parameters

to be estimated where the associated moments in estimation procedure are market clearing conditions.

With the standard estimation with general equilibrium models, the computational bottleneck lies in that we need to satisfy market clearing conditions for each candidate parameter vector. Instead, our suggested method treats market clearing conditions as additional moments. Given the lens of our model, we need to track both of the market clearing prices: wage  $w$  and marginal utility of consumption  $p$  given which we can back out the interest rate through the Euler equation. Thus, we treat  $(p, w)$  as additional parameters to estimate. From our model (given state  $\mathcal{S}$ ),  $p = 1/c(\mathcal{S})$  and  $w = \eta L(\mathcal{S})^{\frac{1}{\lambda}} c(\mathcal{S}) / (1 - \tau^h)$ .

In addition to the empirical moments used in the limited-information Bayesian estimation, we include the following market clearing conditions:

$$\begin{bmatrix} p - 1/c(\mathcal{S}) \\ w - \eta L(\mathcal{S})^{\frac{1}{\lambda}} c(\mathcal{S}) / (1 - \tau^h) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Given candidates of the market clearing prices  $(p, w)$ , we compute the model-generated prices  $(1/c(\mathcal{S}), \eta L(\mathcal{S})^{\frac{1}{\lambda}} c(\mathcal{S}) / (1 - \tau^h))$  after solving the model. Then we can check whether the difference between the model-generated prices and  $(p, w)$  is zero.

In practice, we use the multiple-block Metropolis-Hastings where we break the parameter space into two blocks, one for the price block and the other for the other model parameters. For each iteration, we first update the price block conditional on the previous iteration's value for the price block and the remaining model parameter block. Then we sequentially update the model parameter block conditional on the updated price block. We include more details on the algorithm in Section C of the appendix. As the RWMH chain runs, we obtain the posterior draws that render market clearing conditions closer to zero as well as fitting the target empirical moments closely.

### 4.3 Identification and target moments

The upper bound for fixed cost  $\bar{\zeta}$  is identified using the lumpy investment portion. The convex adjustment cost parameter  $\mu$  is identified from the average investment to capital ratio. Parameter  $\nu$  associated with the constrained investment region is identified from the standard deviation of investment to capital ratio. Private capital share parameter  $\theta$  is identified from the private-to-public capital ratio. Productivity level parameter  $x$  is identified from the high region's output  $y$  portion. Government spending level parameter  $G$  is identified from the government spending to output ratio. Labor disutility parameter  $\eta$  is identified from the employment rate. The elasticity of substitution parameter  $\lambda$  is identified from the difference in private capital stocks between the state with a high level of infrastructure and the state with a low level of infrastructure. We assume that the firm-level production function is identical across firms within a state. As private and public capital are more complementary, the portion of private capital stock in the high infrastructure region is expected to be greater. To the best of our knowledge, our study is the first to use a structural model to identify and estimate the elasticity of substitution between private and public capital at the firm level.

#### 4.3.1 Estimation Results

We apply the multiple-block random-walk Metropolis-Hastings algorithm described in Section C of the appendix with uniform priors to simulate draws from the posterior density  $f(\Theta|\hat{\mathbf{m}})$  given by (2), and the posterior distribution is characterized by a sequence of 2000 draws after a burn-in of 2000 draws.<sup>14</sup>

Table 4 reports the posterior means and the 90% credible intervals of parameters from our estimation. The firm-level elasticity of substitution  $\lambda$  is estimated to be 1.185, which supports the Cobb-Douglas production function as a reason-

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<sup>14</sup>We initialize the chain at the point estimate from particle swarm optimization routine from MATLAB.

Target moment (source)	Data
Lumpy investment portion (Zwick and Mahon (2017))	0.140
Average investment to capital ratio ( $i/k$ ) (Zwick and Mahon (2017))	0.100
Standard deviation of ( $i/k$ ) (Zwick and Mahon (2017))	0.160
Private-to-public capital ratio (Bureau of Economic Analysis)	0.750
High region's private capital $k$ portion (Census Business Dynamics Statistics)	0.830
High region's output $y$ portion (Bennett, Kornfeld, Sichel, and Wasshausen (2020))	0.849
Government spending to output ratio (World Bank Database)	0.155
Total hours (Current Employment Statistics)	0.330

Table 3: Target moments used in estimation

Notes: High region refers to the state with high infrastructure capital stock.

able specification. The productivity of infrastructure abundant region is approximately double that of infrastructure poor region. It is worth noting that we do not consider endogenous evolution of productivity. Overall, the credible intervals are much narrower than the uniform priors, suggesting that the variations from the data is useful to infer the parameters of interest. Table 5 shows the model fit for the targeted moments. The model-generated moments fit the empirical moments from the data reasonably well.<sup>15</sup>

<sup>15</sup>In addition, the market clearing prices are tightly pinned down. When using the posterior mean value for market clearing prices, the market clearing conditions are satisfied with the numerical accuracy of  $e^{-4}$ .

Parameter	Description	Posterior	Prior
		Mean [90% interval]	Uniform distribution [min, max]
$\bar{\xi}$	fixed cost upper bound	0.519 [0.511,0.523]	[0.001,1.900]
$\mu$	convex adjustment cost	3.124 [3.118,3.135]	[0.200,3.500]
$\nu$	constrained investment region	0.0406 [0.0405,0.0407]	[0.001,0.080]
$\theta$	private capital share	0.667 [0.666,0.668]	[0.500,0.999]
$\lambda$	elasticity of substitution	1.185 [1.180,1.190]	[0.300,2.500]
$x$	productivity of high region	2.064 [2.042,2.080]	[0.500,2.500]
$G$	government spending level	0.103 [0.101,0.107]	[0.010,0.400]
$\eta$	labor disutility	2.845 [2.831,2.860]	[2.100,3.500]

Table 4: Estimation results

Target moment	Data	Model
Lumpy investment portion	0.140	0.139
Average investment to capital ratio ( $i/k$ )	0.100	0.100
Standard deviation of ( $i/k$ )	0.160	0.160
Private-to-public capital ratio	0.750	0.798
High region's private capital $k$ portion	0.830	0.870
High region's output $y$ portion	0.849	0.984
Government spending to output ratio	0.155	0.154
Total hours	0.330	0.344

Table 5: Model fit

#### 4.4 External validation with empirical state-level elasticity

As external validation, we compute the state-level elasticity from our model and compare it to the empirical estimate using the state-level data. We find that the state-level input elasticity from our model indicates the complementarity between



private and public capital and this is consistent with the empirical elasticity obtained from the state-level production function estimation.

#### 4.4.1 State-Level Elasticity from the Model

In our model, the infrastructure stock is shared among the firms in the same region. We conduct the state-level aggregation as follows: we fix the firm-level estimates except for the elasticity  $\lambda$  and spatial productivity heterogeneity  $x_1$ .<sup>16</sup> We estimate these two parameters under the state-level production models.<sup>17</sup>

$$\begin{bmatrix} x_1 \left( \theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta) N_1^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}\alpha} l_1^\gamma \\ \left( \theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta) N_2^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}\alpha} l_2^\gamma \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where  $(x_1, \lambda)$  are unknown, while all the other allocations and parameters,  $(y_1, y_2, k_1, k_2, N_1, N_2, l_1, l_2, \theta, \alpha, \gamma)$  are obtained from the estimated baseline model.<sup>18</sup> Using the same nonlinear least squares optimization used in [An, Kangur, and Papageorgiou \(2019\)](#), we get the estimate of the state-level production function  $(x_1, \lambda) = (1.766, 0.349)$ .

If we assume a CRS state-level production function with  $\gamma = 1 - \alpha$ , the estimates are  $(x_1, \lambda) = (1.923, 0.482)$ . Therefore, our model suggests that public capital and private capital are gross complements at the state level.

<sup>16</sup>Since the production function in our model is decreasing returns to scale, there is no guarantee that the firm-level elasticity and productivity is aggregated to have the same value in the state-level.

<sup>17</sup>We cannot identify the public capital stock share,  $\theta$  separately from the elasticity,  $\lambda$  in the state-level model. This is the main reason why we introduce the micro-level heterogeneity in our structural model. Therefore, in the state-level model, we fix the public capital stock share at the firm-level estimate.

<sup>18</sup>The two parameters  $(x_1, \lambda)$  are obtained from the exact identification.

#### 4.4.2 State-Level Elasticity from the Data

Using the state-level data available, we estimate the elasticity of substitution between private and public capital given a CES production technology. We closely follow [An, Kangur, and Papageorgiou \(2019\)](#) in which the elasticity is estimated using the nonlinear least squares using the following:

$$\ln \left( \frac{Y_{it}}{Y_{i,t-1}} \right) = c + (1 - a) \ln \left( \frac{L_{it}}{L_{i,t-1}} \right) + \frac{a}{\psi} \ln \left[ \frac{bK_{it}^\psi + (1 - b)N_{it}^\psi}{bK_{i,t-1}^\psi + (1 - b)N_{i,t-1}^\psi} \right] + (\epsilon_{it} - \epsilon_{i,t-1}).$$

$i$  denotes the state,  $t$  denotes the time, and  $\epsilon$  is the error term.  $Y$  is the output,  $K$  is the private capital stock,  $N$  is the public capital stock, and  $L$  is employment.  $\psi$  is the capital substitution parameter which implies a public-private capital elasticity of substitution (ES) of  $1/(1 - \psi)$ .

Using the state-level data, we compute local estimates of input elasticity. We first compute the net investment on public and private capital stocks. The state-level net public investment is approximated by the portion of aggregate net infrastructure investment, where the weight is obtained by the state-level real public highway infrastructure investment from [Bennett, Kornfeld, Sichel, and Wasshausen \(2020\)](#).<sup>19</sup> This is from the assumption that the infrastructure spending at the state level for each of the different items (e.g., highway, water supply, etc.) is identically distributed across the states. The state-level net private investment is approximated by the portion of aggregate net non-residential fixed investment from NIPA (table 5.2.6), where the weight is obtained by the number of establishments at the state level from the Business Dynamics Statistics (BDS) at the US Census Bureau. This approximation is based on the assumption that the capital stock at each estab-

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<sup>19</sup>The aggregate net infrastructure investment is also from [Bennett, Kornfeld, Sichel, and Wasshausen \(2020\)](#). In the state-level calculation, the weight is computed in the following way:

$$weight_{i,t} = \frac{\text{highway infrastructure investment}_{i,t}}{\sum_i \text{highway infrastructure investment}_{i,t}}$$

lishment does not vary significantly. After we obtain the net investment for public and private capital, we construct public and private capital stocks using the perpetual inventory method. For this approach, the initial capital stocks are needed for both public and private capital stocks. The state-level initial public capital stock is obtained by the portion of the aggregate public capital stock in 1977 from [Bennett, Kornfeld, Sichel, and Wasshausen \(2020\)](#), where the weight is from the highway infrastructure spending in 1977. The state-level initial private capital stock is from the portion of the aggregate private capital stock in 1977 from NIPA (table 4.1), where the weight is from the number of establishments in 1977. All the data is at the annual frequency. All real variables are chained in 2012 dollar value.

	Estimates	90% confidence interval
$a$	0.402	[0.351, 0.453]
$b$	0.070	[0.018, 0.123]
Elasticity of substitution	0.445	[-0.099, 0.989]

Table 6: Results from nonlinear least squares estimation

Notes: Elasticity of substitution is  $\frac{1}{(1-\psi)}$ . Its confidence interval is derived by the delta method.

Table 6 shows the estimation results from nonlinear least squares. The elasticity of substitution between public and private capital is estimated to be 0.445.<sup>20</sup> In other words, the state-level variations indicate the complementarity between private and public capital. However, this result does not imply the complementarity between private and public capital at the firm level. In fact, the private and public capitals can be gross substitutes at the firm level, whereas they are gross complements at the state level. The nature of substitution could flip as the micro-level (firm-level) input elasticities are aggregated up to the state-level as shown in Section 2.

<sup>20</sup>As robustness check, we apply GMM estimation where  $L_{it-2}, K_{it-2}, N_{it-2}$  are used in exogeneity conditions. The elasticity of substitution is estimated to be 0.44. This result is available upon request.

It is worth noting that our model bridges the gap between the firm-level estimates and the state-level estimates. According to our estimates, private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level. At the state level, the elasticity of substitution includes a good public nature of the infrastructure benefiting all firms in a state. Therefore, the non-rivalry of the infrastructure generates the complementarity between the state-level private capital and the public capital.

## 5 Analyses of fiscal multipliers

We analyze the fiscal multipliers of infrastructure investment based on our estimated structural model. We define the fiscal multiplier as follows:

$$\text{Fiscal Multiplier} = \frac{\sum_{t=1}^T \text{Present value of } \Delta x_t}{\sum_{t=1}^T \text{Present value of } \Delta G_t}$$

where  $\Delta x_t$  is the deviation at period  $t$  of the equilibrium allocation of interest from the steady-state level;  $\Delta G_t$  is the fiscal spending shock at period  $t$ .<sup>21</sup> In the short run, we assume  $T = 2$ , and in the long run, we assume  $T = 5$ . In this section, we focus on the impact of a sudden shock in fiscal spending specifically through infrastructure investment. We assume the fiscal spending shock is a one-time unexpected shock (MIT shock) without any persistence. The magnitude of the one-time shock is assumed at 1% of the steady-state output level as in [Ramey \(2020\)](#). We assume that all the fiscal policy shock is financed by a lump-sum tax.<sup>22</sup>

The following laws of motion determine the time path of the public capital

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<sup>21</sup>The shock is deviation from the steady-state level.

<sup>22</sup>In Section D of the appendix, we also consider changes in corporate tax policy on top of the lump-sum taxation for fiscal financing.

stocks after the fiscal spending shock  $\Delta G$  at  $t = 1$ :

$$\begin{aligned}\mathcal{N}_{A,t+1} &= \mathcal{N}_{A,t}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2} \left( \frac{\mathcal{F}_t}{\mathcal{N}_{A,t}} \right)^2 \mathcal{N}_{A,t} \\ \mathcal{N}_{j,t} &= \zeta_j \mathcal{N}_{A,t} \quad \text{for } j \in \{P, G\} \\ F_t &= \begin{cases} F^{ss} + \Delta G & \text{if } t = 1 \\ F^{ss} & \text{otherwise} \end{cases}\end{aligned}$$

where  $F^{ss}$  is the stationary equilibrium level of infrastructure spending.

Figure 1 plots the impulse responses of the fiscal policy shock. The dashed line in each panel shows the government expenditure changes from the steady-state level in percent of the steady-state output. The solid line is the impulse response of the equilibrium allocations.<sup>23</sup>

Private investment contemporaneously decreases. The response of private investment is the outcome of two countervailing forces: 1) increase in the investment incentive with increased infrastructure stock and 2) adjustment in the interest rate that dampens investment (general equilibrium effect; GE effect hereafter). The increase in the investment incentive comes from the imperfect substitution between public and private capital stock. For a simple illustration, we consider a two-period model with the firm-level investment decision where the production functions are the same as in Proposition 1, and investment is subject to the convex adjustment cost. From the first-order condition of the investment, the following equation holds:

$$\underbrace{1 + \mu \left( \frac{k'}{k} - (1 - \delta) \right)}_{\text{marginal cost}} = \underbrace{\overbrace{\frac{1}{1+r}}^{\text{GE channel}} \underbrace{z^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \left( \theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1} - 1} k'^{-\frac{1}{\lambda}} \theta}_{\text{Future MPK}}}_{\text{marginal benefit = discounted future MPK}}$$

<sup>23</sup>The responses of output, consumption, public capital, wage, and government investment decay in slow rates due to the low infrastructure depreciation rate at  $\delta_{\mathcal{N}} = 0.02$ .

The left-hand side of the equation above is the marginal cost of the firm-level investment, and the right-hand side is the marginal benefit. To analyze how the increase in the public capital stock  $N$  affects the marginal benefit of firm-level investment, we take a partial derivative with respect to  $N$ .

$$\frac{\partial}{\partial N} \text{Marginal benefit} = \left( \frac{1}{1+r} \right) \times \frac{\partial}{\partial N} \text{Future MPK} + \overbrace{\text{Future MPK} \times \frac{\partial}{\partial N} \left( \frac{1}{1+r} \right)}^{\text{GE effect}} \quad (3)$$

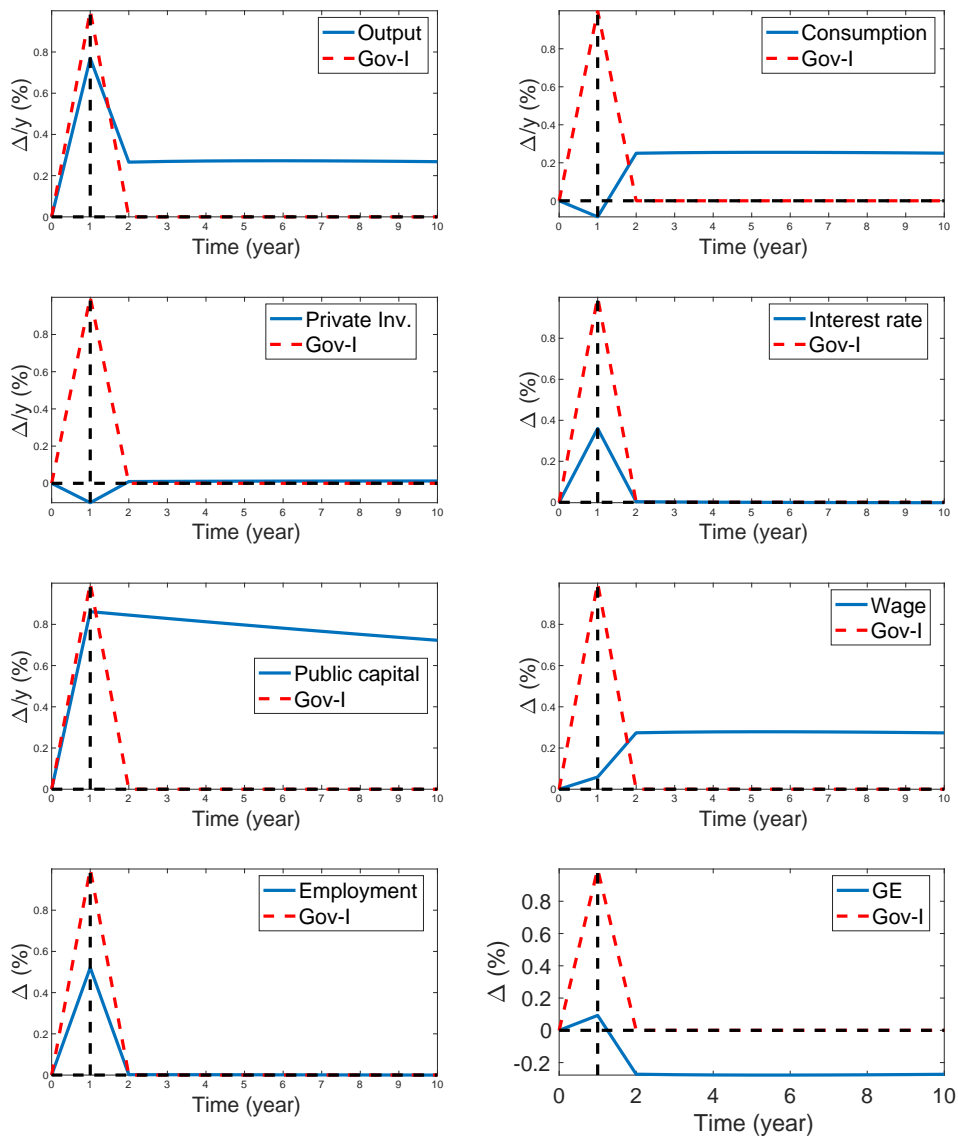
$$\begin{aligned} \frac{\partial}{\partial N} \text{Future MPK} &= \frac{\partial}{\partial N} \left( \theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{1}{\lambda-1}} F(\Theta) \\ &= \left( \frac{1}{\lambda-1} \right) \left( \frac{\lambda-1}{\lambda} \right) (1-\theta)N^{-\frac{1}{\lambda}} \left( \theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{2-\lambda}{\lambda-1}} F(\Theta) \\ &= \frac{1}{\lambda} (1-\theta)N^{-\frac{1}{\lambda}} \left( \theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{2-\lambda}{\lambda-1}} F(\Theta) > 0 \end{aligned} \quad (4)$$

where  $F$  is a function of the parameters,  $\Theta$ . If the elasticity of substitution  $\lambda$  is a finite positive number, the marginal benefit of firm-level investment increases in  $N$  through the increased future marginal productivity of capital, given the general equilibrium effect is fixed. However, if  $\lambda$  goes to infinity, the marginal benefit of investment does not depend on  $N$ . It is worth noting that the marginal benefit increases in  $N$  regardless of whether the public and private capital stocks are gross complements ( $\lambda < 1$ ) or substitutes ( $\lambda > 1$ ).

However, a fiscal policy affects the prices, which we denote as the general equilibrium effect. Regarding this, one of the most important channels is lump-sum taxation to finance infrastructure investment: the household reduces consumption to pay this lump-sum tax. Thus, the marginal utility of contemporaneous consumption increases, leading to an increase in the interest rate in the equilibrium. Then, the heightened interest rate strongly crowds out the firm-level investment despite the increased future MPK as in Equation (4).<sup>24</sup>

<sup>24</sup>Households has no precautionary saving motivation against the aggregate risk, as the economy

Figure 1: The impulse responses to the infrastructure spending shock



In the baseline impulse response, the interest rate increases by 0.39 percent after the infrastructure spending shock. This general equilibrium effect dominates the is abstract from the aggregate uncertainty.

increase in the marginal benefit from the greater public capital stock, leading to a net crowding out of the private investment.

In contrast, the employment response is not completely dampened by the general equilibrium (wage) effect. As the shock hits, employment increases by 0.52% despite the wage increase. Consumption and the GE effect are the mirror image of each other as the GE effect refers to the inverse of consumption under the log utility. In the shock period, consumption decreases strongly due to the lump-sum taxation, but it increases in the following period from the strong consumption smoothing motivation: all the expected future gains out of the infrastructure spending smoothly shift up the consumption level.

## 5.1 The role of elasticity of substitution between private and public capital stocks

The elasticity of substitution between private and public capital stock plays a key role in determining the marginal benefit of firm-level investment given a fiscal expenditure shock. Analytically, the change in the marginal benefit of firm-level investment over the elasticity given the fiscal spending shock can be captured by the cross derivative  $\frac{\partial^2}{\partial \lambda \partial N}$  Marginal benefit in the simple two-period model. Using Equation (3), we have the following equation:

$$\frac{\partial^2}{\partial \lambda \partial N} \text{Marginal benefit} = \frac{\partial}{\partial \lambda} \left[ \underbrace{\frac{1}{\lambda}}_{\text{Direct}} \underbrace{\frac{\left(\frac{1}{1+r}\right) MPK}{\left(\theta k' \left(\frac{N}{k'}\right)^{\frac{1}{\lambda}} + (1-\theta)N\right)}}_{\text{Indirect}} \right] + \frac{\partial}{\partial \lambda} \text{GE effect}$$

As displayed in the equation above, the elasticity of substitution affects the response of marginal benefit through two channels: 1) direct and 2) indirect channels. The direct channel refers to newly added capital being relatively less valu-



able when the public capital stocks are more substitutable with the private capital. The indirect channel refers to a change in marginal benefit of investment due to the change in the relative values of the existing public and private capital stocks. The direct channel predicts the marginal benefit of firm-level investment decreases in the elasticity, while the sign of the indirect channel cannot be analytically determined.<sup>25</sup>

Table 7: Fiscal multipliers with the different elasticities of substitutions

Fiscal multipliers	High ( $\lambda = 3$ )	Estimated ( $\lambda = 1.185$ )	Low ( $\lambda = 0.5$ )
<b>Output</b>			
Short-run	0.6129	1.0416	1.3296
Long-run	0.7734	1.8439	2.6087
<b>Short-run (2 years)</b>			
Consumption	-0.0799	0.1719	0.2553
Investment	-0.2257	-0.0942	0.0532
Labor income	0.3178	0.6590	0.9362
<b>Long-run (5 years)</b>			
Consumption	0.1835	0.9251	1.3078
Investment	-0.3087	-0.0617	0.2242
Labor income	0.4372	1.2904	1.9937

Therefore, the effect of input elasticity on the aggregate marginal benefit needs a quantitative analysis. For this analysis, we separately compute the stationary equilibria under the different elasticities of substitution.<sup>26</sup> Then, we measure the fiscal multipliers by tracking impulse responses of equilibrium allocations separately for these economies.

Table 7 reports the fiscal multipliers for different elasticities of substitutions between public and private capital stocks. As can be seen from the table, the low firm-level elasticity ( $\lambda = 0.5$ ) leads to 28% greater output fiscal multipliers compared to our estimated baseline. Especially when the elasticity is low, the private

<sup>25</sup>The sign of the effect also depends on the firm-level capital stock.

<sup>26</sup>Other parameters are assumed to be at the same level as the baseline estimates except for the prices. Therefore, it is the comparative statics in general equilibrium.

investment is not crowded out even in the presence of the general equilibrium effect. Due to the greater response from the private sector, in the low  $\lambda$  economy, consumption and labor income responses are also greater than in the high  $\lambda$  or baseline economy both in the short run and the long run. The result of low fiscal multipliers for the case of high input substitutability is consistent with the findings of [Ramey \(2020\)](#) and [Chodorow-Reich, Karabarbounis, and Kekre \(2023\)](#).<sup>27</sup>

Depending on the firm-level elasticity level, the fiscal multiplier varies significantly. Therefore, this analysis shows that the sharp measurement of the elasticity of substitution has the first-order importance for the fiscal multiplier analysis.

## 5.2 The role of the general equilibrium effect

In this section, we analyze the role of the general equilibrium effect on fiscal multipliers. For this analysis, we measure the fiscal multipliers by tracking impulse responses of equilibrium allocations to the fiscal spending shock while the prices are exogenously fixed at the stationary equilibrium level (partial equilibrium).

Table 8 reports the fiscal multipliers for different elasticity of substitutions in partial equilibrium. In the absence of the general equilibrium effect, the fiscal spending shock leads to a substantially greater fiscal multiplier than the one with the general equilibrium. Under the baseline policy, the multiplier jumps by around 79% ( $1.04 \rightarrow 1.86$ ) without the general equilibrium effect. Therefore, it is obvious that the general equilibrium effect significantly dampens the fiscal multipliers.

Especially none of the cases leads to a net crowding out of the private investment. Thus, it is followed by the large-scale labor income and consumption multiplier effect. This positive investment response to the fiscal spending shock is explained well by Equation (4).

One important caveat in the analysis is that without the general equilibrium

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<sup>27</sup>Under the estimated traded-nontraded elasticity at 3.17, [Chodorow-Reich, Karabarbounis, and Kekre \(2023\)](#) shows that the output fiscal multiplier of government investment is around 1.24 over the 7-year horizon.

Table 8: Fiscal multipliers in partial equilibrium

Fiscal multipliers	High ( $\lambda = 3$ )	Estimated ( $\lambda = 1.185$ )	Low ( $\lambda = 0.5$ )
<b>Output</b>			
Short-run	0.9012	1.8582	2.2832
Long-run	2.2405	5.0314	6.1574
<b>Short-run (2 years)</b>			
Consumption	-0.1359	0.6052	1.0492
Investment	0.0285	0.1892	0.2247
Labor income	0.5889	1.2935	1.6760
<b>Long-run (5 years)</b>			
Consumption	1.2094	3.4216	4.7162
Investment	0.0745	0.4769	0.5251
Labor income	1.5274	3.4781	4.5102

analysis, the fiscal multiplier can be significantly biased upward. Therefore, the general equilibrium framework is essential for analyzing the fiscal multiplier. However, models with micro-level heterogeneity are often abstract from the general equilibrium effect due to their high computational cost of estimation. We overcome this problem through our estimation strategy in Section 4.2 to obtain the market clearing prices simultaneously with the model parameter estimates.

### 5.3 Heterogeneous fiscal multipliers across the regions

In this section, we analyze the heterogeneous fiscal multipliers of the baseline infrastructure spending policy across Poor and Good states. Table 9 reports the fiscal multipliers for Poor state (column 1), for Good state (column 2), and for the aggregate level (column 3). As there is only a representative household in this economy, the state-level consumption is defined under the following assumptions:

- All the incomes are state-specific, and there is no cross-state transfer.
- Each equity is exclusively owned by the state's household.

- Bond holding and lump-sum subsidies are attributed to each state proportionately to the exogenous fiscal spending ratio.

Given these assumptions, the state-level consumption can be properly defined due to the separate budget clearing across the states. One can introduce two households in the model to capture Poor and Good households separately, but this can be done only at a high computational cost and the model complication.

As can be seen from Table 9, there are large asymmetries in the fiscal multipliers between the two states. In Good state, the output fiscal multiplier exceeds the unity, while Poor state's multiplier is less than one-tenth of the Good state. This is due in part to the productivity difference and to the difference in the public capital stock between the two states.

Table 9: Fiscal multipliers across the states

Fiscal multipliers	Poor state	Good state	Aggregate
<b>Output</b>			
Short-run	0.0819	1.2667	1.0416
Long-run	0.1370	2.2443	1.8439
<b>Short-run (2 years)</b>			
Consumption	-0.5322	0.3749	0.1719
Investment	-0.0104	-0.1139	-0.0942
Labor income	0.2390	0.8019	0.6590
<b>Long-run (5 years)</b>			
Consumption	0.0096	1.2719	0.9251
Investment	-0.0040	-0.0753	-0.0617
Labor income	0.7498	1.5725	1.2904

Importantly, the private investment in Good state is more severely crowded out by the public infrastructure spending than in Poor state. As we will clarify in Table 10, this is due to the large general equilibrium effect in the Good state. However, consumption and labor income multipliers are substantially greater in Good state than in Poor state.

Table 10 reports the fiscal multipliers across the states without the general equilibrium effect. Without the general equilibrium effect, the private investment fiscal multiplier is greater in Good state than in Poor state. This reflects that Good state has higher productivity and greater public capital stocks. Thus, the marginal benefit of investment is greater in Good state if the general equilibrium effect is turned off.<sup>28</sup> However, once the general equilibrium is considered, the Good state's private investment is more severely crowded out than the Poor state's private investment. This shows that the general equilibrium effect asymmetrically affects the Poor state's and the Good state's equilibrium allocations.

Table 10: Fiscal multipliers across the states in partial equilibrium

Fiscal multipliers	Poor state	Good state	Aggregate
<b>Output</b>			
Short-run	0.1441	2.2603	1.8582
Long-run	0.3670	6.1256	5.0314
<b>Short-run (2 years)</b>			
Consumption	-0.5184	0.8700	0.6052
Investment	0.0255	0.2276	0.1892
Labor income	0.1063	1.5735	1.2935
<b>Long-run (5 years)</b>			
Consumption	0.3040	4.1586	3.4216
Investment	0.0667	0.5732	0.4769
Labor income	0.2831	4.2343	3.4781

The consumption multiplier of Good state is also strongly dampened with the general equilibrium effect. In Good state, the short-run consumption multiplier reduces down by around 57% ( $0.87 \rightarrow 0.37$ ), while it reduces down by around 3% ( $-0.52 \rightarrow -0.53$ ) in Poor state.

<sup>28</sup>Equation (4) shows that the marginal benefit of firm-level investment increases in the public capital stock if  $\lambda \in (0, \infty)$ .

## 5.4 The marginal product of private and public capital

In this section, we assess the equilibrium level of the marginal product of private and public capital stock. Table 11 shows the marginal product of private and public capital stocks for the entire economy (column 1), the Good state (column 2), and the Poor state (column 3). In this economy, due to the presence of the capital adjustment cost at the firm level, the marginal product of capital varies across the firms.<sup>29</sup> We use the average marginal product of capital for the analysis.

Table 11: The marginal product of private and public capital

Marginal product of capital (MPK)	Aggregate	Good state	Poor state
Private	0.2840	0.3345	0.0426
Public	0.4799	0.5449	0.1691

The marginal product of public capital stock is substantially higher than the private counterpart. This shows that the current stock of public capital is less than the socially desired level.<sup>30</sup> Moreover, the public-to-private MPK ratio is more than twice greater in the Poor state than in the Good state. This shows that the Poor state's public capital provision shortage is more severe than the Good state's in equilibrium.

## 5.5 The role of time to build

In this section, we analyze the role of time to build on the fiscal multiplier. On top of the one-year time to build in the baseline, we assume there is an extra year of time to build for capital stock to be utilizable after the investment as in [Ramey](#)

<sup>29</sup>The convex adjustment cost depends on the capital stock of the firm, which makes the marginal cost of investment different across the firms. This leads to the heterogeneous marginal product of capital stock in equilibrium.

<sup>30</sup>If there were a competitive market for public capital, the price of the public capital would adjust in the direction to equate the shadow value of private and public capital.

(2020) (two years, in total). Therefore, the law of motion of the public capital stock is as follows:<sup>31</sup>

$$\begin{aligned}\mathcal{N}_{A,t+2} &= \mathcal{N}_{A,t+1}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2} \left( \frac{\mathcal{F}_t}{\mathcal{N}_{A,t+1}} \right)^2 \mathcal{N}_{A,t+1} \\ \mathcal{N}_{j,t+1} &= \zeta_j \mathcal{N}_{A,t+1} \quad \text{for } j \in \{P, G\} \\ F_t &= \begin{cases} F^{ss} + \Delta G & \text{if } t = 1 \\ F^{ss} & \text{otherwise} \end{cases}\end{aligned}$$

where  $F^{ss}$  is the stationary equilibrium level of fiscal spending on infrastructure. Due to the time lag between the fiscal policy shock and the arrival of the public capital stock, there exists a news component in the policy, which will be analyzed further in this section.

Table 12: Fiscal multipliers across the states under time to build of two years

Fiscal multipliers	Poor state	Good state	Aggregate
<b>Output</b>			
Short-run	0.0831	1.1179	0.9213
Long-run	0.1362	2.0521	1.6881
<b>Short-run (2 years)</b>			
Consumption	-0.5586	0.3387	0.1438
Investment	-0.0199	-0.1962	-0.1627
Labor income	0.1995	0.6643	0.5472
<b>Long-run (5 years)</b>			
Consumption	-0.0370	1.1951	0.8636
Investment	-0.0160	-0.1609	-0.1333
Labor income	0.6885	1.4050	1.1542

For this analysis, the fiscal multiplier is measured by the sum of the present values over the first three years for the short run and over the six years for the long

<sup>31</sup>For the consistency in the notation with the previous formulations, we leave the time index of the future public capital stock to be  $t + 1 + s$  where  $s = 1$ .

run after the initial fiscal spending shock.<sup>32</sup>

Table 12 reports the heterogeneous fiscal multipliers across Poor and Good states when there is time to build of two years. The aggregate fiscal multiplier decreases compared to the one without the extended time-to-build assumption (Table 9), consistent with Ramey (2020). In the short (long) run, the output fiscal multiplier decreases by 12% (8%) at the aggregate level. Especially, time to build strongly dampens the fiscal multiplier of Good state: 12% in the short run and 9% in the long run.

To illustrate the mechanism under time to build, Figure 2 plots the impulse responses of equilibrium allocations. Due to the extended time to build, the capitalized government expenditure in the dashed line spikes one year after the beginning of the endogenous responses in the equilibrium allocations. As the fiscal spending shock hits, consumption immediately drops as the lump-sum tax immediately puts downward pressure on the household's consumption. This makes the household more willing to supply the labor. On the other hand, the production side does not face any change in the infrastructure until one year after the shock. Therefore, the increased labor supply at the period of shock ( $t = 1$ ) leads to a lower wage and greater employment. Then, this feeds back into increased output at  $t = 1$ . The interest rate increases as the marginal utility of consumption at  $t = 1$  increases, resulting in a decrease in private investment. After the infrastructure spending becomes capitalized, the demand for labor increases while the willingness for the labor supply decreases (income effect). This leads to an increase in the wage while the employment stays almost unchanged from the stationary equilibrium level.

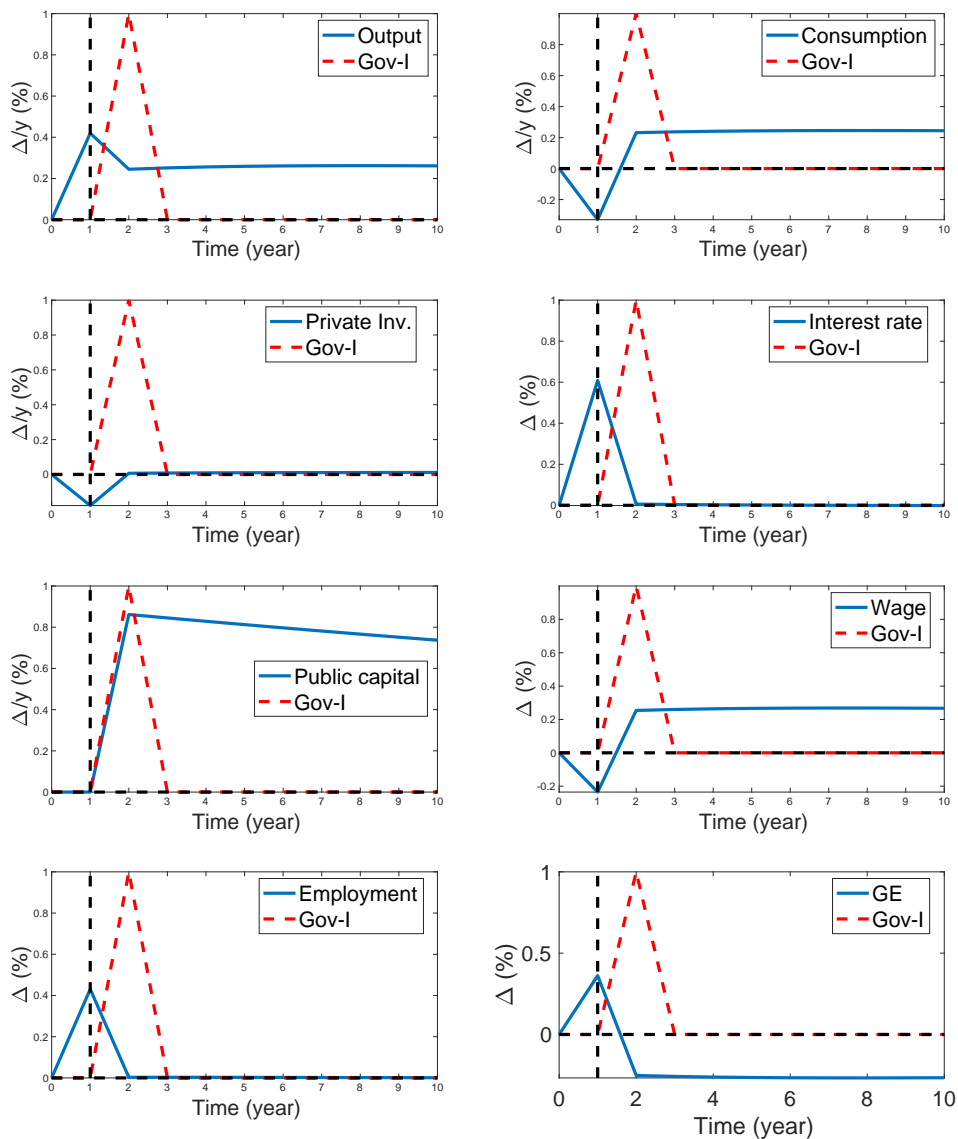
To quantify the general equilibrium effect, we compute the fiscal multipliers under the extended time to build assumption in the partial equilibrium, where the price is fixed at the stationary equilibrium level. Without the general equilibrium

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<sup>32</sup>Previously, it was 2 years for the short run and 5 years for the long run without the extended time to build.



Figure 2: The impulse responses to the fiscal policy shock under time to build of two years



effect, the Good State's output fiscal multiplier is 2.47, which is greater than when there is one year of time-to-build friction (2.2603 in Table 10). The Poor state's fiscal multipliers are also amplified more with the extended time-to-build friction in the

Table 13: Fiscal multipliers across the states under time to build of two years in the partial equilibrium

Fiscal multipliers	Poor state	Good state	Aggregate
<b>Output</b>			
Short-run	0.1503	2.4710	2.0301
Long-run	0.3836	6.5785	5.4015
<b>Short-run (2 years)</b>			
Consumption	-0.5111	0.9130	0.6414
Investment	0.0394	0.3484	0.2897
Labor income	0.1143	1.7653	1.4504
<b>Long-run (5 years)</b>			
Consumption	0.3606	4.4284	3.6506
Investment	0.0835	0.7036	0.5858
Labor income	0.2992	4.5795	3.7604

partial equilibrium, but the difference is not dramatic as in Good state.

This is due to the news effect that allows the agents with the rational expectation to adjust their allocations optimally even before the spending shock is capitalized. However, this effect is dominated by changes in the price once we consider the general equilibrium effect, as can be seen from the large difference between Table 12 and Table 13. The agents' adjustment before the shock capitalization results in wage and interest rate adjustment, dampening the fiscal multiplier even in a greater magnitude than the one-year time-to-build. This is because the interest rate adjustment occurs at one time, and the increased cost of investment at the period before the spending shock leads to a lowered capital stock. Under the real friction such as the convex adjustment cost, the lowered capital stock leads to a greater adjustment cost in the following period when the fiscal spending shock is materialized, leading to a substantially dampened fiscal multiplier. Therefore, this is an outcome of the interaction between the news effect and the real friction.

## 6 Conclusion

This paper analyzes the infrastructure investment multipliers through the lens of an estimated heterogeneous-firm general equilibrium model. In the model, the firm-level production function takes both private and public capital stocks as input factors. The elasticity of substitution between the two input factors is the crucial determinant of the fiscal multiplier through the firm-level investment channel. This elasticity parameter displays a substantial gap between the firm-level and the state-level (aggregate) measurements under the presence of non-rivalry of infrastructure. We theoretically explain the gap and quantitatively support our findings. According to our estimation, private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level.

We use our estimation method that extends the limited-information Bayesian estimation by augmenting market clearing conditions. Especially, the firm-level input elasticity parameter is estimated using the cross-sectional variation in the infrastructure and the private capital stock across the states. Based on the estimated model, the fiscal multiplier is around 1.04 over a 2-year period, which indicates a moderate net economic benefit from the infrastructure investment. When the infrastructure investment is implemented, the aggregate investment is crowded out, and the general equilibrium effect is the key driver for this crowding out.

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