Rising Intangible Capital and the Disappearance of Public Firms[†]

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April 26, 2023

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Abstract

Since the mid-1990s, the number of listed firms in the U.S. has decreased by half. Over the same period, the listed firms' financial disclosure has become significantly more opaque. To explain these observed patterns, we develop a general equilibrium model where the endogenous choices of going public or private and the transparency of the voluntary disclosure are characterized in the analytic form. In the equilibrium, the stock market with the directed search and the private equity market with the random search endogenously co-exist. According to the estimation, the increased intangible share is the key driver of the observed patterns, as sharing knowledge has become significantly costlier. Using the model, we characterize a policymaker's dilemma between maximizing welfare and productivity for the disclosure policy and analyze the optimal policy.

Keywords: Intangible capital, corporate disclosures, technology diffusion.

JEL codes: D24, G24, G38.

[†] We are grateful for Manuel Adelino, Santiago Bazdresch, Harold Cole, Jesús Fernández-Villaverde, Angus Foulis, Luca Guerrieri, Ryo Jinnai, Dirk Krueger, Adriano Rampini, Serene Tan, Skander Van den Heuvel, David N. Weil, and participants of the seminar at the Federal Reserve Board, Lisbon Macro Workshop, Hitotsubashi University, ITAM-PIER conference on macro-finance, and BIS-CEPR WE_ARE_IN Macroeconomics and Finance conference for their constructive comments and discussions. The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. All remaining errors are our own.

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1 Introduction

Since the mid-1990s, the number of listed firms in the U.S. has decreased almost by half. Over the same period, public firms' performance has become increasingly difficult to predict; reports of listed firms have become significantly more opaque. What are the driving forces for these changes? What are their macroeconomic consequences? This paper answers these questions through the lens of a general equilibrium model where a closed-form solution characterizes a rich set of equilibrium allocations. We then use the model to analyze the optimal disclosure regulation based on the equilibrium.

The U.S. Securities and Exchange Commission (SEC, hereafter) requires listed firms to publicly reveal their annual and quarterly financial information and disclose material events such as transactions involving shareholders and insiders. Moreover, public firms are not allowed to selectively disclose materials to some investors (e.g., Regulation Fair Disclosure of 2000) to protect investors and facilitate a fair capital market. However, the cost of disclosure is that it may also reveal crucial information to competitors (Bhattacharya and Ritter, 1983).

Support exists for the notion that private firms' ability to avoid public disclosures is an important factor in their decision to stay private. For example, Dambra, Casares Field, and Gustafson (2015) study the effect of Title I of the JOBS Act (Jumpstart Our Business Startups Act), which disburdens the IPO process by exempting emerging growth companies from certain accounting and disclosure requirements, and allows issuers to disclose information exclusively to investors, but not competitors, until the IPO becomes likely to succeed. They find that the act increased the volume of IPOs by 25% compared to their previous level; and this increase is concentrated in firms with a high cost of disclosure, such as firms in the tech sector. Aghamolla and Thakor (2022) exploit a shock to disclosure requirements in the biopharmaceutical industry to show that increased mandatory disclosure requirements for private firms significantly increases their propensity of going public.

Our hypothesis is that, given its nature, *intangible capital* is one of the most fragile input factors to the information disclosure. While companies may keep patents and trade secrets, some ideas are simply non-excludable: Once information about intangible capital is revealed, then it is easily mimicked. [1]

By estimating our model on U.S. firm data from 1992 to 2016, we show that the increased importance of intangible capital in production is one of the critical factors driving public firms' disappearance. As firms start adopting more intangible capital, which is subject to imitation risk to a greater extent, they have a stronger incentive to conceal information, leading to an increased tendency to remain privately held. This technological change also leads to less transparent reporting for firms that stay public, as observed in the data. The estimated model also predicts that *access to funds by private investors* has become easier, contributing to the reduction of public firms. We show that the disappearance of public firms and overall greater opacity in financial markets substantially reduce productivity and technological diffusion across firms.

The baseline version of our model envisions a key trade-off between mandated transparency and market output and productivity. We capture the government's regulation on information revelation by including a mandated minimum level of disclosure for listed firms in the model. Using the closed-form characterization of the equilibrium allocations, we are able to analyze globally how information regulation affects welfare, productivity, and output. In the baseline model, stricter regulation increases the welfare of risk-averse investors through more transparent information in the reports. Stricter regulation can, however, crowd out voluntary disclosure and even backfire through the extensive-margin channel, as more firms tend to stay in the private equity market, which is more opaque[2]. This reduces the technology diffusion

[1]We refer to those components of intangible capital that are not necessarily patentable or patented yet. Most importantly, early stages innovation and R&D, but also certain novel business methods and organizational innovations, branding and marketing strategies, employee training, proprietary information and trade secrets such as some formulas, customer lists, processes, and generally, firms' strategies and intentions that a public firm cannot selectively disclose.

[2]This is one of the core issues SEC is concerned about. For example, in a February 2017 speech,

across firms, which is followed by lower productivity and output.[3] We characterize this trade-off for the policy maker between welfare and productivity. The trade-offs make it hard for the policymaker to consider a dramatic shift in policy in a certain direction. Our estimation captures that disclosure policy has become stricter in the recent period. Moreover, we show that even small magnitudes in policy shift have a meaningful effect on the transparency of the reports, as much as a 9.4% increase in the transparency measure.

We start by collecting and analyzing macro and micro-level empirical evidence on which our structural framework is based. On the macroeconomic side, we revisit some well-known facts in the literature; the level of intangible capital has risen, and the number of public firms has decreased almost by 50% in the U.S. since 1996. Then, using U.S. Compustat data and data on earnings surprises from I/B/E/S, we construct a transparency measure of the firm-level disclosure. Our measure shows that the average transparency has significantly declined over the same period of the two aforementioned trends.

To investigate the relationship between transparency and intangible capital further we run a panel regression of the transparency on intangible capital with firm-level controls and fixed effects. We find that our measure of transparency, which is based on professional analysts' forecast errors, is significantly negatively correlated with the firm level of intangible capital. This cross-sectional fact constitutes an important bridge to link the two macro facts: rising intangible capital and declining transparency. We interpret the result in the following way: the negative relationship between intangible and transparency, proxied by forecast accuracy can be due to two

SEC Commissioner Kara Stein posed a question regarding additional disclosures and regulation around private market investment: "We also need to understand why more companies are staying private for longer periods of time. Should we apply enhanced disclosure laws to these private companies? Or perhaps they require a unique set of rules." See "The Markets in 2017: What's at Stake?" Commissioner Kara M. Stein, SEC website, https://www.sec.gov/news/speech/stein-secspeaks- whats-at-stake.html

[3]We characterize the space of parameters for which this is true. In some parametric regions, this dilemma may not show up; all of the welfare, productivity, and output can improve by the change in the information regulation, as the divine coincidence in the literature on monetary policy.

reasons: one, firms with high levels of intangible tend to be less transparent and, therefore, more difficult to forecast. Two, it may be that, given a certain level of disclosure and transparency, firms with high intangible capital are inherently more challenging to forecast due to their nature. Using our model, we set out to disentangle the two forces and their effect.

In order to analyze these empirical patterns and their impact on the macroeconomic allocations in a unified framework, we introduce a general equilibrium model of heterogenous firms where financing decisions are endogenous. In the model, ex-ante homogeneous firms choose whether to go public or private, the level of intangible capital stock, and the transparency of their intangible capital. The intangible capital is subject to diffusion to other firms as an externality in the form of productivity gain. If a firm goes private, transparency is minimal, and there is no technology diffusion to the other firms. Due to the externality, a firm with a greater intangible capital stock has less incentive to choose high transparency. However, a risk-averse representative household values transparency, giving better funding opportunities for firms with greater transparency. This generates a clear trade-off in choosing low transparency: it reduces intangible leakage, but it worsens funding opportunities. In equilibrium, the transparency distribution and the portion of private firms are determined at the price and externality level where all firms become indifferent.

A policymaker sets a mandated minimum transparency level, and all the listed firms need to sustain a greater transparency level than the mandated level. Therefore, a higher mandated transparency level decreases the incentive to go public, leading to more private firms in the equilibrium. However, a stricter policy would lower uncertainty for investors, achieving greater welfare. Therefore, the policymaker also faces a sharp trade-off.

To evaluate the consequences of the information disclosure policy, we provide three criteria: output, productivity, and investors' welfare[4]. In the estimated model,

[4]The mission of SEC is "The mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation." See "Our Goals", SEC website,

a regulation policy can achieve only higher output and productivity or higher welfare, which shows the policymaker's dilemma. From the perspective of the protection of investors, we find the recent regulation has substantially improved welfare. However, we also document that it has led to a substantial loss in productivity in the production sector.

One of the advantages of our model is that these decisions have a closed-form solution, which allows us to characterize the model and optimal policy globally and cleanly. The model resembles Burdett and Mortensen (1998), as it characterizes the general equilibrium distribution of endogenous objects in closed form. In their model, the wage distribution is endogenously determined, as the model captures the endogenous wage postings from the firm side. Similarly, in our model, a risk-averse representative household with CARA utility endogenously chooses the amount of funding for each transparency level.

Contribution and literature Our paper delivers two main contributions to the literature. First, we provide a theoretical and quantitative model framework that analyzes the effect of rising intangible capital on the firm-level financing decision[5]. Using the estimated model, we show that the rising intangible has been the key driver of the disappearing public firms. Also, the qualitative aspect of our model is worth highlighting as it allows closed-form characterization of rich equilibrium allocations, including the distribution of public and private firms. This tractability promotes the transparent illustration of endogenous mechanisms in our model. Also, it enables a fast and accurate quantitative analysis.[6]

Second, we bring a novel policy angle, information regulation, to the table and

https://www.sec.gov/our-goals.

[5]Kahle and Stulz (2017) hinted at the possibility of the role of intangible capital in the observed declining trends of listed firms. However, the structural analysis of the channel has been missing in the literature.

[6]The portion of public firms are often substantially smaller than the private firms in many countries. Then, a computation error of 0.1% in the portion of public firms is a substantially large error. Therefore, a highly-computational model is easily subject to a high approximation error in capturing the portion of large firms.

analyze its macroeconomic trade-off. From the tractable general equilibrium model, we show that in a reasonable range of parameters, a policymaker faces a dilemma between welfare and productivity. We believe the closed-form characterization of our model would serve as a useful tool for future research on the information regulation policy.

Two strands of the literature are closely related to this paper. The first is the literature that studies the rising importance of intangible capital. It was only around a decade ago that intangible capital was first recognized as an important macroeconomic factor that affects economic growth and the business cycle. For example, McGrattan and Prescott (2010) and McGrattan (2020) highlight the importance of intangible capital as a key input factor for production and show how mismeasurement of intangible capital may mislead the neoclassical model predictions in terms of economic growth. Relatedly, Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2014) modeled plant-level intangible capital as an important input for production. Mainly, their intangible capital refers to organizational capital that is partly firm-specific and partly embodied in key labor inputs.

We contribute to this literature by analyzing a novel macroeconomic implication of the rising share of intangible capital. The intangible capital has become an important source of competitiveness, leaving firms to put a great effort into R&D or developing a productive corporate culture. However, the intangible capital has a strong spillover effect, which can benefit competitors besides the owner firm. Therefore, the rising importance of intangible capital has naturally increased a firm's incentive to stay opaque in its disclosure. Using our model, we theoretically and quantitatively analyze how this change affects the macroeconomy in terms of welfare and productivity.

The second literature is about the disappearance of the listed firms. Different explanations have been put forward to shed light on this issue. For example, Gao, Ritter, and Zhu (2013) point to the increase in mergers and acquisitions among U.S. firms; Doidge, Karolyi, and Stulz (2017) conjecture that as markets have become more globally integrated, the net benefits of going public in the U.S. versus in other markets have decreased; Ewens and Farre-Mensa (2020) argue that the deregulation of securities laws (NSMIA 1996) improved the private equity market, which reduced the incentives for firms to go public.

In this paper, we propose a complementary explanation. We argue that the rise of intangible capital, especially the components of intangible capital that could benefit competitors besides the owner firm, has increased the cost of disclosing information and made staying private more attractive. The estimated model also predicts that access to funds by venture capital firms, private equity funds, and other private investors has become easier.

2 Empirical analysis

In this section, we empirically analyze the observed patterns in the number of listed firms and the intangible capital stocks. In the firm-level analysis, the key variables are intangible capital stock and firm transparency. We first explain our measurement of these two variables. Then, using the measured allocations, we analyze the time-series patterns at the macro and sector levels and the cross-sectional relationships at the firm level.

2.1 Data and Measurement

In this section, we explain how we measure the intangible capital stock of public firms and firm transparency.

We use firm level data on public U.S. firms from Compustat covering the period from 1985 to 2016 to measure firm-level intangible capital stock. Our baseline measure of internally generated intangible capital is the sum of two components: (i) estimated knowledge capital, calculated using research and development expenditure (XRD); and (ii) estimated organizational capital, calculated using selling, general, and administrative expenses (XSGA). The measure is constructed using the perpetual inventory method, which aggregates net investment flows over the life of the firm:

$$\begin{aligned} \text{[Knowledge capital]}: \quad & k^G_{i,t} = (1-\delta_G)\, k^G_{i,t-1} + R\&D_{it}, \\ \text{[Organizational capital]}: \quad & k^O_{i,t} = (1-\delta_O)\, k^O_{i,t-1} + \gamma_O SG\&A_{it}, \end{aligned}$$

where $R\&D$ is research and development expenditure expenditure; $SG\&A$ is selling, general, and administrative expenses. All the intangible flow variables are deflated by the price of intellectual property products from National Income and Product Accounts data (NIPA Table 1.1.9, line 12). δ_G and δ_O are the depreciation rates.[7] γ_O is the fraction of $SG\&A$ expenditure that adds to the intangible capital stock. We assume $\gamma_O = 0.20$ following Falato et al. (2022). All the empirical results are robust over other reasonable choices of this parameter level.

Then, we calculate the net change in the acquired amount of intangibles from the changes in the book values of intangibles after the amortization, using Compustat variables INTAN and AM. We obtain the acquired intangible stock $k^B_{i,t}$, applying the perpetual inventory method to the deflated net change in the intangibles.

Our final measure of firm-level intangible capital stock $k^I_{i,t}$ is obtained by combining the internally generated intangible stocks and the acquired intangibles stocks:

$$k^I_{i,t} = k^G_{i,t} + k^O_{i,t} + k^B_{i,t}$$

In order to get a measure of firms' transparency, we leverage information on earnings surprises. Every quarter, professional financial analysts produce and disseminate forecasts of firms' earnings, based on their timely access to all available information on and off the balance sheet. Our assumption is that it is easier to forecast firms that

[7] We use $\delta_G = \delta_O = 0.15$, which is around the levels estimated in the literature (Corrado, Hulten, and Sichel, 2009).

disclose more information, and so we can proxy firm transparency using the accuracy of these forecasts.[8]

Specifically, earnings surprise $ES_{i,j,t}$ is defined as the difference between a firm's announced actual earnings per share $e_{t,i}$ and the earnings forecast per share $\epsilon_{i,j,t}$ made by an analyst for that firm, normalized by the price of a share $P_{i,t}$:

$$ES_{i,j,t} := \frac{\epsilon_{i,j,t} - e_{i,t}}{P_{i,t}}$$

where t is the indicator of a quarter; i and j are firm and analyst indicators, respectively. Thus, the surprise is measured at the analyst-firm level.

The data on analysts' forecasts come from the Institutional Brokers' Estimate System (I/B/E/S). The dataset collects quarterly estimates made by professional financial analysts on the future earnings of publicly traded companies. We closely follow Dellavigna and Pollet (2009) for the detailed steps of the earnings surprise calculation.

Then, we define two different proxies for the transparency of the balance sheet at the firm level. The first is the inverse of the median absolute value of earnings surprises:

$$Transparency^1_{i,t} := \frac{1}{median(|ES_{i,j,t}|)}$$

This proxy is based on the intuition that more transparent firms have lower absolute earnings surprise, on average.[9]

Our second proxy is the inverse of the variance of earnings surprises:

$$Transparency^2_{i,t} := \frac{1}{var(ES_{i,j,t})}$$

[8] Forecast errors may be influenced by other factors as well, namely, idiosyncratic and aggregate risk, analysts coverage and effort, and varying inherent difficulty of the task. We discuss how these factors influence our analysis in the next sections.

[9] The median is used instead of the average, this is to rule out the outlier's level effect on the average.

The intuition behind this proxy is that more transparent firms have lower dispersion in the earnings surprise among the analysts, on average. Therefore, the second proxy is calculated only for firms with multiple analysts' forecasts available in the data. In our dataset, the average number of analysts covering a firm is three.

2.2 Trends in the number of listed firms, intangible capital, and transparency

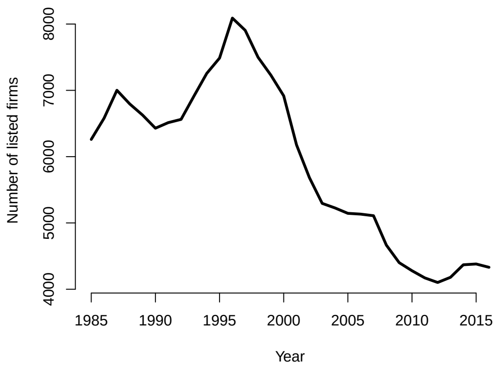
Figure 1 plots the time series of the variables of interest from 1985 until 2015. Panel (1a) plots the number of listed firms in the U.S. The data is from the World Development Indicators (WDI) by World Bank.[10] As shown in the figure, there has been a gradually rising trend in the number of listed firms until the mid-1990s. Then, after the peak in the mid-1990s, the number of listed firms steeply declined to almost half the level at the peak year: 8,090 listed firms in 1996 reduced to 4,102 listed firms in 2012. Panel (1b) shows that listed firms have been declining not only in absolute number, but also as a share of all firms in the U.S.

Panel (1c) shows the time series of the ratio between the total intangible capital stock of public non-financial corporations and GDP. Over the thirty years, the ratio has dramatically increased from 10% to 50%. This shows how fast intangible capital in the U.S. has grown.

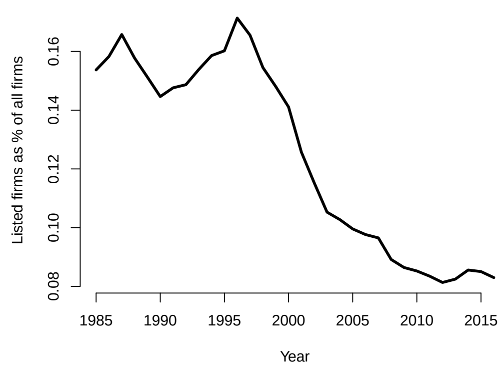
Lastly, panel (1d) shows the time series of the cross-sectional average transparency. The overall patterns of both transparency measures closely mimic the one in the number of listed firms: transparency has increased until the mid-1990s and decreased after the peak in 1996[11]. The time-series correlation between the transparency measure and the number of listed firms is 0.80 for the first measure and 0.61 for the second mea-

[10] The number of listed firms in WDI is only negligibly different from the one in the Compustat data.

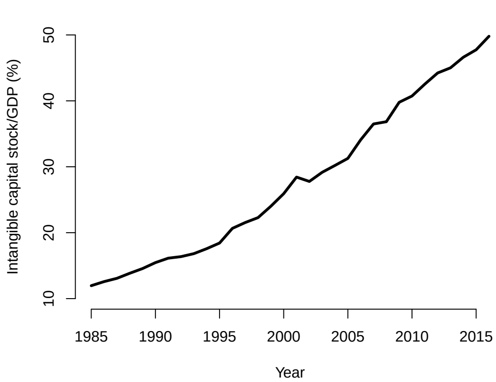
[11] Recessions and especially the Great Recession represent a big shocks to earnings surprises. In order to take that into account, we also measure average transparency by excluding recession periods as measured by the NBER, and we still find that average transparency has been declining.



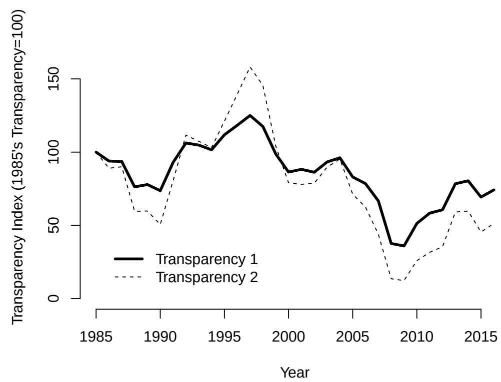
(a) Number of listed firms



(b) Listed firms as share of all firms



(c) Intangible/GDP



(d) Average transparency

Figure 1: **Time series of aggregate variables.**
This figure shows the trend in the number and share of listed firms, intangible capital, and firm transparency in the U.S. Data comes from Compustat, I/B/E/S, and the World Development Indicators. See Section 2.1 for details on measurement.

sure, and all are statistically significant. This co-movement between the number of listed firms and the average transparency is the key motivation of this paper: what drives such co-movements?

2.3 The trends by industries

In this section we show trends in the number of firms, intangible capital, and transparency by macro industries. Panel 2a shows the trend in the number of firms for the Information and Services sector, excluding trade and transportation, Manufacturing, and other sectors (Trade and transportation, Agriculture and Mining, Construction). All sectors show an initial increase and then a decline after the mid-nineties, although the decline is much more pronounced in the information and service sector. Panel 2b shows the *intangible intensity*, defined as the ratio of intangible asset to total intangible and tangible asset values, for the same industries. Manufacturing had historically a higher intangible intensity, which has been slightly taken over in the early 2000s by the service sector.

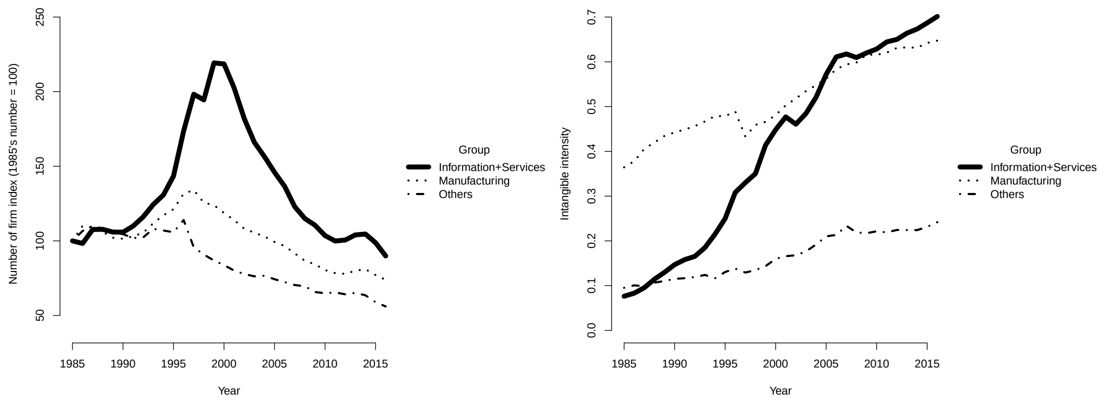
Finally, Figure 3 shows our transparency measures for the information and service sector, compared to all other sectors. The information and service sector has a lower transparency over the entire period, and both time series of transparency for all sectors have also declined over time.

In Appendix A, we show the trends for more disaggregated industries. We also report the trends in intangible capital using internally generated R&D only, so that the numbers on intangible intensity can be compared to the ones produced by the BEA.

2.4 Cross-sectional evidence

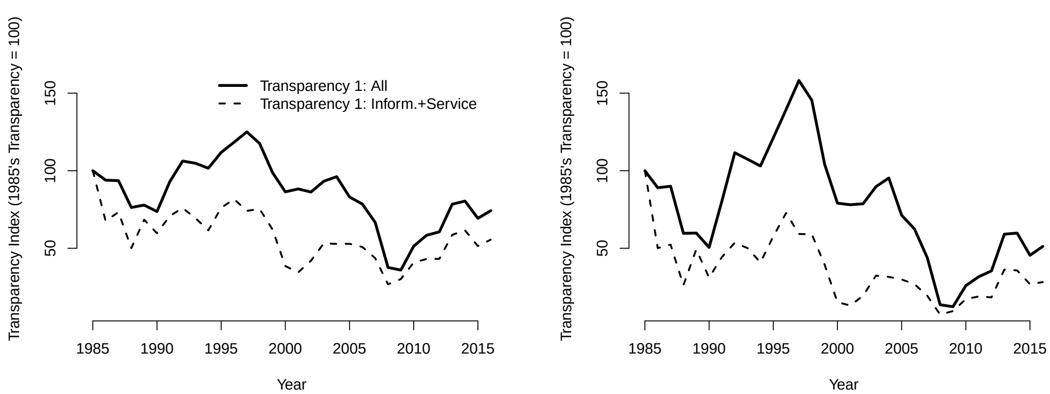
In this section we describe cross-sectional evidence that links high reliance on intangible capital with the value of earning surprises. We run the following regression on our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least one analysts is available:

$$\mathrm{y}_{f,t} = \theta_t + FEs + \beta \times \text{Intangible over total assets}_{f,t} + \gamma \times X_{f,t} + \varepsilon_{f,t}$$



(a) Number of listed firms (b) Intangibles intensity

Figure 2: **Number of listed firms and intangible intensity by industry.** This figure shows the trend in the number of listed firms and intangible capital intensity in the U.S. Intangible intensity is defined as the ratio of intangible asset to total intangible and tangible asset values. The groups are defined as Information and Services, excluding trade and transportation, Manufacturing,and other sectors (Trade and transportation, Agriculture and Mining, Construction). Data comes from Compustat. See Section 2.1 for details on measurement.



(a) Transparency 1 (b) Transparency 2

Figure 3: **Time series of transparency for information and service industries.**

This figure shows the trend in in transparency for information and service industries compared to all other industries. Information and Services excludes trade and transportation. Data comes from Compustat and I/B/E/S. See Section 2.1 for details on measurement.

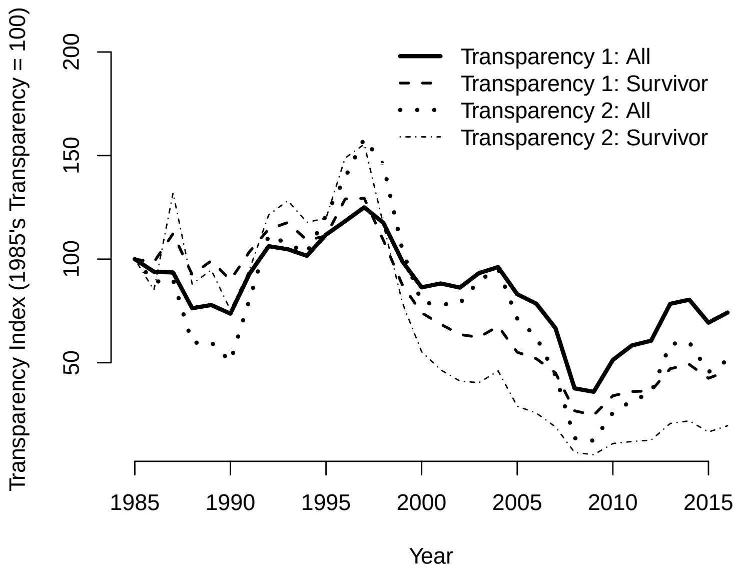


Figure 4: **Time series of transparency: all firms vs. survivors**. This figure shows the trend in in transparency for all firms vs. survivors. Data comes from Compustat and I/B/E/S. See Section 2.1 for details on measurement.

where $y_{f,t}$ is either our first or second transparency measure as described in section 2.1. θ_t are year fixed effects and FEs include industry fixed effects. $X_{f,t}$ represents firm controls. [12] The firm-level controls include book-to-market ratio, return on asset, leverage (total debt over total asset), tangible capital, sales, age (from the IPO year), and the number of analysts. Intangible and tangible capital are normalized by the total asset. Since information on firms' characteristics is only available at the fiscal year level, we average all observations of earnings surprises for a given firm in a given fiscal year.

Table 1 reports the results. The regressions show that intangible capital and transparency are inversely related, i.e., firms that have a higher share of intangible capital compared to their size are more difficult to forecast. Specifically, an increase in one standard deviation in intangible capital over assets decreases the value of

[12] Firm-level controls and regression specifications are based on Li (2010) and Bird, Karolyi, and Ruchti (2017).

the first transparency by 0.31 standard deviations, and the variance of the second transparency measure by 0.3 standard deviations

We interpret the result in the following way. Given the inclusion of year fixed effects and the number of analysts covering a given firm, we can exclude the effect of a gradual worsening of analysts' ability and effort, analysts' coverage, and common changes in idiosyncratic and aggregate risk. Therefore, we can directly link the rise in intangible capital with a decline in the ability of the market to forecast a firm. This relationship can be due to two reasons: one, firms with high intangible intensity tend to be less transparent, and, therefore, more difficult to forecast. Two, it may be that, given a certain level of disclosure and transparency, firms with high intangible intensity are inherently more challenging to forecast due to their nature. We include both possibilities in our model and set out to disentangle the two effects using our structural estimation.

	Dependent Variables:	
	Transparency 1	Transparency 2
$Intangible_{it}$	-0.31 (0.049)	-0.303 (0.096)
Industry FE	Yes	Yes
Year FE	Yes	Yes
Controls	Yes	Yes
Two-way cluster	Yes	Yes
Observations	256,962	256,962
R^2	0.275	0.289

Table 1: Regression of transparency proxies on intangibles

This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least one analysts is available:

$$\mathrm{y}_{f,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{f,t} + \gamma \times X_{f,t} + \varepsilon_{f,t}$$

where $y_{f,t}$ is either the inverse absolute value of earning surprises from the consensus, or the inverse of variance of earning surprises when more than one analyst forecast is present. θ_t are year fixed effects and FEs include industry fixed effects. $X_{f,t}$ represents firm controls.

3 Baseline model

In this section, we introduce a general equilibrium model, where the firm-level equilibrium allocations are characterized in the closed form. Using this model, we will qualitatively and quantitatively analyze the cause and consequences of the empirical patterns we have shown in the previous section.

We consider a stand-in household and a continuum of measure one of the ex-ante homogeneous firms. The model is static.[13] A representative household decides its asset portfolio and consumes the payouts from the portfolio. An entrepreneur decides in which market the firm operates between the public and non-listed markets. If a firm is listed, the entrepreneur chooses the disclosure level of the firm's intangible capital to the public, which we define as transparency. On the other hand, the entrepreneur does not disclose any intangible to the public if a firm is private.

3.1 Household

A stand-in household decides the asset portfolio and consumes the portfolio return. The household is given a wealth level $a > 0$. The household is risk-averse, and the utility takes the following constant absolute risk aversion form (CARA):

$$u(C) = -e^{-\Lambda C}$$

where $\Lambda > 0$ is the absolute risk aversion parameter.

In the listed market, the household forms a belief about the return $\widetilde{r}(q)$ based on a balance sheet information of a listed firm with a transparency level q and the

[13] The model is intended to capture an equilibrium that is formed over long years. Therefore, the dynamic aspect is abstracted. Also, the static setup gives a great degree of tractability in the model, as will be described in the equilibrium analysis.

mandated transparency level $\overline{q}$. The belief about the return is assumed as follows:

$$\begin{aligned}\widetilde{r}(q) &\sim_{iid} N\left(\overline{r}(q), \frac{1}{\xi + (\overline{q} + q)\psi}\right) \\ \text{s.t.} \quad \overline{r} &= \frac{\pi(q)}{P(q)}\end{aligned}$$

where $q \in [0, 1 - \overline{q}]$ is a transparency of the balance sheet information;[14] $\overline{q}$ is the mandated transparency required by the policy maker; ξ is the baseline information level a household has about both listed and non-listed firms; $\psi > 0$ is the marginal contribution of transparency to the household's information about the listed firm.[15] $\pi(q)$ is the profit of the firm with transparency q; $P(q)$ is the price of the firm with transparency q.

In the non-listed equity market, the household forms the following belief about the non-listed firms:

$$\begin{aligned}\widetilde{r}^N &\sim_{iid} N\left(\overline{r}^N, \frac{1}{\xi}\right) \\ \text{s.t.} \quad \overline{r}^N &= \frac{\pi^N}{P^N}\end{aligned}$$

where π^N and P^N are the profit and price of a non-listed firm. As non-listed firms do not disclose any information publicly, the household does not distinguish a non-listed firm from another.

[14] The range of transparency is assumed at our convenience. However, the qualitative and quantitative results of this paper are unaffected from this normalization assumption.

[15] We regard ξ and ψ as the functions of a structural parameter θ, the share of intangible capital in the production function. Intuitively, the importance of intangible capital in the production function affects the information quality household has about each firm. We do not impose any structural assumption on these two functions. Instead, we identify the level of ξ and ψ in our estimation using the firm-level data. Then, in the quantitative analysis, we assume ξ and ψ are simultaneously affected by the variation in θ.

Then, the household solves the following portfolio choice problem:

$$\max_{x(q),x^N} \quad \mathbb{E}(-e^{-\Lambda C})$$
$$\text{s.t. } C = \int x(\widetilde{q})\widetilde{r}(\widetilde{q})d\widetilde{q} + x^N\widetilde{r}^N, \quad \int x(\widetilde{q})d\widetilde{q} + x^N = a,$$

where $x(q)$ is the funding supply for firms with transparency level q; x^N is the funding supply for non-listed firms. As the model does not include the inter-temporal decision of the household, all the payoffs from the equity investment are consumed.

3.2 Technology

A measure one of the ex-ante homogeneous firms produces output using two inputs: tangible (k_T) and intangible capital (k_I). In this economy, there are two types of production technologies. One is listed firms' production technology, and the other is non-listed firms' production technology.

3.2.1 Production function of listed firms

A listed firm i operates using the following production function.

$$f^L(k_i^T, k_i^I, q_i; \overline{q}, \Phi^{ex}) = z(k_i^T)^\alpha(k_i^I(1-\overline{q}-q_i))^\theta(\Phi^{ex})^\gamma$$

where $\overline{q}$ is the mandated portion of intangible disclosure imposed by the policy maker; q_i is the voluntarily disclosed portion of intangible; Φ^{ex} is the shared intangible capital from all other firms; z is a constant aggregate productivity level; α and θ are the tangible and intangible shares. γ is the scale parameter for the externality. We assume $\alpha + \theta + \gamma \leq 1$.

Importantly, we assume the revealed portion of intangible capital disappears from the private intangible stock. This assumption is to let the revealed intangible capital be symmetrically used between the disclosing firms and the free-riding firms without double counting. If this symmetry is not guaranteed, partial knowledge sharing needs to be specified, which requires an additional intensive margin in the shared information. We simplify the model by assuming pure symmetry to avoid such complications.

We assume that a listed firm's disclosed portion of intangibles can range from q to 1, which does not rule out the possibility of publicly sharing nearly all intangibles. Therefore, the intangible in this model does not include patents or intellectual properties that are legally protected in terms of ownership. Therefore, we treat these assets as tangible assets.[16]

We assume a firm i's disclosed intangible q_i is perfectly substitutable by the other disclosed intangible. Therefore, the shared intangibles are aggregated in the following additive form:

$$\Phi^{ex} = \int_0^1 1_{\{i \in \text{Listed}\}} \times k_{I,i} \left(\underbrace{\overline{q}}_{\text{Disclosure mandated by the policy maker}} + \underbrace{q_i}_{\text{Voluntary disclosure}} \right) di$$

A firm chooses first the voluntary disclosure level of the intangible before the operation. The choice problem of voluntary disclosure is elaborated on in the following section.[17] The ex-post profit of a firm with voluntary transparency q_i is obtained after taking out the operational costs $rk_i^T + pk_i^I$ from the revenue:

$$\pi(q_i; \overline{q}, \Phi^{ex}) := \max_{k_i^T, k_i^I} \quad z(k_i^T)^{\alpha}(k_i^I(1 - \overline{q} - q_i))^{\theta}(\Phi^{ex})^{\gamma} - rk_i^T - pk_i^I$$

where r is the capital rental rate, and p is the $R\&D$ cost per unit of intangible capital. For the notational brevity, we assume r and p already include the depreciation rates.

[16]Given these assets are even used as collateral in reality, the exclusion of them from the definition intangible is desired for the focus of this paper. In our estimation, we target the intangible share calculated based on the expenditures rather than the stock. Therefore, the protected intangible assets, such as patent do not significantly affect the main results.

[17]The assumption of timing is solely for the descriptive purpose. Even if the decision of input levels and the disclosure level occur simultaneously, the model stays unaffected.

3.2.2 Production function of non-listed (private) firms

If a firm is private, it does not disclose the intangible capital publicly. The production function of a non-listed firm i is as follows:

$$f^N(k_i^T, k_i^I; \Phi^{ex}) = z(k_i^T)^\alpha (k_i^I)^\theta (\Phi^{ex})^\gamma$$

Except for the disclosure of the intangible capital, the production function is assumed to take the same form and parameters as the one for the listed firms. The profit is also symmetrically defined as listed firms:

$$\pi^N(\Phi^{ex}) := \max_{k_i^T, k_i^I} \quad z(k_i^T)^\alpha (k_i^I)^\theta (\Phi^{ex})^\gamma - rk_i^T - pk_i^I$$

3.3 Financial markets

In this section, we characterize the financial market in the model. The funding supply is driven by the representative household's portfolio choice problem. The funding demand is determined by each firm's value maximization problem.

3.3.1 Funding supply: The household's mean-variance portfolio

From the $i.i.d$ assumption of the stock return uncertainty, the consumption (income) satisfies

$$C \sim N\left(\int x(\widetilde{q})\overline{r}(\widetilde{q})d\widetilde{q} + x^N\overline{r}^N, \int x(\widetilde{q})^2 \frac{1}{\xi + \psi(\overline{q} + q)} d\widetilde{q} + (x^N)^2 \frac{1}{\xi}\right).$$

Then, the investors' expected utility maximization problem is translated into the following form:[18]

$$\max_{\int x(\widetilde{q})d\widetilde{q}+x^N=a} \quad -e^{-\Lambda\left(\int x(\widetilde{q})\frac{\pi(\widetilde{q})}{P(\widetilde{q})}d\widetilde{q}+x^N\frac{\pi^N}{P^N}-\frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q}-\frac{\Lambda}{2}(x^N)^2\frac{1}{\xi}\right)}.$$

After a strictly-increasing (log) transformation, the problem reduces down to

$$\max_{\int x(\widetilde{q})d\widetilde{q}+x^N=a} \quad \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{P(\widetilde{q})}d\widetilde{q}+x^N\frac{\pi^N}{P^N}-\frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q}-\frac{\Lambda}{2}(x^N)^2\frac{1}{\xi}.$$

The first-order condition with respect to $x(q)$ yields

$$\frac{\pi(q)}{P(q)}-\Lambda x^*(q)\frac{1}{\xi+\psi(\overline{q}+q)}-\mu=0,$$

where μ is the Lagrange multiplier of the wealth constraint. From this equation, we can derive the following supply curve of funding for the listed market:

$$x^*(q)=\frac{\pi(q)/P(q)-\mu}{\Lambda/(\xi+\psi(\overline{q}+q))},$$

where $x^*(q)$ is the funding supply in a dollar amount for firms with the transparency level q. So, the household is willing to invest $\frac{\pi(q)/P(q)-\mu}{\Lambda\frac{1}{\xi+\psi(\overline{q}+q)}}$ in the firms with transparency level q.

Similarly, from the first-order condition with respect to x^N, the funding supply curve for non-listed firms is characterized as follows:

$$x^{N*}=\frac{\pi^N/P^N-\mu}{\Lambda/\xi}.$$

[18] The derivation of the mean-variance portfolio objective function is as follows: consider a random variable, $y\sim N(\mu_y,\sigma_y^2)$. Then,

$$\mathbb{E}(-e^{-\Lambda y})=-\mathbb{E}(e^{-\Lambda y})=-e^{-\Lambda(\mu_y-\frac{\Lambda}{2}\sigma_y^2)}.$$

The last equation is derived from the moment generating function of the normal distribution.

From this point on, we assume the representative household has a large enough wealth a, as our interest is not on the household's constrained optimization. Thus, $\mu = 0$.

3.3.2 Funding demand: Listed firms' value maximization

A manager of a firm chooses where to operate to maximize the firm's price. The price is interchangeable with the value of a firm. The decision problem of where to operate is characterized as follows:

$$\max\{\max_{q \in [0, 1-\overline{q}]} P(q), P^N\}.$$

where $P(q)$ is the price of the firm operating in the listed market with the transparency level at q. P^N is the price of a non-listed firm.

In the funding market for the listed firms, the price of a firm, $P(q)$, is determined at the level where funding supply in the number of firms $\frac{x^*(q)}{P(q)}$ meets the funding demand in the number of firms $\mathcal{M}(q)$. Thus, the market-clearing condition is as follows:

$$\frac{x^*(q)}{P(q)} = \mathcal{M}(q).$$

Recall that a manager needs to determine the transparency level, after going on the listed market:

$$\max_{q \geq 0} P(q).$$

Given the funding demand and the market-clearing condition, this problem is equivalent to the following form:

$$\max_{q\geq 0}\sqrt{\frac{\pi(q)}{\Lambda\frac{\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}}}$$

which is equivalently transformed to

$$\max_{q\geq 0}\frac{\pi(q)}{\frac{\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}}.$$

Now, we define a net funding intensity $\phi^L(q)$ as follows:

$$\phi^L(q) := \frac{\xi+\psi(\overline{q}+q)}{\mathcal{M}(q)}$$

Therefore, a listed firm's problem can be summarized as the following form:

$$J^L(\mathcal{M}) = \max_{q}\max_{k_T,k_I} \quad \left(zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I\right)\phi^L(q)$$
$$\text{s.t. } \phi^L(q) = \frac{\xi+\psi(\overline{q}+q)}{\mathcal{M}(q)}$$

where J^L is the value of a listed firm given the distribution of listed firm $\mathcal{M}$.[19] The solution to this problem characterizes the funding demand in the listed market.

3.3.3 Financial market for non-listed firms

The price of a non-listed firm, P^N, is determined at the level where funding supply in the number of firms, $\frac{x^{N*}}{P^N}$ is matched with the demand in a frictional private equity market. Especially, we assume the congestion among non-listed firms generates the attrition in the funding opportunity in the following way:

$$\frac{1}{\nu_N}\frac{x^{N*}}{P^N} = M_N$$

[19]Note that the price P is not identical to J^L, as one is a monotonically transformed version of the other. Specifically, $\Lambda P(q)^2 = J^L(q)$.

where, M_N is the total number of non-listed firms. $\nu_N > 1$ is a structural parameter that captures the congestion effect in the non-listed financial market.

Then, we define a net funding intensity $\phi^N(q)$ as follows:

$$\phi^N := \xi/(\nu_N M_N).$$

A non-listed firms' problem can be written down as follows, similar to the listed firms' problem:

$$\begin{aligned} J^N(\mathcal{M}_N) = \max_{k_T, k_I} \quad & \left(z k_T^{\alpha} (k_I)^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^N \\ \text{s.t. } & \phi^N = \xi/(\nu_N M_N). \end{aligned}$$

3.4 Summary of a firm's problem

A firm's manager decides whether to go listed or non-listed before the operation. If a firm becomes non-listed, the manager does not have to worry about the leakage of their intangibles through disclosure. However, investors penalize the opacity of the non-listed firms by allowing only a low funding intensity.

If a firm becomes public, the manager should decide the level of transparency $q \geq 0$. If too many firms choose the same transparency level, it will decrease the firm's value in the listed market due to demand-side competition.

A firm's problem could be summarized as follows:

$$\begin{aligned} \text{[Entry decision]} \quad & V(\mathcal{M}, M_N) = \max\{J^L(\mathcal{M}), J^N(M_N)\} \\ \text{[Listed firm]} \quad & J^L(\mathcal{M}) = \max_{q} \max_{k_T, k_I} \ \left(z k_T^{\alpha} (k_I(1 - \overline{q} - q))^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^L(q) \\ & \text{s.t. } \phi^L(q) = \frac{\xi + \psi(\overline{q} + q)}{\mathcal{M}(q)} \\ \text{[Non-listed firm]} \quad & J^N(M_N) = \max_{k_T, k_I} \ \left(z k_T^{\alpha} (k_I)^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^N \\ & \text{s.t. } \phi^N := \xi/(\nu_N M_N). \end{aligned}$$

4 Equilibrium

Here we define an equilibrium where the economy is given total intangible capital reserve K^I (fixed aggregate intangible supply). This endogenously determines the R&D cost of intangible capital p. The R&D cost is not a price for a trade. Instead, it is a cost that increases if all the other firms increase their spending in R&D. This captures the intuition that developing new knowledge is harder if more firms seek new knowledge. The rental rate for the tangible capital r is exogenously given.

Definition 1. *A collection of functions* $(k_T, k_I, q, \mathcal{M}, M_N, p, P, P^N, x^*, x^{N*}, \Phi^{ex})$ *is an equilibrium if*

1. $\left(x^*, x^{N*}\right)$ *solves the household's problem.*

2. $(k_T(q,\mathcal{M}), k_I(q,\mathcal{M}), q(\mathcal{M}))$ *solves the listed firm's problem.*

3. *The measure of listed firms choosing a transparency level* q *is consistent with* $\mathcal{M}(q)$ *for all* $q \in [0, 1-\overline{q}]$.

4. *The measure of non-listed firms is* M_N *and satisfies*

$$\int_0^{1-\overline{q}} \mathcal{M}(q)dq + M_N = 1$$

5. *R&D cost of intangible capital* p *is determined by the following equation:*

$$K^I = \int_0^1 k_{I,i} di$$

6. *Aggregate shared knowledge satisfies*

$$\Phi^{ex} = \int_0^1 1_{\{i \in Listed\}} \times k_{I,i}(\overline{q} + q_i) di$$

7. Financial market is cleared:

$$\frac{x^*(q)}{P(q)} = \mathcal{M}(q) \quad and \quad \frac{1}{\nu_N}\frac{x^{N*}}{P^N} = M_N$$

8. Indifference in the extensive-margin decision:

$$P(q) = P^N,\ for\ \forall q \in [0, 1-\overline{q}]$$

With the endogenously determined distribution $\mathcal{M}$ of firms for each q, we can re-write the market-clearing condition for intangible capital and the externality condition using $\mathcal{M}$. In the definition, each firm is aggregated along with index $i \in [0,1]$. Instead, we aggregate firms over the distribution of firms at each q. This is doable since $\mathcal{M}$ is endogenously obtained, and k_I is also a function of q and $\mathcal{M}$. Therefore, we re-write those two conditions in the following way.

$$K^I = \int_0^{1-\overline{q}} k_I(q,\mathcal{M})\mathcal{M}(q)dq$$

$$\Phi^{ex} = \int_0^{1-\overline{q}} k_I(q,\mathcal{M})(\overline{q}+q)\mathcal{M}(q)dq :$$

Among all possible equilibria, we are interested in the non-degenerate equilibrium where all the homogeneous firms use mixed strategies over the transparency level q. The mixed strategy leads to the distribution of firms at each level of q. In the equilibrium, this distribution needs to be consistent with the distribution that a firm takes as a given state variable.

In the following section, we analytically characterize the equilibrium allocations in this economy.

4.1 A listed firm's decision

First, we solve a listed firm's problem backward from the decision on the transparency level and the other allocations. Then, we solve the firm's decision on which financial market to go between the public and private market.

Given a net funding intensity function, ϕ^L and the externality, Φ^{ex}, a listed firms' firm's problem is characterized as follows:

$$\max_{q} \quad \overbrace{\left[\max_{k_T, k_I} \left(z k_T^{\alpha} (k_I(1-\overline{q}-q))^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^L(q) \right]}^{\text{Interim problem}}$$
$$\text{s.t. } \phi^L(q) = \frac{\xi + \psi(\overline{q}+q)}{\mathcal{M}(q)}$$

From the optimality conditions of the interim problem, we can derive the relationship among the transparency q, the regulation parameter $\overline{q}$, and the intangible capital k_I. The relationship is formally stated in the following proposition:

Proposition 1. *(Intangibles and the transparency)*
Given $\alpha + \theta < 1$, $k^I(q, \mathcal{M}; \overline{q})$ decreases in both q and $\overline{q}$. Specifically,

$$k_I(q, \mathcal{M}; \overline{q}) = \left(\left(\frac{\alpha z (\Phi^{ex})^{\gamma}}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right) (1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}$$

Proof.
See Appendix C.1. ■

If a firm is in a state where the knowledge has to be transparently revealed to the public, it naturally disincentivizes the firm to accumulate less knowledge. Therefore, the marginal increase in voluntary or mandatory transparency leads to a marginal decrease in the deployment of intangible capital stock. This result is consistent with the empirical observation we document in the empirical analysis section.

Moreover, the incentive to reveal the information interacts with the importance of the intangible in the production function, θ. When the intangible becomes more important in production, the negative association between the intangible and transparency strengthens. In other words, given the fixed intangible capital stock, a greater θ is associated with a stronger incentive to conceal the information (lower q). Proposition 2 theoretically shows this relationship.

Proposition 2 (Intangible share and the transparency)**.**
Given $\alpha + \theta < 1$, the sensitivity of $k^I(q, \mathcal{M}; \overline{q}, \theta)$ to the changes in q and $\overline{q}$ increases in θ.

Proof.
See Appendix C.2. ■

Then, from the optimality condition with respect to the transparency, q, we can characterize an ordinary differential equation (ODE) where the function of interest is the net funding intensity function $\phi(q)$. The ODE is specified in Appendix C.3. Solving the ODE, we characterize the transparency distribution $\mathcal{M}$ in the analytic form. We state the analytic form of $\mathcal{M}$ in the following proposition:

Proposition 3. *(Transparency distribution)*
The unnormalized probability density function $\mathcal{M}$ of transparency q has the following analytic form:

$$\mathcal{M}(q) = (\xi + \psi(\overline{q} + q))\,(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\phi^N}.$$

Proof.
See Appendix C.3. ■

In the multiplicative form of the closed-form endogenous distribution in Proposition 3, each component is directly interpretable.[20]

$$\mathcal{M}(q) = \underbrace{(\xi + \psi(\overline{q} + q))}_{\text{funding supply}} \underbrace{(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}}_{\text{funding demand}} \underbrace{\frac{1}{\phi^N}}_{\text{eq. normalizer}} .$$

The first component is the household's preference for transparent firms. For a higher q, the household is willing to provide greater funding to the firm. It generates an incentive for a firm to choose high q. In contrast, the second term captures firms' incentive to reveal less information. This is consistent with the intuition that a greater revelation only benefits competitors at the firm's own cost. The third term is the equilibrium object that balances the measure of listed and non-listed firms.

The following corollary establishes that the equilibrium distribution is unique for the given support of the transparency $[0, 1 - \overline{q}]$.

Corollary 1. *(Uniqueness of the transparency distribution)*
Given the support $[0, 1 - \overline{q}]$, the equilibrium unnormalized probability density function $\mathcal{M}$ is unique.

Proof. The result is immediate from the uniqueness of the ODE solution that satisfies the boundary condition. ■

The probability density function $\mathcal{M}(q)$ belongs to a variant of a well-known class of density functions: Beta distribution. In the following corollary, we prove that $\mathcal{M}(q)$ follows a shifted truncated beta distribution and provide the closed-form characterization of the net funding intensity of the private firms, ϕ^N. For the brevity of notation, I define $B := \frac{\theta}{1-\alpha-\theta}$.

Corollary 2. *(Truncated normalized Beta distribution)*
The gross transparency, $y := q + \overline{q}$, follows a truncated normalized Beta distribution

[20]It is worth noting that the endogenous distribution is independent of the productivity level z. Thus, the firm-level productivity heterogeneity does not matter in this setup. In the quantitative analysis, we normalize the productivity z at 1.

where the shape parameters are $B+1$ and 2, and the support is $[\overline{q}, 1]$.

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1-\overline{q}]\}}{1 - M_N} \times Beta\,(B+1, 2)\,,$$

where $B = \frac{\theta}{1-\alpha-\theta}$.

Proof.
See Appendix C.4. ∎

It is worth noting that the probability density of q depends on the net funding intensity of non-listed firms, ϕ^N. This net funding intensity is determined by the following identity that requires the total measure of firms is unity:

$$\frac{1}{\phi^N}\int_0^{1-\overline{q}} (\xi + \psi(\overline{q}+q))(1-\overline{q}-q)^B dq = 1 - \left(\frac{\xi}{\nu_N \phi^N}\right). \tag{1}$$

Equivalently, we can write down the identity in terms of the mass of non-listed firms as follows:

$$\psi\frac{\nu_N}{\xi} M_N \int_0^{1-\overline{q}} \left(\frac{\xi}{\psi} + (\overline{q}+q)\right)(1-\overline{q}-q)^B dq = 1 - M_N. \tag{2}$$

Therefore, we have the following closed-form solution for M_N:

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}\int_0^{1-\overline{q}}(\frac{\xi}{\psi} + (\overline{q}+q))(1-\overline{q}-q)^B dq}. \tag{3}$$

Equation (3) is the fundamental component of the model, which captures how the total measure of non-listed firms, M_N, behaves when the policy parameter $\overline{q}$ changes. Using Corollary 2, we can integrate out the $M(q)$ in the right-hand side of the equation in the following steps, using $y = \frac{1-q-\overline{q}}{1+\xi/\psi} \in [0, \frac{1-\overline{q}}{1+\xi/\psi}]$:

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\int_0^{\frac{1-\overline{q}}{1+\xi/\psi}}(1-y)(y)^B dy}$$

Then, we divide the numerator and the denominator by a beta function, $\mathcal{B}(B+1,2)$.[21]

$$M_N = \frac{1/\mathcal{B}(B+1,2)}{1/\mathcal{B}(B+1,2) + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)\int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} y^B(1-y)dy}$$

We integrate the denominator using the cumulative distribution function of beta distribution, F:

$$M_N = \frac{1/\mathcal{B}(B+1,2)}{1/\mathcal{B}(B+1,2) + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}F\left(\frac{1-\overline{q}}{1+\xi};B+1,2\right)}$$

By multiplying $\mathcal{B}(B+1,2)$ on the numerator and the denominator, we obtain the following analytic form:

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi};B+1,2\right)} \tag{4}$$

Equation (4) characterizes the measure of private firms in the analytic form. Importantly, the equation does not include either the price of the intangible or the externality. That is, the measure of private firms is independently determined from the general equilibrium effects and externality. The intuition behind this result is that both the productivity shift through the externality and the general equilibrium effect uniformly affect the operating profit of each firm, so they do not affect the decision of how to finance their operating activities.[22] Due to this separation, a measure of private firm M_N is determined directly by Equation (4). M_N determines the funding intensity of private firm ϕ^N. Then, from Proposition 3, the distribution of firms over transparency is also independently determined from the general equilibrium effect

[21]The beta function is defined as follows:

$$\mathcal{B}(a,b) := \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} = \int_0^1 x^{a-1}(1-x)^{b-1}dx$$

[22]For the same logic, the heterogeneous firm-level productivity does not affect the analytic form in the current setup.

and the externality. Therefore, the mandated transparency $\overline{q}$ affects the firm-level distribution directly through Equation (4) without any feedback effects in the general equilibrium. Also, it is worth noting that Equation (4) theoretically predicts that M_N increases in $\overline{q}$.

Proposition 4. *(The relationship between disclosure regulation and the measure of listed firms)*
M_N strictly increases in $\overline{q} \in [0, 1]$.

Proof.
See Appendix C.5 ■

As the policy maker requires a stricter disclosure regulation on the financial information, the measure of non-listed firms increases. This is because a firm does not internalize the productivity gain from the shared information. As can be observed from Equation (2), the measure of non-listed firms is independent of the externality effect, Φ^{ex}.

However, the total measure of listed or non-listed firms cannot solely serve as an objective of the disclosure regulation. The desired objective is stated clearly in the following mission of the SEC in the U.S.: "The mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation."[23] Consistent with the view of the SEC, we investigate the effect of regulation on the investors' welfare, productivity, and output in the following section.

4.2 The scoreboards: Welfare, productivity, and output

In this section, we define the three objectives of the disclosure regulation: welfare, productivity and output. First, we define the welfare measure. Besides the performance of the firms, the investor values the transparency of the disclosed information, as it

[23]The mission is from https://www.sec.gov/our-goals.

is helpful for its portfolio. The representative investor's utility can be monotonically transformed into the following mean-variance form:

$$
\begin{aligned}
&Objective_{welfare} = \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{p(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{\Lambda}{2}\int \frac{x(\widetilde{q})P(q)\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}\frac{x^N\nu_N P^N M_N}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{\Lambda}{2}\int \frac{\frac{\pi(q)/P(q)}{\Lambda/(\xi+\psi(\overline{q}+q))}P(q)\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}\frac{\frac{\pi^N/P^N}{\Lambda/\xi}\nu_N P^N M_N}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{1}{2}\int \pi(\widetilde{q})\mathcal{M}(\widetilde{q})d\widetilde{q} - \frac{1}{2}\nu_N\pi^N M_N. \\
&= \frac{1}{2}\int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \frac{\nu_N}{2}M^N\pi^N. \qquad (5)
\end{aligned}
$$

Therefore, the welfare measure is equivalent to the expected profit in equilibrium.

The second measure is the productivity in the production sector that is defined as follows:

$$
\begin{aligned}
Objective_{productivity} &= (\Phi^{ex})^\gamma \\
&= \left(\int_0^{1-\overline{q}}(\overline{q}+q)k_I(q,\mathcal{M};\overline{q})\mathcal{M}(q)dq\right)^\gamma,
\end{aligned}
$$

where $\widetilde{A} := \left(\left(\frac{\alpha z}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)$. The productivity is identical to the externality effect, which is the total shared knowledge in the economy. From the regulator's perspective, there is a trade-off in the productivity measure for increasing the strictness of the disclosure requirement. For higher $\overline{q}$, the amount of shared information is greater, while the pool of listed firms to share the information shrinks due to the firm-level extensive-margin responses. Also, the size of intangible k_I to be shared declines as in Proposition 1.

The third measure is the aggregate output in the economy. The output measure is defined in the following form:

$$Objective_{output} = \int_0^{1-\overline{q}} zk_T(q)^{\alpha}(k_I(q)(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}M(q) + zk_{DT}^{\alpha}k_{DI}^{\theta}(\Phi^{ex})^{\gamma}M_N$$

In the quantitative analysis, we will quantitatively analyze the variation in these three measures.

5 Quantitative analysis

Using the model we developed in the theory section, we conduct a quantitative analysis of the macroeconomic effects resulting from the increasing significance of intangible assets and the impact of information regulation policies. We estimate our model using data from two distinct periods. The first period, spanning from 1992 to 1996, serves as our baseline, while the second period, from 2012 to 2016, is considered as the post-change period. As our model is static, we cannot examine the dynamic response that may have occurred immediately after a change in structural parameters. Therefore, we compare a period just before the year of the dramatic shift in the number of listed firms with a period several years after the change to assume that it has reached a stationary level.

5.1 Estimation

In this section, we elaborate on how we fit the firm-level data into the model. The core parameters to be estimated are the following:

$$\{\overline{q}, \theta, \xi, \psi, \nu_N\},$$

where $\overline{q}$ is the mandated transparency of disclosure; θ is the intangible capital share; ξ is the baseline information level a household has about both listed and non-listed firms, ψ is the transparency's contribution to the household's information about listed firms; and ν_N is the efficiency parameter of the private equity market.

To generate our baseline estimates, we match the average target moments between 1992 and 1996. For the estimates of the post-change periods, we match the average target moments between 2012 and 2016. The target moments and simulated moments are reported in Table 2. The parameter $\overline{q}$ is identified based on the adjusted fraction of listed firms out of the total number of firms with more than 100 employees. To account for mergers and acquisitions (M&As) by another public firm (Doidge, Karolyi, and Stulz, 2017) we adjusted the target fraction of listed firms. Starting from 1975, we sequentially updated the exit rate, which is the number of delistings minus M&As, over the number of listed firms, plus new entries and minus M&As. Our adjustments show that the total drop in listed firms was about 52%, but after accounting for M&As, the drop is only 31%. This means that the adjusted fraction of listed firms went from 11.08% in the baseline period to 7.60% in the post-change period. Regarding the share of intangible capital, θ is identified from the intangible to tangible ratio.

Since in the model the households form a belief on a stock return that follows a normal distribution:

$$\widetilde{r}(q) \sim N\left(\overline{r}(q), \frac{1}{\xi + \psi(\overline{q} + q)}\right).$$

Analysts' forecast dispersion is a natural data counterpart to the dispersion in the ex-ante stock return. Specifically, earnings surprise is defined as:

$$ES(q) := \overline{r}(q) - \widetilde{r}(q) \sim N\left(0, \frac{1}{\xi + \psi(\overline{q} + q)}\right).$$

Hence, we identify ψ and ξ with the firms' average standard deviation of the returns of all firms and the top 1% opaque firms, respectively. Lastly, ν_D for the baseline period is identified using the 30% fraction of private firms that get funded, and for the post-change period, we use the 4 percentage points estimate of improvement in the private equity market friction following Ewens and Farre-Mensa (2020).

We use the method of simulated moments to estimate the parameters. The weight matrix is chosen to be an identity matrix. However, the choice of the weight matrix is not an issue in our estimation, as the parameters are exactly identified at the level where the level of moments is exactly matched.

Moments	Data	Model	Reference
Baseline (1992 $\sim$ 1996)			
Fraction of listed after M&A adj. (%)	11.08	11.08	Compustat & BDS
(*cf. without M&A adj. (%)*)	(8.30)		
Intangible Exp./Sale (%)	2.906	2.906	Compustat
Average $sd(\widetilde{r})$ (%)	12.53	12.53	Compustat
Average $sd(\widetilde{r})$ of top 1% (%)	25.52	25.52	Compustat
Portion of funded non-listed firms (%)	30.30	30.00	Ewens and Farre-Mensa (2020)
Post-change periods (2012 $\sim$ 2016)			
Fraction of listed after M&A adj. (%)	7.60	7.60	Compustat & BDS
(*cf. without M&A adj. (%)*)	(4.01)		
Intangible Exp./Sale (%)	5.356	5.356	Compustat
Average $sd(\widetilde{r})$ (%)	28.00	28.00	Compustat
Average $sd(\widetilde{r})$ of top 1% (%)	84.81	84.81	Compustat
Portion of funded non-listed firms (%)	34.30	34.00	Ewens and Farre-Mensa (2020)

Table 2: Fitted Moments

Table 3 reports the estimated parameters. In the post-change period, the estimated mandated transparency parameter, $\overline{q}$, slightly increased, indicating that information regulation has become stricter, consistent with the intended direction of the reform. The share of intangible assets, θ, has increased by approximately 50%, reflecting the significant rise in the importance of intangible input. The baseline information level a household has about both listed and non-listed firms, ξ, has decreased substantially, and the transparency's contribution to the household's information about the listed firms, ψ, has decreased, both changes indicating an increase in the return variance on both listed and non-listed markets. Furthermore, the friction parameter ν_N has decreased, indicating an improvement in the private equity market, which reflects the impact of the National Securities Markets Improvement Act of 1996 (Ewens and Farre-Mensa, 2020).

Parameters	Description	Value
Baseline (1992 $\sim$ 1996)		
$\overline{q}$	Mandated transparency	0.981
θ	Intangible share	0.029
ξ	Baseline information level	25.520
ψ	Transparency's contribution to listed firms information	38.539
ν_N	PE market friction	3.300
Post-change periods (2012 $\sim$ 2016)		
$\overline{q}$	Mandated transparency	0.995
θ	Intangible share	0.054
ξ	Baseline information level	1.390
ψ	Transparency's contribution to listed firms information	11.394
ν_N	PE market friction	2.915

Table 3: Estimated parameters

Besides the estimated parameters, we fix the following parameters before the estimation:

$$\{\alpha, \gamma, K^I\}.$$

Capital share, α, is set to be 0.50. Because our model is abstract from a labor input, the capital share in the model needs to be interpreted as an after-labor-adjustment capital share, as in the following formulation:

$$\begin{aligned} Ak^{\alpha} &= \max_{L} \widetilde{A} k^{\widetilde{\alpha}} L^{\epsilon} - wL \\ &= (1-\epsilon)\widetilde{A}^{\frac{1}{1-\epsilon}} \left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}} k^{\frac{\widetilde{\alpha}}{1-\epsilon}} = Ak^{\frac{\widetilde{\alpha}}{1-\epsilon}} \end{aligned}$$

where $A = (1-\epsilon)\widetilde{A}^{\frac{1}{1-\epsilon}} \left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}}$. Therefore, our model's α is equivalent to a standard model's $\frac{\widetilde{\alpha}}{1-\epsilon}$. We assume $\widetilde{\alpha} = 0.2$, and $\epsilon = 0.6$, leading to $\alpha = 0.50$. The public intangible share, γ, is assumed to be equal to the private intangible share, θ. The total intangible capital stock, K^I, is normalized to 1.

Parameters	Description	Value
α	Capital share	$0.30 - \theta$
γ	Public intangible share	$= \theta$
r	Rental rate tangible capital plus depreciation	0.10
K^I	Total intangible supply	1
z	TFP level	1

Table 4: Fixed parameters

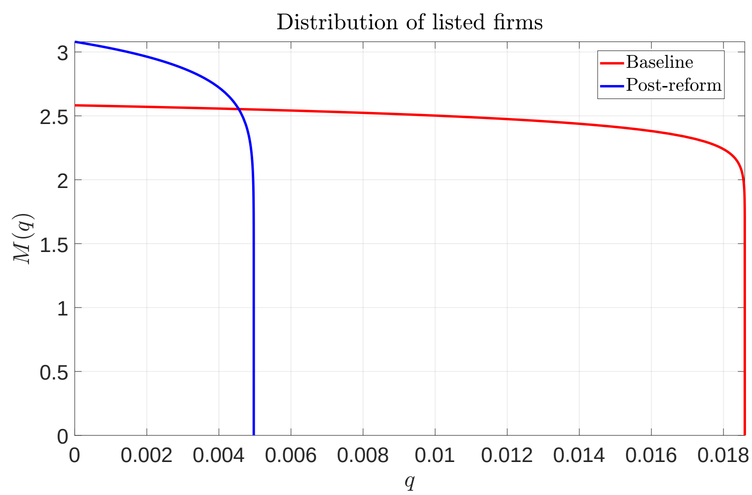


Figure 5: Distribution of listed firms over transparency

Figure 5 shows the non-normalized distribution of listed firms over the transparency level for the baseline and the post-change periods. The distribution shrinks in the post-change period due to the reduced number of listed firms, and shifts leftward, indicating a decrease in the average transparency level.

5.2 Decomposition analysis

In this section, we calculated the average contributions of each parameter to the decrease in the measure of listed firms and the decrease in the average transparency. We obtained these contributions by first keeping the estimated parameters at their baseline values and changing only one parameter to its post-change value to obtain the counterfactual measure of listed firms and average transparency if only that specific parameter changed. Second, we kept the estimated parameters at their post-change values and changed only one parameter to its baseline value to obtain the counterfactual measure of listed firm and average transparency if only that specific parameter remained at the baseline value. We performed this calculation for all five estimated parameters and then averaged both numbers from each parameter to obtain the average contributions to the decrease in the measure of listed firms and the decrease in the average transparency. Table 5 reports the results of the decomposition analysis in annualized percentage.[24]

		Contribution to the change:			
Param.	Channel	#listed	transparency	productivity	welfare
	Total change (p.a.)	-1.88	-8.04	-0.42	-1.42
$\overline{q}$	SEC regulation (p.a.)	-6.22	0.03	-0.25	0.20
θ	Raising intangible share (p.a.)	-0.89	0.00	-0.37	-0.80
ξ	Baseline information level (p.a.)	8.62	-3.85	0.34	-0.92
ψ	Harder to forecast public firms (p.a.)	-3.72	-4.22	-0.16	0.16
ν_N	PE market friction (p.a.)	-0.56	0.00	-0.02	-0.59

Table 5: Decomposition of the channels in the macroeconomic changes

Table 5 presents the results of a decomposition analysis examining the factors contributing to the observed decline in the number of listed firms over the past two decades. The analysis reveals that the percentage of listed firms decreased from 11.08% in the baseline period to 7.60% in the post-change period, representing a 31% drop over 20 years, with an average annual change of -1.88. Furthermore, transparency, productivity, and welfare have also exhibited annual changes of -8.04, -0.42, and -1.42, respectively.

[24]The two periods of comparison are 20 years apart from each other. So, we annualized the total change by a division of 20.

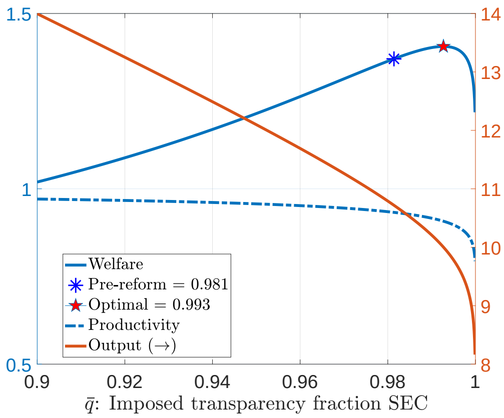
The decomposition analysis identifies several factors contributing to the observed decline in the number of listed firms. Specifically, the stricter SEC regulation accounted for the majority of the change, contributing -6.22 percentage points. The rising share of intangible capital, in conjunction with the changes in both the transparency's contribution to listed firms information and the improvements in private market funding, contributed in -0.89, -3.72, and -0.56 percentage points, respectively. On the contrary, the decline in the household's baseline information level about listed and non-listed firms contributed positively to the change by 8.62 percentage points.

Overall, the results suggest that the drivers of lowered transparency, productivity, and welfare are largely attributed to the stricter SEC regulation, increased share of intangible capital and overall greater opacity in financial markets. These results provide valuable insights for policymakers and market participants seeking to understand the underlying factors contributing to the decline in the number of listed firms and the associated macroeconomic implications.

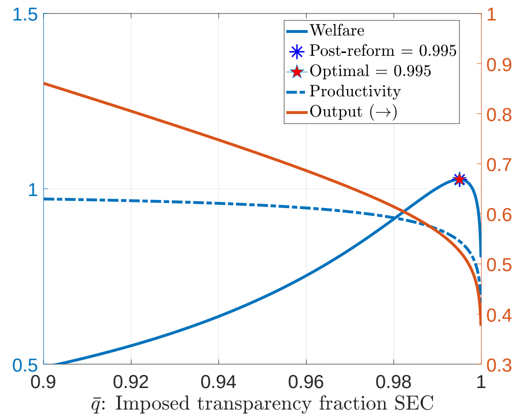
5.3 Optimal Policies

In this section, we use the proposed model to analyze the optimal level of imposed transparency for welfare maximization. As shown in the previous section, the policy maker can choose the imposed transparency level $\bar{q}$. However, since welfare is obtained from the utility maximization problem of the household, $\bar{q}$ will have two effects on welfare. On the one hand, lower imposed transparency increases the measure of listed firms that will have more access to finance relative to private firms, increasing output and consumption. On the other hand, lower imposed transparency also increases the output's variance, lowering the welfare of the risk-averse household. Hence there is a trade-off between the level of consumption and its volatility. In Figure 6, we show the Laffer-type curve for the transparency policy for both periods.

The estimated level of transparency in the pre-change period is 0.981 (Table 3) and the optimal level is 0.993, suggesting the mandated transparency was below



(a) Baseline estimation



(b) Post-change estimation

Figure 6: Optimal level of mandated transparency

the optimal level in the pre-change period. In the post-change period estimation, the results suggest that both the estimated and the optimal level of transparency increased to 0.995 (Table 3). It is worth mentioning that output and productivity also show an inverted-U shape.[25] This property of the model suggests that depending on the value of the estimated parameters, moving $\bar{q}$ towards the welfare-optimal point could increase both output and productivity as well, achieving a *divine coincidence*. With the current estimated parameters, such a *divine coincidence* happens when $\bar{q}$ is above the welfare optimal point: Decreasing $\bar{q}$ toward the optimal would increase welfare, output, and productivity.

6 Concluding remarks

This paper analyzes how the rising intangible has affected the secular trend of disappearing public firms and declining transparency of the financial report and its macroeconomic impact. From the empirical analysis, we show that the number of listed firms and the average transparency (the inverse of the earnings forecast dis-

[25]Figure 6 only shows the region where output and productivity decrease monotonically with respect to $\bar{q}$.

persion) of listed firms' reporting have substantially decreased in recent years. Then, from the cross-sectional regression of earning surprises on the firm-level intangible intensity, we document a negative correlation between the transparency and the intangible intensity of a firm. To theoretically and quantitatively analyze the observed patterns in the data, we develop a heterogenous-firm equilibrium model where the firm-level decision of going public or private and the distribution of rich equilibrium allocations are characterized in a closed form. Using the model, we theoretically show that the number of listed firms decreases in the strictness of the SEC's requirement on the listed firms' financial disclosure. From the estimated model, we quantitatively show that the rising importance of intangible capital is the crucial driving factor of the recent changes. Furthermore, the change has led to significant losses in productivity and welfare.

Then, we analyze the optimal regulation policy and on which side the current policy parameter is located with respect to the optimal level in terms of output, productivity, and welfare. Each of the macroeconomic allocations displays the inverted-U-shaped changes over the shifts in the regulation parameter. According to the estimated model, the SEC regulation parameter has been at a level lower than the optimal level in terms of welfare, while it is at a greater level than the optimal levels for productivity and output. The recent changes in the disclosure regulation, including Sarbanes-Oxley Act, slightly contributed to the trend of disappearing listed firms and a drop in productivity. However, it has significantly contributed to mitigating the welfare loss dominantly driven by the rising intangibles.

Our approach broadens the scope of structural policy analysis to the regulation on information disclosure. Our analytical framework serves as a useful tool to analyze the impact of information regulation change on firm financing decisions, productivity, and welfare.

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A Additional figures

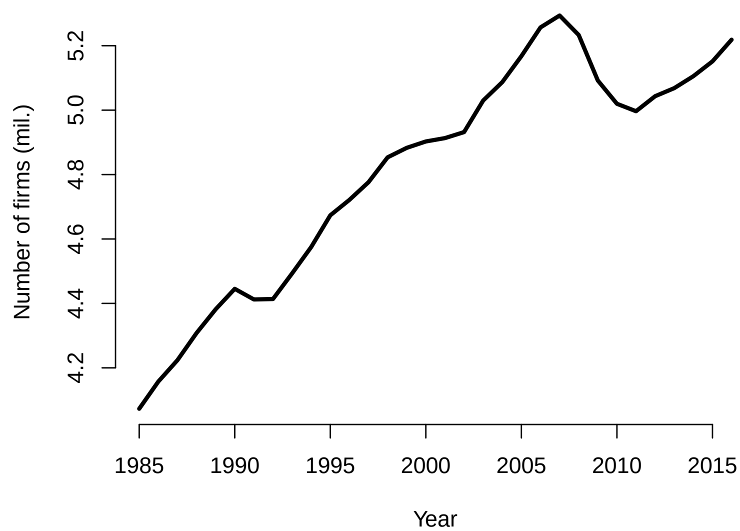
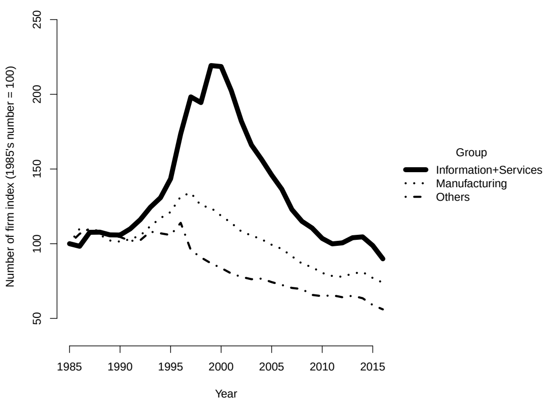
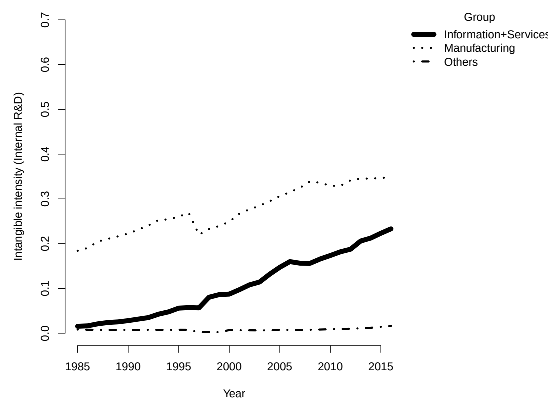


Figure A.1: Number of all firms



(a) Number of listed firms



(b) Intangibles intensity

Figure A.2: **Number of listed firms and intangible intensity by industry (internal R&D only).**
This figure shows the trend in the number of listed firms and intangible capital intensity in the U.S. Intangible intensity is defined as the ratio of intangible asset, excluding acquired intangible and organizational capital, to total intangible (again excluding acquired intangible and organizational capital) and tangible asset values.The groups are defined as Information and Services, excluding trade and transportation, Manufacturing,and other sectors (Trade and transportation, Agriculture and Mining, Construction). Data comes from Compustat. See Section 2.1 for details on measurement.

Figure A.3: Trends in the number of public firms by industry

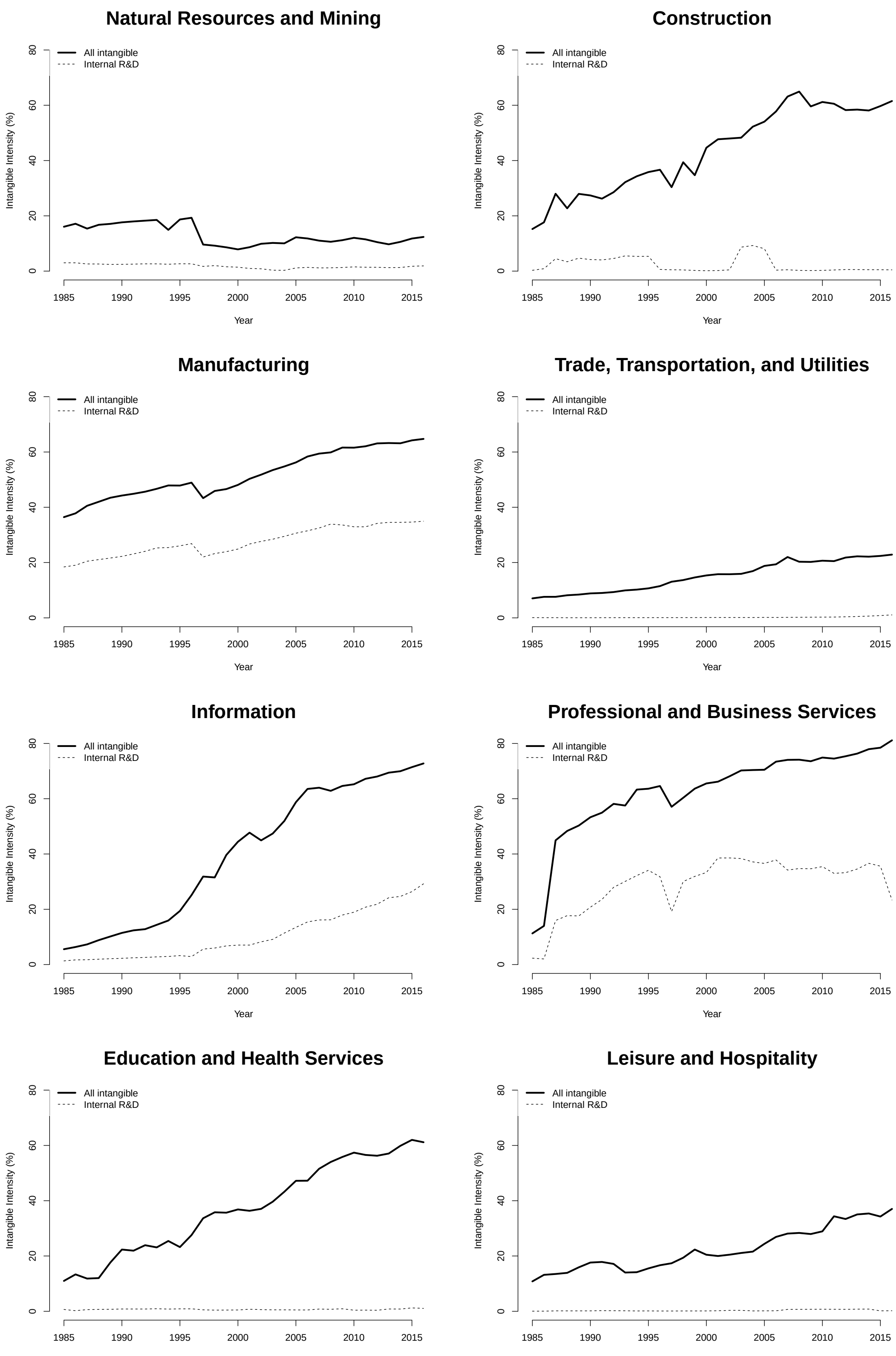
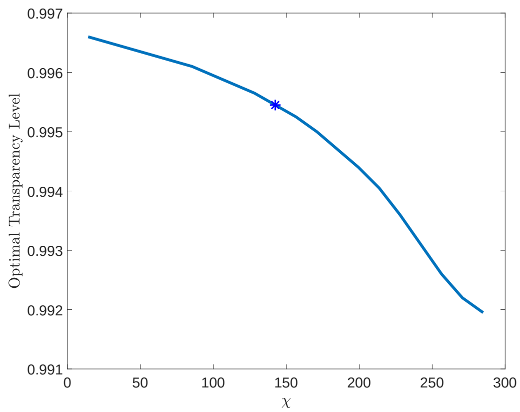
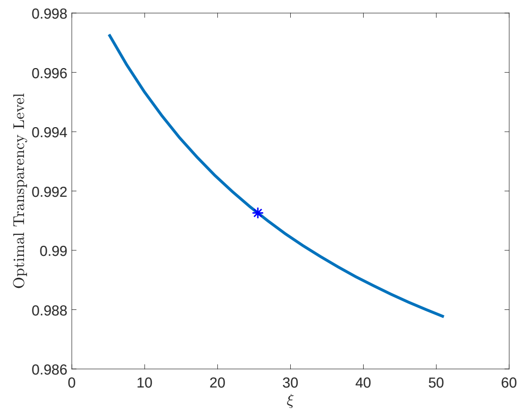


Figure A.4: Trends in intangible intensity by industry

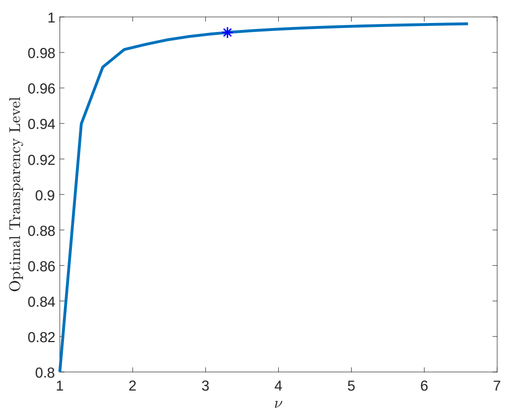
B Comparative Statics



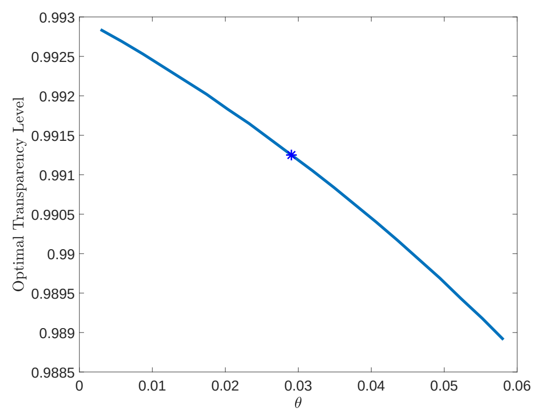
(a) Dispersion of stock returns χ



(b) Uncertainty private returns ξ

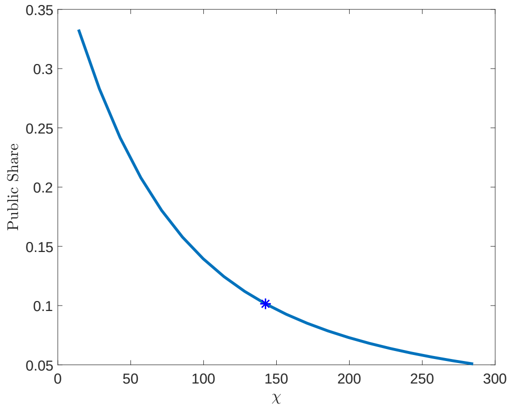


(c) Congestion private funds ν_N

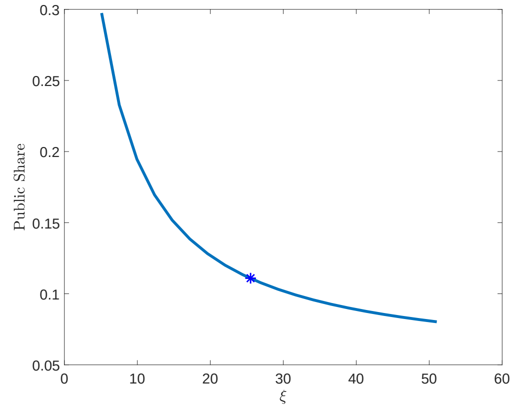


(d) Intangible share θ

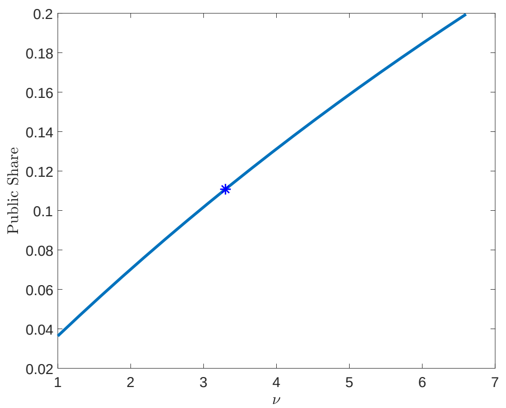
Figure B.5: Comparative statics on optimal transparency level with respect to each parameter. We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting optimal transparency level.



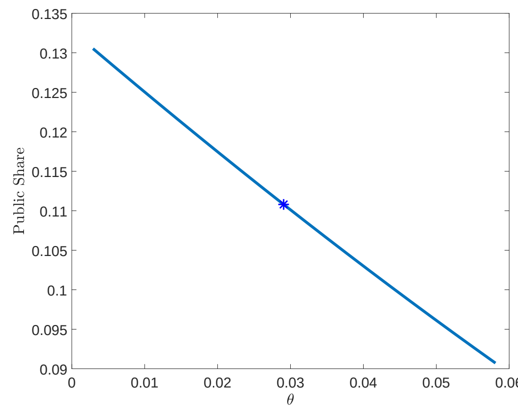
(a) Dispersion of stock returns χ



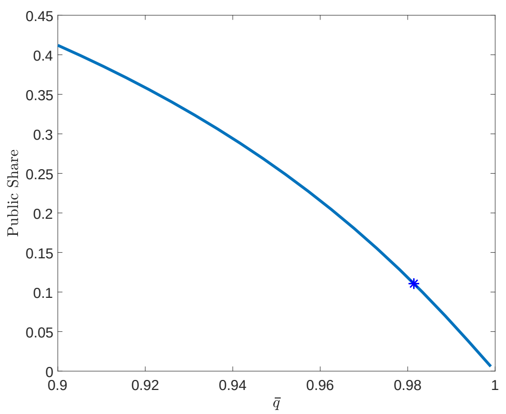
(b) Uncertainty private returns ξ



(c) Congestion private funds ν_N



(d) Intangible share θ



(e) Mandated minimum transparency $\bar{q}$

Figure B.6: Comparative statics on fraction of listed firms with respect to each parameter. We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting number of listed firms.

C Proofs

C.1 Proof for Proposition 1

Proposition 1. *(Intangibles and the transparency)*
Given $\alpha + \theta < 1$, $k^I(q, \mathcal{M}; \overline{q})$ decreases in both q and $\overline{q}$. Specifically,

$$k_I(q, \mathcal{M}; \overline{q}) = \left(\left(\frac{\alpha z(\Phi^{ex})^\gamma}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right) (1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}$$

Proof.
From FOC

$$\begin{aligned}
[k_T]: \quad & z\alpha k_T^{\alpha-1}(k_I(1-\overline{q}-q))^\theta(\Phi^{ex})^\gamma = r \\
[k_I]: \quad & z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^\gamma(1-\overline{q}-q) = p \\
& + \left(z k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^\gamma - r k_T - p k_I \right) \phi'^{L}(q) = 0
\end{aligned}$$

From the first-order conditions with respect to k_T and k_I, we obtain

$$\frac{r}{p} = \left(\frac{\alpha}{\theta} \right) \frac{k_I}{k_T}.$$

Substituting this relation into the first-order condition with respect to k_T, we get

$$r = \alpha z \left(\frac{\alpha p}{\theta r} \right)^{\alpha-1} (k_I)^{\alpha+\theta-1}(1-\overline{q}-q)^\theta(\Phi^{ex})^\gamma.$$

Thus,

$$k_I = \left(\left(\frac{\alpha z(\Phi^{ex})^\gamma}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right) (1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}} = A(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}},$$

where $A := \left(\left(\frac{\alpha z(\Phi^{ex})^\gamma}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right)$. As $\alpha+\theta < 1$, the proposition is immediate from the last equation. ■

C.2 Proof for Proposition 2

Proposition 2 (Intangibles and the transparency)**.**
Given $\alpha + \theta < 1$, the sensitivity of $k^I(q, \overline{q}, \theta)$ to the changes in q and $\overline{q}$ increases in θ.

Proof.

$$
\begin{aligned}
\frac{\partial}{\partial \theta}\left|\frac{\partial}{\partial q} log(K_I)\right| &= \frac{\partial}{\partial \theta}\left|\frac{\partial}{\partial q}\left(\frac{\theta}{1-\alpha-\theta}\right) log(1-q-\overline{q})\right| \\
&= \frac{\partial}{\partial \theta}\left(-1 + \frac{1-\alpha}{1-\alpha-\theta}\right)\frac{1}{1-q-\overline{q}} \\
&= \frac{1-\alpha}{(1-\alpha-\theta)^2}\frac{1}{1-q-\overline{q}} > 0
\end{aligned}
$$

■

C.3 Proof for Proposition 3

Proposition 3. *(Transparency distribution)*
The probability density function $\mathcal{M}$ of transparency q has the following closed form:

$$
\mathcal{M}(q) = (\xi + \psi(\overline{q} + q))\,(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\phi^N}.
$$

Proof.

We derive the following equations using the first-order condition with respect to q:

$$
\begin{aligned}
\frac{\phi'^L(q)}{\phi^L(q)} &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I} \\
&= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{(1-\alpha-\theta)zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}} \\
&= \frac{\theta}{1-\alpha-\theta}\left(\frac{1}{1-\overline{q}-q}\right)
\end{aligned}
$$

From $\frac{\partial}{\partial q} log(\phi^L(q)) = \frac{\phi'^L(q)}{\phi^L(q)}$, the solution of the first-order differential equation is as follows:

$$\phi^L(q) = (1-\overline{q}-q)^n\,\widetilde{C},$$

for some $n \in \mathbb{R}$ and some $\widetilde{C} \in \mathbb{R}$. From the indifference condition in the equilibrium, $\pi^L(q)\phi^L(q)$ does not depend on q.

$$\pi^L\phi^L(q) = \left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^n\,\widetilde{C}$$

Therefore,

$$n = -\frac{\theta}{1-\alpha-\theta}$$

This leads to $\phi^L(q) = (1-\overline{q}-q)^{-\frac{\theta}{1-\alpha-\theta}}\,\widetilde{C}$.

Then, the distribution of listed firms is as follows:

$$\begin{aligned}\mathcal{M}(q) &= (\xi+\psi(\overline{q}+q))/\phi^L(q)\\ &= (\xi+\psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\widetilde{C}}.\end{aligned}$$

From the indifference condition between listed and non-listed,

$$\begin{aligned}\phi^N &= \frac{\pi^L(q)\phi^L(q)}{\pi^N}\\ &= \frac{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^{-\frac{\theta}{1-\alpha-\theta}}\,\widetilde{C}}{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}\Phi^{\gamma}\right)}\\ &= \widetilde{C}\end{aligned}$$

Therefore, $\mathcal{M}(q) = (\xi+\psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\widetilde{\phi^N}}$

In the equilibrium, $\phi^N(=\widetilde{C})$ is determined at the level where the following equation holds:

$$\int_0^{1-\overline{q}} \mathcal{M}(q)dq = 1 - M_N.$$

■

C.4 Proof for Corollary 2

Corollary 1. *(Truncated Beta distribution)*

The gross transparency, $y := q + \overline{q}$, follows a truncated Beta distribution where the shape parameters are 2 and $B+1$, and the support is $[\overline{q}, 1]$.

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1-\overline{q}]\}}{1 - M_N} \times Beta\,(B+1, 2)\,,$$

where $B = \frac{\theta}{1-\alpha-\theta}$.

Proof.

We define $M_Y(y)$ as the probability density function of the random variable $y = \frac{1-q-\overline{q}}{1+\xi/\psi}$.

$$M_Y(y) \propto (1-y)y^B \quad \text{and} \quad y \in \left[0, \frac{1-\overline{q}}{1+\xi/\psi}\right].$$

Also, $\int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} M_Y(y)dy = 1 - M_N$. Therefore, $y \sim \frac{\mathbb{I}\{q \in [0,1-\overline{q}]\}}{1-M_N} \times Beta\,(B+1, 2)$. ■

C.5 Proof for Proposition 4

Proposition 4. *(The relationship between disclosure regulation and the measure of listed firms)*

M_N strictly increases in $\overline{q} \in (0, 1)$.

Proof.

We have

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2} \mathcal{B}(B+1, 2) F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)}$$

F decreases in $\overline{q}$, and M_N decreases in F. Thus, M_N increases in $\overline{q}$. ■