# Rising Intangible Capital and the Disappearance of Public Firms[†]

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**Abstract**

Since 1996, the number of listed firms in the U.S. has decreased by around 50%. Using U.S. Compustat and earnings surprises from I/B/E/S data, we document that the financial reports of listed firms required by the U.S. Securities and Exchange Commission's (SEC) regulation have become significantly less transparent over the same period. To theoretically and quantitatively analyze these secular trends, we develop a heterogenous-firm equilibrium model where endogenous exit and entry in the public equity market, intangible capital stock level, and intangible capital's transparency are characterized by closed-form solutions. In the model, each listed firm's publicly disclosed intangible is diffused to other firms' productivity as an externality. In the estimated model, the increased intangible share, together with the recent stricter disclosure requirement, has substantially decreased the average transparency of the financial disclosure and the number of listed firms. Finally, we characterize a policy maker's dilemma between maximizing productivity and welfare. We conclude that stricter regulation improves welfare under the sacrifice of productivity.

**Keywords**: Intangible capital, Initial public offerings, Corporate disclosures, Transparency, Technology diffusion.

**JEL codes**: D24, G24, G38.

[†] The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. All remaining errors are our own.

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# 1 Introduction

The number of listed firms in the US has decreased by around 50%, and total capital raised annually in the private capital market is now larger than the amount raised in the public markets (Bauguess, Gullapalli, and Ivanov (2015)). Different explanations have been put forward to shed light on this issue: for example, Gao, Ritter, and Zhu (2013) point to the increase in mergers and acquisitions among US firms; Doidge, Karolyi, and Stulz (2013) conjecture that as markets have become more globally integrated, the net benefits of going public in the US versus in other markets have decreased; Ewens and Farre-Mensa (2020) argue that the deregulation of securities laws changed the going-public versus staying-private trade-off.

In this paper, we propose an alternative and complementary explanation. Going public in the US dramatically changes the amount of publicly available information to both investors and competitors, while there are almost no disclosure requirements for privately held firms in the US. We argue that the rise of intangible capital, especially the components of intangible capital that could benefit competitors besides the owner firm, has increased the cost of disclosing information and made staying private more attractive. At the same time, we observe that (i) the SEC has required higher levels and more transparent reporting, (ii) access to funds by venture capital firms, private equity funds, and other private investors has become easier.

In order to analyze these trends and their impact on capital allocation, productivity, and welfare in a unified framework, we introduce a general equilibrium model of heterogenous-firm investment and capital financing decisions. In the model, firms choose exit and entry in the public equity market, the level of intangible capital stock, and the transparency of their intangible capital. One of the advantages of our model is that these decisions have a closed-form solution, which allows us to characterize the model and optimal policy cleanly.

In the model, the key trade-off in a firm's decision to go public or stay private is between a better financing condition and a loss in the intangible capital by sharing it with the other competing firms. Specifically, the shared intangible capital is diffused to the other firms' productivity as an externality. There is also a trade-off in the policy maker's perspective, as a stricter policy would facilitate the knowledge spillover across the firms and lower uncertainty for investors. However, it disincentivizes the firms to stay in the public equity market.

More transparency naturally benefits investors but, in equilibrium, discourages the listing decision and might end up causing a less than optimal level of capital and for remaining firms to be on average overall less transparent. Indeed, our empirical analysis suggests that transparency has been declining over time. Using the estimated model, we find that increased intangible share together with stricter disclosure requirements that only apply to the public market has substantially decreased the average transparency of the financial disclosure and the number of listed firms.[1]

To evaluate the consequences of the information disclosure policy, we provide three criteria: output, productivity, and welfare.[2] In the estimated model, a regulation policy can achieve only either higher output and productivities or higher welfare, which shows the policy maker's dilemma. From the perspective of the protection of investors, we find the recent regulation has substantially improved welfare. However, we also document that it has led to a substantial loss in productivity in the production sector.

**Literature and contribution** There are two strands of the literature closely related to this paper. The first is the literature that studies the rising importance of intangible capital. It was only around a decade ago that intangible capital was first recognized as an important macroeconomic factor that affects economic growth and the business cycle. For example, McGrattan and Prescott (2010) and McGrattan (2020) highlight the importance of intangible capital as a key input factor for production and show how mismeasurement of intangible capital may mislead the neoclassical model predictions in terms of economic growth. Relatedly, Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2014) modeled plant-level intangible capital as an important input for production. Mainly, their intangible capital refers to organizational capital that is partly firm-specific and partly embodied in key labor inputs.

Despite the consensus on the rising importance of intangible capital in the modern economy, there is still no consensus on the measurement of intangible capital due to its complex intangibility nature. Among the key papers measuring intangible capital, Corrado, Hulten, and Sichel (2009) showed that around 30% of intangible capital in the U.S. is organizational capital, the single largest category of intangible capital. These are consistent results with the notion that intangible capital is hard to trade as it is less valued outside of the plant it belongs to and thus hard to be collateralized. Recent papers in the literature applied the perpetual inventory method to U.S. Compustat data to capitalize the intangible-related expenditures (Eisfeldt and Papanikolaou (2014); Peters and

[1]This is one of the core issue SEC is concerned about. For example, in a February 2017 speech, SEC Commissioner Kara Stein posed a question regarding additional disclosures and regulation around private market investment: "We also need to understand why more companies are staying private for longer periods of time. Should we apply enhanced disclosure laws to these private companies? Or perhaps they require a unique set of rules." See "The Markets in 2017: What's at Stake?" Commissioner Kara M. Stein, SEC website, https://www.sec.gov/news/speech/stein-secspeaks- whats-at-stake.html

[2]The mission of SEC is "The mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation". See "Our Goals", SEC website, https://www.sec.gov/our-goals.

Taylor (2017)). Those expenditures include the R&D cost and a portion of SG&A expenditure. Ewens, Peters, and Wang (2020) measures the intangible capital stock with intangible capital price considered using the equity price and the exit price. We also followed this approach to compute the intangible capital stock.

We contribute to this literature by analyzing a novel macroeconomic implication of the rising share of intangible capital. The intangible capital has become an important source of competitiveness, leaving firms to put a great effort into R&D or developing a productive corporate culture. However, the intangible capital has a strong spillover effect, which can benefit competitors besides the owner firm. Therefore, the rising importance of intangible capital has naturally increased a firm's incentive to stay opaque in its financial disclosure. Using our model, we theoretically and quantitatively analyze how this change affects the macroeconomy in terms of welfare and productivity.

Another strand of the literature our paper stands on is a subfield of corporate finance that studies a firm-level financing decision. Doidg, Karolyi, and Stulz (2017) highlighted that the number of listed firms in the U.S. has become abnormally lower than the other countries since 1997. One of the closest papers to ours is Ewens and Farre-Mensa (2020), which studies the effect of the National Securities Markets Improvement Act (NSMIA) of 1996 on the firm-level decision to go public or stay private. In contrast to the focus on the increase in the available private capital on the decreasing number of listed firms, our paper studies how the regulation on information disclosure affects the number of listed firms.

Our contribution to this literature is on the quantification of the SEC's regulation level and analyzing its effect on the financing decision of firms. Our approach enables a rich discussion on how financial disclosure regulation can affect macroeconomic allocations.

# 2 Empirical Analysis

## 2.1 Data and Measurement

Our main source of information on the number of US publicly traded firms and their characteristics is Compustat. Our baseline measure of internally generated intangible capital is the sum of two components: (i) estimated knowledge capital, calculated using research and development expenditure; and (ii) estimated organizational capital, calculated using selling, general, and administrative expenses. The measure is constructed using the perpetual inventory method, which aggregates net investment flows over the life of the firm[3]:

$$K_{i,t}^{G} = (1 - \delta_G)\, K_{i,t-1}^{G} + \gamma_G R\&D_{it}$$
$$K_{i,t}^{O} = (1 - \delta_O)\, K_{i,t-1}^{O} + \gamma_O SG\&A_{it}$$

where $R\&D$ is research and development expenditure expenditure, $SG\&A$ is selling, general, and administrative expenses, both deflated by the price of intellectual property products; and the depreciation rates $\delta_G$ and $\delta_O$ are taken from Ewens, Peters, and Wang (2020).

In order to get measures of firms' transparency, we leverage information on earnings surprises. Following Dellavigna and Pollet (2009), earnings surprises are defined as the difference between a firm's earnings announcement and the earnings forecasts made by analysts for that firm, normalized by the price of a share. Data on analysts' forecasts come from the Institutional Brokers' Estimate System (I/B/E/S). The dataset collects quarterly estimates made by stock analysts on the future earnings for publicly traded companies.

We use two different proxies for firm trasparency. The first is the inverse of the absolute value of earnings surprises from the consensus analyst forecast, i.e. the median forecast among all the analysts that make a forecast in the last 30 calendar days before the earning announcement. Let $e_{t,k}$ be the earnings per share announced in quarter $t$ for company $k$ and $\hat{e}_{t,k}$ be the corresponding consensus analyst forecast. Indicate by $P_{t,k}$ the price of the shares of company $k$ in quarter. Earnings surprise from the consensus is given by:

$$\widehat{ES}_{t,k} := \frac{e_{t,k} - \hat{e}_{t,k}}{P_{t,k}}$$

Assuming that more transparent firms have on average lower earning surprise, our first proxy is then:

$$t_{k,t}^{1} := \frac{1}{|\widehat{ES}_{t,k}|}$$

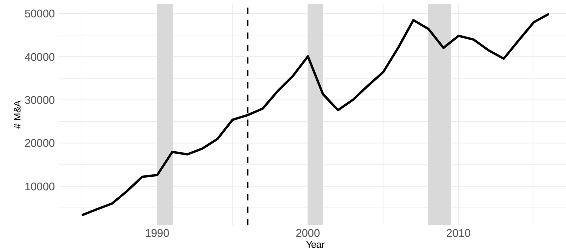
Our second measure takes advantage of the fact that I/B/E/S collects multiple estimates made by different analysts for each firm, up to 35 and on average 3 in our dataset. Let $e_{t,k,j}$ be the earning forecast made by analyst $j$, then earning surprise for each firm-analyst is:

$$ES_{t,k,j} := \frac{e_{t,k} - e_{t,k,j}}{P_{t,k}}$$

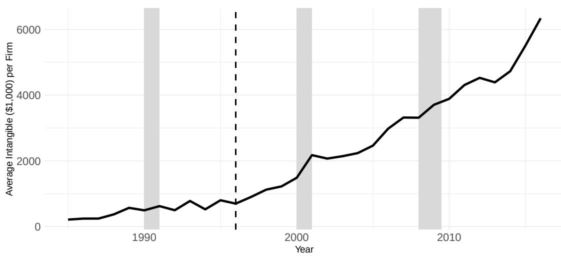
Under the assumption that more transparent firms have lower disagreement among ana-

[3]see for example Cockburn and Griliches (1988), Eisfeldt and Papanikolaou (2013, 2014), Ewens, Peters, Wang (2020), Hall, Mairesse, and Mohnen (2010), Hulten and Hao (2008)

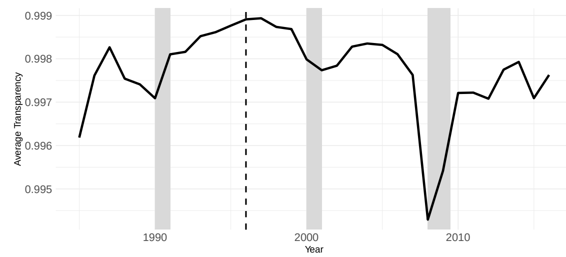

(a) Number of listed Firms




(b) Total number of mergers




(c) Average intangible




(d) Average transparency

Figure 1: Timeseries plot of aggregate corporate variables

lysts, our second trasnparency proxy is:

$$t_{k,t}^{2} := \frac{1}{var\left(ES_{t,k,j}\right)}$$

## 2.2 Trends in Intangible Capital and Transparency

Figure (1) shows the change in the aggregate variables among publicly listed firms from 1980 through 2016. As seen in panel (a), the number of listed firms started to decrease from 1996, which the dotted vertical line indicates. Before 1996, there was no such trend of decreasing listed firms. One of the explanations for the decrease of listed firms points to the higher number of mergers and acquisitions observed in the U.S. (Gao, Ritter, and Zhu (2013)). Panel (b) shows the number of M&A, which has been increasing over the whole sample period. We focus instead on an alternative and complementary explanation: Panel (c) shows the average intangible capital stock has increased sharply from the middle of the 1990s. At the same time, the average transparency level of corporate's quarterly report has decreased from the middle of 1990, as can be seen in panel (d). Recessions and especially the Great Recession represent a big shocks to earnings surprises. In order to take that into account, we also measure average transparency by excluding recession periods as measured by the NBER, and we still find that average transparency has been declining.

# 3 Baseline Model

We consider a stand-in household and a continuum of measure one of firms that are ex-ante homogeneous. The model is static. A representative household decides its asset portfolio and consumes the payouts from the portfolio. A manager of a firm decides whether to go listed or to go private. If a firm is listed, the manager chooses the disclosure level of the firms' intangible capital to the public, which we define as the transparency. If a firm is private, the manager does not disclose any intangible to the public.

## 3.1 Household

A stand-in household decides the asset portfolio and consumes the portfolio return. The household is given with the wealth level $a > 0$. The household is risk-averse, and the utility takes the following constant absolute risk aversion form (CARA):

$$u(C) = -e^{-\Lambda C}$$

where $\Lambda > 0$ is the absolute risk aversion parameter.

In the listed market, the household forms a belief on the return $\widetilde{r}(q)$ based on a balance sheet information of a listed firm with a transparency level $q$. The belief on the return is assumed as follows:

$$\widetilde{r}(q) \sim N(\overline{r}(q), (\overline{q} + q)^{-\chi})$$
$$\text{s.t.} \quad \overline{r} = \frac{\pi(q)}{P(q)}$$

where $q \geq 0$ is a transparency of the balance sheet information; $\overline{q}$ is the mandated transparency required by SEC; $\pi(q)$ is the profit of the firm with transparency $q$; $P(q)$ is the price of the firm with transparency $q$. Due to opacity in the information, a household believes probabilistic return, where the variance decreases in transparency $q$. $\chi$ is a structural parameter that governs the variance of the listed firms' return. We interpret this parameter as the illegibility of the financial information. As the illegibility of financial information increases, the variance of return increases.[4]

[4] This is because $q + \overline{q} \leq 1$.

In the OTC market, the household forms the following belief on the non-listed firms:

$$\widetilde{r}^D \sim N(\overline{r}^D, \frac{1}{\xi})$$
$$\text{s.t.} \quad \overline{r}^D = \frac{\pi^D}{P^D}$$

where $\xi$ is a structural parameter that governs the variance of the non-listed firms' return. $\pi^D$ and $P^D$ are the profit and price of a non-listed firms. As non-listed firms do not disclose any information publicly, the household do not distinguish a non-listed from another.

## 3.2 Technology

An ex-ante homogeneous firm produces output using two inputs: tangible ($k_T$) and intangible capital ($k_I$). Tangible's share in the production is $\alpha$, and the intangible's share is $\theta$, where $\alpha + \theta < 1$. In this economy, there are two types of production technologies. One is listed firms' production technology and the other is non-listed firms' production technology.

### 3.2.1 Production function of listed firms

While tangible capital is a standard input in production, intangible capital is different. The intangible is shared with other firms without additional cost if it is disclosed. Due to this externality, a firm in our model is reluctant to reveal much of their own intangible capital because they can free ride on the readily available shared knowledge already in the market. However, the trade-off is that if they conceal their intangible capital, investors discount their value due to the opaque information about the firm. Therefore, if a firm decides to be listed, the firm should decide how much intangible capital to reveal ($q$) to the public. Also, there is a required level of information disclosure ($\overline{q}$) imposed by the SEC for the listed firms. The unit of $q$ is assumed to be the same as the intangible capital stock.

The production function of a listed firm is as follows:

$$z k_T^{\alpha} (k_I(1 - \overline{q} - q))^{\theta} (\Phi^{ex})^{\gamma}$$

where $z$ is the aggregate TFP; $\overline{q}$ is the exogenous minimum level of intangible disclosure imposed by SEC; $q$ is the disclosed amount of intangible (the transparency level); $\Phi^{ex}$ is the shared intangible capital from all other firms. $r$ is the capital rental rate, and $p$ is the $R\&D$ cost per unit of intangible capital. $\gamma$ is the scale parameter for the externality. If $\gamma$ is beyond a certain threshold, the equilibrium does not exist due to a divergent externality effect. In particular, only when $\alpha + \theta + \gamma \leq 1$, the equilibrium exits, which we will formally discuss after we define the equilibrium.

Ex-post profit $\pi(q)$ is obtained after taking out the operational costs $rk_T + pk_I$ from the revenue:

$$\pi(q) := \max_{k_T, k_I} \quad zk_T^{\alpha}(k_I(1 - \overline{q} - q))^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I$$

We assume a firm $i$'s disclosed intangible $q_i$ is perfectly substitutable by the other disclosed intangible. Therefore, the shared intangible are aggregated in the following way:[5]

$$\Phi^{ex} = \int_0^1 k_{I,i} \left( \underbrace{\overline{q}}_{\text{Disclosure mandated by SEC}} + \underbrace{q_i}_{\text{Voluntary disclosure}} \right) di$$

### 3.2.2 Production function of non-listed (private) firms

If a firm is private, they do not need to publicly disclose their intangible capital. The production function of a listed firm is as follows:

$$zk_T^{\alpha}(k_I)^{\theta}(\Phi^{ex})^{\gamma}$$

Except for the disclosure of the intangible capital, the production function is assumed to take the same form and the same parameters. The profit is also similarly defined as listed firms:

$$\pi^D := \max_{k_T, k_I} \quad zk_T^{\alpha}(k_I)^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I$$

## 3.3 Financial Markets

In this section, we characterize the financial market in the model. The capital supply is driven by the representative household's portfolio choice problem. The investment demand is determined by each firm's value maximization problem.

[5] We also consider a finite elasticity of substitution, $\lambda < \infty$, across the disclosed information of each firm as an extension of the baseline model. However, the main results of this paper stay unaffected over different choices of $\lambda$.

### 3.3.1 Capital supply: The household's mean-variance portfolio

A household solves the following maximization problem:

$$\max_{x(q),x^D} \quad \mathbb{E}(-e^{-\Lambda C})$$
$$\text{s.t. } C = \int x(\tilde{q})\tilde{r}(\tilde{q})d\tilde{q} + x^D\tilde{r}^D, \quad \int x(\tilde{q})d\tilde{q} + x^D = a$$

where $x(q)$ is the capital supply for firms with transparency level $q$ and $x^D$ is the capital supply for non-listed firms. Then, the investors' utility maximization problem is translated into the following form:

$$\max_{\int x(\tilde{q})d\tilde{q}+x^D=a} \quad -e^{-\Lambda\left(\int x(\tilde{q})\frac{\pi(\tilde{q})}{P(\tilde{q})}d\tilde{q}+x^D\frac{\pi^D}{P^D}-\frac{\Lambda}{2}\int x(\tilde{q})^2(\bar{q}+q)^{-\chi}d\tilde{q}-\frac{\Lambda}{2}(x^D)^2\frac{1}{\xi}\right)}$$

This is equivalent to

$$\max_{\int x(\tilde{q})d\tilde{q}+x^D=a} \quad \int x(\tilde{q})\frac{\pi(\tilde{q})}{P(\tilde{q})}d\tilde{q} + x^D\frac{\pi^D}{P^D} - \frac{\Lambda}{2}\int x(\tilde{q})^2(\bar{q}+q)^{-\chi}d\tilde{q} - \frac{\Lambda}{2}(x^D)^2\frac{1}{\xi}$$

The first-order condition with respect to $x(q)$ yields

$$\frac{\pi(q)}{P(q)} - \Lambda x^*(q)(\bar{q}+q)^{-\chi} - \mu = 0$$

where $\mu$ is the Lagrange multiplier of the wealth constraint. From this equation, we can derive the following supply curve of investment for the listed market:

$$x^*(q) = \frac{\pi(q)/P(q) - \mu}{\Lambda(\bar{q}+q)^{-\chi}}$$

where $x^*(q)$ is the capital supply in a dollar amount for firms with the transparency level $q$. So, the household is willing to invest $\frac{\pi(q)/P(q)-\mu}{\Lambda(\bar{q}+q)^{-\chi}}$ in the firms with transparency level $q$.

Similarly, from the first-order condition with respect to $x^D$, the capital supply curve for non-listed firms is characterized as follows:

$$x^{D*} = \frac{\pi^D/P^D - \mu}{\Lambda/\xi}$$

From this point on, we assume the representative household has a large enough wealth $a$, as our interest is not on the household's constrained optimization. Thus, $\mu = 0$.

### 3.3.2 Investment demand: Listed firms' value maximization

The price of a firm, $P(q)$, is determined at the level where capital supply in the number of firms, $\frac{x^*(q)}{P(q)}$ meets the investment demand in the number of firms $\mathcal{M}(q)$. Thus, the market-clearing condition is as follows:

$$\frac{x^*(q)}{P(q)} = \mathcal{M}(q)$$

In the model, a firm's value is identical to the price of the firm. Therefore, a manager of a firm chooses the transparency level to maximize the price of the firm:

$$\max_{q \geq 0} P(q)$$

Given the investment demand and the market-clearing condition, this problem is equivalent to the following form:

$$\max_{q \geq 0} \sqrt{\frac{\pi(q)}{\Lambda(\overline{q}+q)^{-\chi}\mathcal{M}(q)}}$$

which is equivalent to

$$\max_{q \geq 0} \frac{\pi(q)}{(\overline{q}+q)^{-\chi}\mathcal{M}(q)}$$

Now, we define a net funding rate $\phi^L(q)$ as follows:

$$\phi^L(q) := \frac{(\overline{q}+q)^{\chi}}{\mathcal{M}(q)}$$

Therefore, a listed firm's problem can be summarized as the following form:

$$J^L(\mathcal{M}) = \max_{q,k_T,k_I} \quad \left(zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I\right)\phi^L(q)$$
$$\text{s.t. } \phi^L(q) = (\overline{q}+q)^{\chi}/\mathcal{M}(q)$$

where $J^L$ is the value of a listed firm given the distribution of listed firm $\mathcal{M}$. In equilibrium, the value is equal to the price of each of the listed firms: $J^L(\mathcal{M}) = P(q(\mathcal{M}))$ for $\forall q \geq 0$. The solution to this problem characterizes the investment demand of each firm.

### 3.3.3 Financial market for non-listed firms

The price of a non-listed firm, $P^D$, is determined at the level where capital supply in the number of firms, $\frac{x^{D*}}{P^D}$ is matched with the demand in a frictional OTC market. Especially, we assume the congestion among non-listed firms generates the attrition in the funding opportunity in the following way:

$$\frac{x^{D*}}{P^D} = M_D^{\nu_D}, \quad \nu_D > 1$$

where, $M_D$ is the total number of non-listed firms. $\nu_D > 1$ is a structural parameter that captures the congestion effect in the OTC market. Thus, $M_D > M_D^{\nu_D}$.

Then, we define a net funding rate $\phi^D(q)$ as follows:

$$\phi^D := \xi / M_D^{\nu_D}.$$

A non-listed firms' problem can be written down as follows, simliar to the listed firms' problem:

$$\begin{aligned} J^D(\mathcal{M}_{\mathcal{D}}) = \max_{k_T, k_I} \quad & \left( z k_T^{\alpha} (k_I)^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^D \\ & \text{s.t. } \phi^D = \xi / M_D^{\nu_D} \end{aligned}$$

## 3.4 A firm's problem

A firms' manager should decide whether to go listed or non-listed before the operation. If a firm becomes non-listed, the manager does not have to worry about the leakage of their intangible through the disclosure. However, investors penalize the opacity of the non-listed firms by allowing only a low funding rate.

If a firm becomes public, the manager should decide the level of transparency $q \geq 0$. If there are too many firms to choose the same transparency level, it will decrease the value of the firm in the listed market.

A firm's problem could be summarized as follows:

$$
\begin{aligned}
&\text{[Entry decision]} \quad && V(\mathcal{M}, M_D) = \max\{J^L(\mathcal{M}), J^D(M_D)\} \\
&\text{[Listed firm's problem]} \quad && J^L(\mathcal{M}) = \max_{q,k_T,k_I} \left( z k_T^{\alpha} (k_I(1-\bar{q}-q))^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^L(q) \\
& && \text{s.t. } \phi^L(q) = (\bar{q}+q)^{\chi} / \mathcal{M}(q) \\
&\text{[Non-listed firm's problem]} \quad && J^D(M_D) = \max_{k_T,k_I} \left( z k_T^{\alpha} (k_I)^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^D \\
& && \text{s.t. } \phi^D = \xi / M_D^{\nu_D}
\end{aligned}
$$

# 4 Equilibrium

Here we define an equilibrium where the economy is given with total intangible capital reserve $K^I$ (fixed aggregate intangible supply). This endogenously determines the R&D cost of intangible capital $p$. Here, the R&D cost is not a price for a trade. Instead, it is a cost that increases if all the other firms invest in their own R&D. This is because developing new knowledge is harder if more firms are seeking new knowledge. The rental rate for the tangible capital $r$ is exogenously given.

**Definition 1.** *A collection of functions* $(k_T, k_I, q, \mathcal{M}, M_D, p, \Phi^{ex})$ *is an equilibrium if*

1. $(k_T(q, \mathcal{M}), k_I(q, \mathcal{M}), q(\mathcal{M}))$ *solves the listed firm's problem.*

2. *The measure of listed firms choosing a transparency level* $q$ *is consistent with* $M(q)$ *for all* $q \in [0, 1-\bar{q}]$.

3. *The measure of non-listed firms is* $M_D$ *and satisfies*

$$\int_0^{1-\bar{q}} \mathcal{M}(q) dq + M_D = 1$$

4. *R&D cost of intangible capital* $p$ *is determined by the following equation:*

$$K^I = \int_0^1 k_{I,i} di$$

5. *Aggregate shared knowledge satisfies*

$$\Phi^{ex} = \int_0^1 k_{I,i}(\bar{q} + q_i) di$$

6. *All the firm prices are identical:*

$$P(q) = P^D, \text{ for } \forall q \in [0, 1 - \overline{q}]$$

With the endogenously determined distribution $\mathcal{M}$ of firms for each $q$, we can re-write the market-clearing condition for intangible capital and the externality condition using $\mathcal{M}$. In the definition, each firm is aggregated over the index $i \in [0, 1]$. Instead, we aggregate firms over the distribution of firms at each $q$. This is doable since $\mathcal{M}$ is endogenously obtained, and $k_I$ is also a function of $q$ and $\mathcal{M}$. Therefore, we re-write those two conditions in the following way.

$$I = \int_0^{1-\overline{q}} k_I(q, M) M(q) dq$$

$$\Phi^{ex} = \int_0^{1-\overline{q}} k_I(q, M)(\overline{q} + q) M(q) dq :$$

Among all possible equilibria, we are interested in the non-degenerate equilibrium where all the homogeneous firms use mixed strategies over the transparency level $q$. The mixed strategy leads to the distribution of firms at each level of $q$. And in the equilibrium, this distribution needs to be consistent with the distribution that a firm takes as a given state variable.

In the following section, we analytically characterize the equilibrium allocations in this economy.

## 4.1 A listed firm's decision

First, we solve a listed firm's problem backward from the decision on the transparency level and the other operating allocations. Then, we solve the firm's decision on which financial market to go between the public and private market.

Given a net funding rate function, $\phi^L$ and the externality, $\Phi^{ex}$, a listed firms' firm's problem is characterized as follows:

$$\max_{q, k_T, k_I} \left( z k_T^{\alpha} (k_I(1 - \overline{q} - q))^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^L(q)$$

From the optimality conditions of the problem, we can derive the relationship among the transparency $q$, the regulation parameter $\overline{q}$ and the intangible capital $k_I$. The relationship is formally stated in the following proposition:

**Proposition 1.** *(Intangibles and the transparency)*
*Given $\alpha + \theta < 1$, $k^I(q, \overline{q})$ decreases both in $q$ and in $\overline{q}$.*

*Proof.*
See Appendix A.1. ■

Then, from the optimality condition with respect to the transparency, $q$, we can characterize an ordinary differential equation where the function of interest is the net funding rate function $\phi(q)$. The ODE is specified in Appendix A.2. By solving the ODE, we characterize the closed form of the transparency distribution $\mathcal{M}$. We formally state the closed form of $\mathcal{M}$ in the following proposition:

**Proposition 2.** *(Transparency distribution)*
*The probability density function $\mathcal{M}$ of transparency $q$ has the following closed form:*

$$\mathcal{M}(q) = (\overline{q} + q)^{\chi} (1 - \overline{q} - q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}} \frac{1}{\phi^D}.$$

*Proof.*
See Appendix A.2. ■

The probability density function $\mathcal{M}(q)$ belongs to a variant of a well-known class of density function: Beta distribution. In the following corollary, we prove that $\mathcal{M}(q)$ follows a shifted truncated beta distribution and provide the closed-form characterization of the net funding rate of the private firms, $\phi^D$. For the brevity of notation, I define $B := \frac{\theta(\alpha+\theta)}{1-\alpha-\theta} + 1$.

**Corollary 1.** *(Truncated normalized Beta distribution)*
*The gross transparency, $y := q + \overline{q}$, follows a truncated normalized Beta distribution where the shape parameters are $\chi + 1$ and $B + 1$, and the support is $[\overline{q}, 1]$.*

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1 - \overline{q}]\}}{1 - M_D} \times Beta\left(\chi + 1, B + 1\right),$$

*where $B = \frac{\theta(\alpha+\theta)}{1-\alpha-\theta}$.*

*Proof.*
See Appendix A.3. ■

Even if we obtain the the closed form of the transparency distribution $\mathcal{M}$, the distribution is not readily matched with the data counterpart, as the firm-level transparency distribution is not observed. Instead, the residualized variance of the stock return $(\overline{q} + q)^{-\chi}$ can be measured from the variance of earnings' surprise. To discipline the parameters based on the proper observable counterparts in the data, we analyze the distribution of stock return variance $\widehat{M}$ instead of $\mathcal{M}$. The closed form of $\widehat{M}(\widehat{q})$, where $\widehat{q} = (\overline{q} + q)^{-\chi}$ is stated in the following corollary. As the return variance decreases in the transparency, we name $\widehat{q}$ the opacity of the firms' information.

**Corollary 2.** *(Opacity distribution)*
*The probability density of return variances, $\widehat{M}(\widehat{q})$ has the following closed form:*

$$\widehat{M}(\widehat{q}) = \frac{1}{\chi\phi^D}\widehat{q}^{-\frac{1}{\chi}-2}\left(1 - \widehat{q}^{-\frac{1}{\chi}}\right)^B \text{ for } \forall\widehat{q} \in [1, \overline{q}^{-\chi}]$$

*where $B = \frac{\theta(\alpha+\theta)}{1-\alpha-\theta}$.*

*Proof.*
See Appendix A.4. ■

In the proof, we use a useful property that $\widehat{q}$ is a monotone transformation of $q$. This relationship also helps characterize the corresponding boundaries of the support. Specifically, The probability density $\widehat{M}(\widehat{q})$ takes a bounded support, $[1, \overline{q}^{-\chi}]$.

It is worth noting that the probability density of $q$ or $\widehat{q}$ depends on the net funding rate of non-listed firms, $\phi^D$. This net funding rate is determined at the following identity that requires total measure of firms is unity:

$$\frac{1}{\phi^D}\int_0^{1-\overline{q}}(\overline{q} + q)^{\chi}(1 - \overline{q} - q)^B dq = 1 - \left(\frac{\phi^D}{\xi}\right)^{-\frac{1}{\nu_D}}. \tag{1}$$

Equivalently, we can write down the identity in terms of the mass of non-listed firms as follows:

$$\frac{1}{\xi}M_D^{\nu_D}\int_0^{1-\overline{q}}(\overline{q} + q)^{\chi}(1 - \overline{q} - q)^B dq = 1 - M_D. \tag{2}$$

The equation (2) is the fundamental component of the model, which captures how the total measure of non-listed firms, $M_D$, behaves when the policy parameter $\overline{q}$ changes. Using Corollary 1, we can integrate out the $M(q)$ in the laft-hand side of the equation in the following steps, using $y = q + \overline{q}$:

$$\frac{1}{\xi}M_D^{\nu_D}\int_{\overline{q}}^{1}(y)^{\chi}(1 - y)^B dy = 1 - M_D.$$

Then, we devide the both sides by a beta function, $\mathcal{B}(\chi+1, B+1)$.[6]

$$\frac{1}{\xi} M_D^{\nu_D} \frac{1}{\mathcal{B}(\chi+1,B+1)} \int_{\overline{q}}^{1} (y)^{\chi}(1-y)^{B} dy = \frac{1}{\mathcal{B}(\chi+1,B+1)}(1-M_D).$$

We integrate the left-hand side using the cumulative distriubtion function of beta distribution, $F$:

$$\frac{1}{\xi} M_D^{\nu_D} \left(1 - F\left(\overline{q}; \chi+1, B+1\right)\right) = \frac{1}{\mathcal{B}(\chi+1,B+1)}(1-M_D).$$

By rearranging the terms, we obain the following equation:

$$\xi M_D^{-\nu_D}(1-M_D) = \mathcal{B}\left(\chi+1, B+1\right)\left(1 - F\left(\overline{q}; \chi+1, B+1\right)\right)$$

It is worth noting that the right-hand side stricly decreases in $\overline{q}$. This characterization theoretically predicts that $M_D$ decreases in $\overline{q}$.

**Proposition 3.** *(The relationship between disclosure regulation and the measure of listed firms) $M_D$ strictly increases in $\overline{q} \in [0,1]$.*

*Proof.*
See Appendix A.5 ■

Therefore, as SEC requires a stricter disclosure regulation on the financial information, the measure of non-listed firms increases. This is because a firm does not internalize the productivity gain from the shared information. As can be observed from the equation (2), the measure of non-listed firms is independent from the externality effect, $\Phi^{ex}$.

However, the total measure of listed or non-listed firms cannot solely serve as a objective of the information regulation. The objective is stated clearly in the following mission of SEC: "the mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation."[7] Therefore, the effect of regulation on he investors' welfare and the productivity needs to be investigated.

[6]The beta function is defined as follows:

$$\mathcal{B}(a,b) := \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} = \int_0^1 x^{a-1}(1-x)^{b-1} dx$$

[7]The mission is from https://www.sec.gov/our-goals.

## 4.2 The scoreboards: Welfare, productivity, and output

In this section, we define the three objectives of the information disclosure policy: welfare, productivity and output. First, we define the welfare measure. Besides the performance of the firms, the investor values the transparency of the disclosed information, as it is helpful for its portfolio. The representative investors utility can be monotonely transformed into the following mean-variance form:

$$\begin{aligned} Objective_{welfare} &= \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{p(\widetilde{q})}d\widetilde{q} + x^D\frac{\pi^D}{P^D} - \frac{\Lambda}{2}\int x(\widetilde{q})^2(\overline{q}+\widetilde{q})^{-\chi}d\widetilde{q} - \frac{\Lambda}{2}(x^D)^2\frac{1}{\xi} \\ &= \int M(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + M^D\pi^D - \frac{1}{2}\int\frac{\pi(\widetilde{q})}{M(\widetilde{q})}d\widetilde{q} - \frac{1}{2}\frac{\pi^D}{M_D}. \end{aligned} \tag{3}$$

The welfare measure increases in the expected profits and decreases in the ex-ante variance of the profits.

The second measure is the productivity in the production sector. The productivity measure is defined as follows:

$$Objective_{productivity} = \Phi^{ex} = \left(\int_0^{1-\overline{q}}(\widetilde{A}(\overline{q}+q)(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\mathcal{M}(q))^{\frac{\lambda-1}{\lambda}}dq\right)^{\frac{\lambda}{\lambda-1}\times\frac{1-\alpha-\theta}{1-\alpha-\theta-\gamma}}, \tag{4}$$

where $\widetilde{A} := \left(\left(\frac{\alpha z}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)$. The productivity is identical to the externality effect, which is the aggregated shared knowledge in the economy. $\lambda$ is the elasticity of substitution across the knowledge shared from each firm. In the baseline quantitative analysis, we assume $\lambda = \infty$ so the knowledge is perectly substitutable across the firms. As a robustness check, we investigate the macroeconomic implications under $\lambda < \infty$. In the regulator's perspective, there is a trade-off in the productivity measure for increasing the strictness of the disclosure requirement. For higher $\overline{q}$, the amount of shared information is greater, while the pool of listed firms to share the information shrink due to the firm-level extensive-margin responses.

The third measure is the aggregate output in the economy. The output measure is defined in the following form:

$$Objective_{output} = \int_0^{1-\overline{q}} k_T(q)^\alpha(k_I(q)(1-\overline{q}-q))^\theta(\Phi^{ex})^\gamma M(q) + k_{DT}^\alpha k_{DI}^\theta(\Phi^{ex})^\gamma M_D$$

As the regulation parameter $\overline{q}$ increases, producitivty varies, and total measure of listed firms changes. Therefore, the aggregate output can potentially feature a nonlinear curve over the variation in $\overline{q}$. In the quantitative analysis, we will quantitatively analyze the variation in these three measures.

# 5 Quantitative analysis

Based on the model we developed in the theory section, we quantitatively analyze the macroeconomic implications of the disclosure regulation parameter, $\overline{q}$, and how the role changes as the share of intangible capital rises. First we estimate our model based on two different periods. One is from 1992 to 1996, which is the baseline period, and the other is the post-reform period, from 2012 to 2016. As our model is static, we cannot analzye the dynamic response that might have happened right after the reform in 1997. Therefore, we analyze a period just before the reform as a baseline and compare this with a period several years after the reform to assume that it has reached a stationary level.

## 5.1 Estimation

In this section, we elaborate on how we fit the firm-level data into the model. The core parameters to be estimated are as follows:

$$\{\overline{q}, \theta, \chi, \xi, \nu_D\}$$

where $\overline{q}$ is the mandated transparency of disclosure; $\theta$ is the intangible share; $K^I$ is the total intangible capital stock; $\chi$ is the residualized return variance parameter of listed firms; $\xi$ is the residualized return variance parameter of non-listed firms, which we interpret as illegibility of the financial information; $\nu_D$ is the congestion parameter of non-listed firms.

The baseline estimates are from matching the average target moments between 1992 and 1996. The estimates of the post-reform periods are from matching the average target moments between 2012 and 2016. The target moments and the simulated moments are reported in Table 1. The identification strategy of the parameter $\overline{q}$ comes from the fraction of listed firms out of total firms. As studied in the model section, the higher required transparency increases incentive to stay in the OTC market. $\theta$ is from the intangible to tangible ratio. $\chi$ is identified from the funding rate variation by matching the ex-ante intangible-to-profit ratio.

In the model, the households form a belief on a stock return that follows a normal distribution:

$$\widetilde{r}(q) \sim N(\overline{r}(q), (\overline{q} + q)^{-\chi})$$

Analysts' forecast dispersion is a natural data counterpart to the dispersion in the ex-ante stock return, $q^{-\chi}$. Specifically, earnings surprise is defined as:

$$ES(q) := \overline{r}(q) - \widetilde{r}(q) \sim N(0, (\overline{q} + q)^{-\chi}).$$

Once $\chi$ is identified, a firm $i$'s transparency level can be recovered as:

$$\hat{q}_i = \text{var}(ES_i)^{-\frac{1}{\chi}}.$$

$\xi$ is determined by matching the top 1% opaque firms' average forecast variance normalized by the mean forecast variance. This identification strategy relies on an assumption that non-listed firms' opacity is at a similar level with the most opaque firms among the listed. $\nu_D$ is identified from the exit rate of firms, $1 - (M_D^\nu + (1 - M_D)) = M_D - M_D^\nu$.

We use the method of simulated moments to estimate the parameters. We are currently working on including more relelvant moments to closely fit the data in to the model.

Table 1: Fitted Moments

| Moments | Data | Model | Reference |
|---|---|---|---|
| **Baseline** (1992 $\sim$ 1996) | | | |
| Fraction of listed firms with 100+ emp. (%) | 8.02 | 8.02 | Compustat & Census BDS |
| Intangible/Tangible (%) | 27.31 | 27.31 | Compustat |
| Intangible/Profit (%) | 36.20 | 36.20 | Compustat |
| Top 1% normalized forecast variance | 55.56 | 55.56 | Compustat |
| Exit rate of firms with 100+ emp. (%) | 0.887 | 0.887 | Census BDS |
| **Post-reform periods** (2012 $\sim$ 2016) | | | |
| Fraction of listed firms with 100+ emp. (%) | 4.39 | 4.39 | Compustat & Census BDS |
| Intangible/Tangible (%) | 36.34 | 36.34 | Compustat |
| Intangible/Profit (%) | 44.37 | 44.37 | Compustat |
| Top 1% normalized forecast variance | 51.54 | 51.54 | Compustat |
| Exit rate of firms with 100+ emp. (%) | 0.548 | 0.548 | Census BDS |

Table 2 reports the estimated parameters. The estimated mandated transparency parameter, $\overline{q}$, is higher in the post-reform period, which implies the SEC regulation has become stricter, which is consistent with the SEC's intended direction of the reform. The share of intangible, $\theta$, is around 50% greater in the post-reform period. This is consistent with the empirical fact that the importance of intangible input has risen significantly. The exponent parameter for the inverse return variance of listed firms, $\chi$ has decreased, and the inverse return variance of non-listed firms, $\xi$ has increased over the same period. And the friction parameter $\nu_D$ has increased, which implies competition for funding in the OTC market has increased.

Table 2: Estimated parameters

| Parameters | Description | Value |
|---|---|---|
| **Baseline** (1992 $\sim$ 1996) | | |
| $\bar{q}$ | Mandated transparency | 0.751 |
| $\theta$ | Intangible share | 0.137 |
| $\chi$ | Inverse return variance (exponent) of listed firms | 9.478 |
| $\xi$ | Inverse return variance of non-listed firms | 0.513 |
| $\nu_D$ | Congestion parameter of OTC market | 1.116 |
| **Post-reform periods** (2012 $\sim$ 2016) | | |
| $\bar{q}$ | Mandated transparency | 0.866 |
| $\theta$ | Intangible share | 0.182 |
| $\chi$ | Inverse return variance (exponent) of listed firms | 11.728 |
| $\xi$ | Inverse return variance of non-listed firms | 0.406 |
| $\nu_D$ | Congestion parameter of OTC market | 1.128 |

Besides the estimated parameters, we fix the following parameters before the estimation.

$$\{\alpha, \gamma, K^I\}$$

Capital share, $\alpha$, is set to be 0.50. Because our model is abstract from a labor input, the capital share in the model needs to be interpreted as an after-labor-adjustment capital share, as in the following formulation:

$$\begin{aligned} Ak^{\alpha} &= \max_L \widetilde{A}k^{\widetilde{\alpha}}L^{\epsilon} - wL \\ &= (1-\epsilon)\widetilde{A}^{\frac{1}{1-\epsilon}} \left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}} k^{\frac{\widetilde{\alpha}}{1-\epsilon}} = Ak^{\frac{\widetilde{\alpha}}{1-\epsilon}} \end{aligned}$$

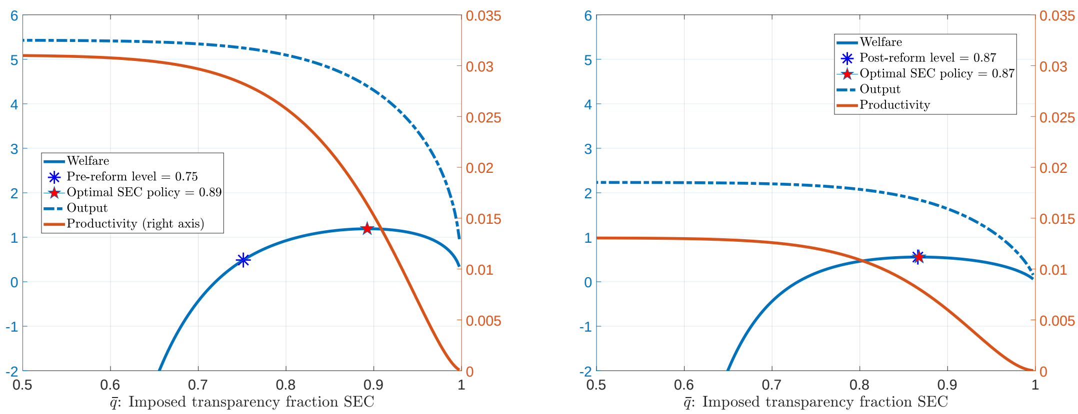
where $A = (1-\epsilon)\widetilde{A}^{\frac{1}{1-\epsilon}} \left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}}$. Therefore, our model's $\alpha$ is equivalent to a standard model's $\frac{\widetilde{\alpha}}{1-\epsilon}$. We assume $\widetilde{\alpha} = 0.2$, and $\epsilon = 0.6$, leading to $\alpha = 0.50$. Public intangible share, $\gamma$, is assumed to be equal as the private intangible share, $\theta$.

## 5.2 Optimal Policies

In this section, we use the proposed model to analyze the welfare optimizing level of imposed transparency. As shown in the previous section, the SEC can choose the imposed transparency level $\bar{q}$. However, since welfare is obtained from the utility maximization

Table 3: Fixed parameters

| Parameters | Description | Value |
|---|---|---|
| $\alpha$ | Capital share | 0.50 |
| $\gamma$ | Public intangible share | $= \theta$ |
| $r$ | Rental rate tangible capital | 0.02 |
| $K^I$ | Total intangible supply | 1 |
| $z$ | TFP level | 1 |




(a) Pre-reform estimation (b) Post-reform estimation

Figure 2: Optimal SEC transparency level

problem of the household, $\bar{q}$ will have two effects on welfare. On the one hand, lower imposed transparency increases the measure of listed firms which will have more access to finance relative to the private firms, increasing output and consumption. However, on the other hand, lower imposed transparency also increases the output's variance, lowering the welfare of the risk-averse household. Hence there is a trade-off between the level of consumption and its volatility. In Figure 2 we show the Laffer-type curve for the transparency policy for both periods.

The estimated level of transparency in the pre-reform period is 0.75 (Table 2) and the optimal level is 0.89. Suggesting the SEC transparency requirements were below the optimal level in the pre-reform period. In the post-reform period estimation, the results suggest that while the optimal level decreased slightly to 0.87, the estimated level increased to 0.87 (Table 2), shortening the gap between the optimal and estimated values. Moreover, although welfare at the optimal points decreased in the post-reform period, we can see that welfare at the estimated values slightly increased. Also, it is worth mentioning that output and productivity also show an U-inverse shape.[8] This property of the model suggests that depending on the value of the estimated parameters, moving $\bar{q}$ towards the welfare-optimal point could increase both output and productivity as well, achieving a *divine coincidence*. With the current estimated parameters, such *divine coincidence* happens when $\bar{q}$ is above the welfare optimal point: decreasing $\bar{q}$ towards the optimal would increase welfare, output and productivity.

# 6 Conclusion and future works

This paper studies how the U.S. Securities and Exchange Commission's (SEC) regulation on financial disclosure affects the number of listed firms, productivity, and welfare. Also, the focus of this paper includes how this effect changes when the importance of intangible capital increases. The empirical analysis shows that the number of listed firms and the average transparency (the inverse of the earnings forecast dispersion) of listed firms' financial reports has substantially decreased since the National Securities Markets Improvement Act of 1996. To theoretically and quantitatively analyze these secular trends, we develop a heterogenous-firm equilibrium model where endogenous exit and entry in the public equity market, the level of intangible capital stock, and the transparency of intangible capital are characterized in a closed-form. Using the model, we theoretically show that the number of listed firms decreases in the strictness of SEC's requirement on the listed firms' financial disclosure. From the estimated model, we quantitatively show that the strict SEC requirement decreases the average transparency of listed firms, consistent with the empirical observation. Then, we analyze the optimal regulation policy and on which side the current policy parameter is located with respect to the optimal level in terms of output, productivity, and welfare. According to the estimated model, the post-reform regulation level is almost at the optimum with respect to welfare, while the reform incurs a substantial loss in productivity and output.

Our approach broadens the scope of structural policy analysis to the regulation on information disclosure. In future work, we will quantitatively decompose the observed changes into the channel of the rising importance of intangible and the channel of the regulation change.

[8] Figure 2 only shows the region where output and productivity decrease monotonically with respect to $\bar{q}$.

# References


Atkeson, Andrew and Patrick J. Kehoe. 2005. "Modeling and Measuring Organization Capital." *Journal of Political Economy* 113 (5):1026–1053.

Bauguess, Scott, Rachita Gullapalli, and Vladimir Ivanov. 2015. "Capital Raising in the U.S.: An Analysis of the Market for Unregistered Securities Offerings, 2009-2014." *U.S. Securities and Exchange Commision* .

Corrado, Carol, Charles Hulten, and Daniel Sichel. 2009. "Intangible Capital and U.S. Economic Growth." *Review of Income and Wealth* 55 (3):661–685.

Dellavigna, Stefano and Joshua M. Pollet. 2009. "Investor Inattention and Friday Earnings Announcements." *The Journal of Finance* 64 (2):709–749.

Doidg, Craig, G. Andrew Karolyi, and Rene M. Stulz. 2017. "The U.S. listing gap." *Journal of Financial Economics* 123 (3):464–487.

Doidge, Craig, George Andrew Karolyi, and Rene M. Stulz. 2013. "Financial Globalization and the Rise of IPOs Outside the U.S." *Journal of Financial Economics* 110 (3):464–487.

Eisfeldt, Andrea L. and Dimitris Papanikolaou. 2014. "The Value and Ownership of Intangible Capital." *American Economic Review* 104 (5):189–94.

Ewens, Michael and Joan Farre-Mensa. 2020. "The Deregulation of the Private Equity Markets and the Decline in IPOs." *The Review of Financial Studies* 33 (12):5463–5509.

Ewens, Michael, Ryan Peters, and Sean Wang. 2020. "Measuring Intangible Capital with Market PricesMeasuring Intangible Capital with Market Prices." *NBER* 25960.

Gao, Xiaohui, Jay R. Ritter, and Zhongyan Zhu. 2013. "Where Have All the IPOs Gone?" *Journal of Financial and Quantitative Analysis* 48 (6):1663–1692.

McGrattan, Ellen R. 2020. "Intangible capital and measured productivity." *Review of Economic Dynamics* 37 (1):S147–S166.

McGrattan, Ellen R. and Edward C. Prescott. 2010. "Technology Capital and the US Current Account." *American Economic Review* 100 (4):1493–1522.

Peters, Ryan and Lucian A. Taylor. 2017. "Intangible capital and the investment-q relation." *Journal of Financial Economics* 123 (2):225–440.

# A Appendix

## A.1 Proof for Proposition 1

**Proposition 1.** *(Intangibles and the transparency)*
*Given $\alpha + \theta < 1$, $k^I(q, \overline{q})$ decreases both in $q$ and in $\overline{q}$.*

*Proof.*
From FOC

$$
\begin{aligned}
[k_T]: \quad & z\alpha k_T^{\alpha-1}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma} = r \\
[k_I]: \quad & z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}(1-\overline{q}-q) = p \\
[q]: \quad & -z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}\phi^L(q)k_I \\
& + \left(zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I\right)\phi'^L(q) = 0
\end{aligned}
$$

From the first-order conditions with respect to $k_T$ and $k_I$, we obtain

$$
\frac{r}{p} = \left(\frac{\alpha}{\theta}\right)\frac{k_I}{k_T}.
$$

Substituting this relation into the first-order condition with respect to $k_T$, we get

$$
r = \alpha z \left(\frac{\alpha p}{\theta r}\right)^{\alpha-1}(k_I)^{\alpha+\theta-1}(1-\overline{q}-q)^{\theta}(\Phi^{ex})^{\gamma}.
$$

Thus,

$$
k_I = \left(\left(\frac{\alpha z(\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}} = A(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}},
$$

where $A := \left(\left(\frac{\alpha z(\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)$. As $\alpha + \theta < 1$, the proposition is immediate from the last equation. ∎

## A.2 Proof for Proposition 2

**Proposition 2.** *(Transparency distribution)*
*The probability density function $\mathcal{M}$ of transparency $q$ has the following closed form:*

$$
\mathcal{M}(q) = (\overline{q}+q)^{\chi}(1-\overline{q}-q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\frac{1}{\phi^D}.
$$

*Proof.*
We derive the following equations using the first-order condition with respect to $q$:

$$\begin{aligned}\frac{\phi'^L(q)}{\phi^L(q)} &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}-rk_T-pk_I} \\ &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{(1-\alpha-\theta)zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}} \\ &= \frac{\theta}{1-\alpha-\theta}\left(\frac{1}{1-\overline{q}-q}\right)\end{aligned}$$

From $\frac{\partial}{\partial q}log(\phi^L(q)) = \frac{\phi'^L(q)}{\phi^L(q)}$, the solution of the first-order differential equation is as follows:

$$\phi^L(q) = (1-\overline{q}-q)^n\,\widetilde{C},$$

for some $n \in \mathbb{R}$ and some $\widetilde{C} \in \mathbb{R}$. From the indifference condition in the equilibrium, $\pi^L\phi^L(q)$ does not depend on $q$.

$$\pi^L\phi^L(q) = \left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha}A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^n\,\widetilde{C}$$

Therefore,

$$n = -\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}$$

This leads to $\phi^L(q) = (1-\overline{q}-q)^{-\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\,\widetilde{C}$.

Then, the distribution of listed firms is as follows:

$$\begin{aligned}\mathcal{M}(q) &= (\overline{q}+q)^{\chi}/\phi^L(q) \\ &= (\overline{q}+q)^{\chi}\,(1-\overline{q}-q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\,\frac{1}{\widetilde{C}}.\end{aligned}$$

From the indifference condition between listed and non-listed,

$$
\begin{aligned}
\phi^D &= \frac{\phi^L(q)}{\phi^D} \\
&= \frac{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^{-\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\widetilde{C}}{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}\Phi^{\gamma}\right)} \\
&= \widetilde{C}
\end{aligned}
$$

Therefore, $\mathcal{M}(q) = (\overline{q}+q)^{\chi}(1-\overline{q}-q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\frac{1}{\widetilde{\phi^D}}$

In the equilibrium, $\phi^D(=\widetilde{C})$ is determined at the level where the following equation holds:

$$\int_0^{1-\overline{q}} \mathcal{M}(q)dq = 1 - M_D.$$

■

## A.3 Proof for Corollary 1

**Corollary 1.** *(Truncated Beta distribution)*
*The gross transparency, $y := q + \overline{q}$, follows a truncated Beta distribution where the shape parameters are $\chi + 1$ and $B + 1$, and the support is $[\overline{q}, 1]$.*

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1-\overline{q}]\}}{1 - M_D} \times Beta\left(\chi+1, B+1\right),$$

*where $B = \frac{\theta(\alpha+\theta)}{1-\alpha-\theta}$.*

*Proof.*
We define $M_Y(y)$ as the probability density function of the random variable $y = q + \overline{q}$.

$$M_Y(y) \propto y^{\chi}(1-y)^B \quad \text{and} \quad y \in [\overline{q}, 1].$$

Also, $\int_{\overline{q}}^1 M_Y(y)dy = 1 - M_D$. Therefore, $y \sim \frac{\mathbb{I}\{q\in[0,1-\overline{q}]\}}{1-M_D} \times Beta\left(\chi+1, B+1\right)$. ■

## A.4 Proof for Corollary 2

**Corollary 2.** *(Opacity distribution)*

*The probability density of return variances,* $\widehat{M}(\widehat{q})$ *has the following closed form:*

$$\widehat{M}(\widehat{q}) = \frac{1}{\chi\phi^D}\widehat{q}^{-\frac{1}{\chi}-2}\left(1-\widehat{q}^{-\frac{1}{\chi}}\right)^B \textit{ for } \forall\widehat{q}\in[1,\overline{q}^{-\chi}]$$

*where* $B = \frac{\theta(\alpha+\theta)}{1-\alpha-\theta}$.

*Proof.*
From $\widehat{q} = (q+\overline{q})^{-\chi}$,

$$\frac{d\widehat{q}}{dq} = -\chi(q+\overline{q})^{-\chi-1}.$$

As $\widehat{q}$ is the strictly monotone trasformation of $q$, the following equation holds:

$$\begin{aligned}
\widehat{M}(\widehat{q}) &= \mathcal{M}(q)\left|\frac{dq}{d\widehat{q}}\right| \\
&= (\overline{q}+q)^{\chi}(1-\overline{q}-q)^{\frac{\theta(\alpha+\theta)}{1-\alpha-\theta}}\frac{1}{\phi^D}\left(\frac{1}{\chi}\right)(q+\overline{q})^{\chi+1} \\
&= \widehat{q}^{-1}\left(1-\widehat{q}^{-\frac{1}{\chi}}\right)^B\frac{1}{\phi^D}\left(\frac{1}{\chi}\right)\widehat{q}^{-\frac{1}{\chi}-1} \\
&= \frac{1}{\chi\phi^D}\widehat{q}^{-\frac{1}{\chi}-2}\left(1-\widehat{q}^{-\frac{1}{\chi}}\right)^B \text{ for } \forall\widehat{q}\in[1,\overline{q}^{-\chi}]
\end{aligned}$$

■

## A.5 Proof for Proposition 3

**Proposition 3.** *(The relationship between disclosure regulation and the measure of listed firms)* $M_D$ *strictly increases in* $\overline{q}\in(0,1)$.

*Proof.*
The following equation holds for all $\overline{q}\in(0,1)$:

$$\xi M_D^{-\nu_D}(1-M_D) = \mathcal{B}(\chi+1,B+1)\left(1-F(\overline{q};\chi+1,B+1)\right).$$

We take the derivative with respect to $\overline{q}$ on both sides:

$$\xi\left(-\nu_D M_D^{-(\nu_D+1)} - (1-\nu_D)M_D^{-\nu_D}\right)\frac{dM_D}{d\overline{q}} = -\mathcal{B}(\chi+1,B+1)f(\overline{q};\chi+1,B+1)$$

where $f$ is the probability density function corresponding to the cumulative distribution function, $F$. Then, by rearranging the terms in the left-hand side of the equation,

$$\underbrace{-\xi M_D^{-\nu_D}}_{<0}\underbrace{\Big(\nu_D(M_D^{-1}-1)+1\Big)}_{>0\ (\because 0<M_D<1)}\frac{dM_D}{d\bar{q}} = \underbrace{-\mathcal{B}(\chi+1,B+1)f\left(\bar{q};\chi+1,B+1\right)}_{<0}.$$

Therefore, we conclude

$$\frac{dM_D}{d\bar{q}} = \frac{\overbrace{-\mathcal{B}(\chi+1,B+1)f\left(\bar{q};\chi+1,B+1\right)}^{<0}}{\underbrace{-\xi M_D^{-\nu_D}\Big(\nu_D(M_D^{-1}-1)+1\Big)}_{<0}} > 0.$$

■